

# Three-Way Handshake Methodology to Share Information between Administrator and Multiple Users using Matrix Transformation

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**Abstract:** In this paper, 'three-way handshake methodology' is employed to authenticate sharing information between multiple different users and administrator. Also the methodologies to share the information using matrix transformation with symmetric, hybrid symmetric and asymmetric procedures are given to ensure high security.

**Key Words:** Information Security, Matrix Transformation, Symmetric and Asymmetric Keys, Key Sharing, Three-Way handshake, Administrator, Multiple Users.

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## 1. Introduction

To share information between two or more users, both public key and authentication are employed to maintain secrecy and privacy. In order to have more reliability to share information between two or more users, the complexity of decoding procedure of general cipher text for any intruder need to be nondeterministic. Factorization of any integer into prime factors is in general not identified even now whether it is polynomial or not, because there is no efficient polynomial time algorithm available to find factorization of randomly assigned integer. We use this fact and employ a scheme to use matrix transformation as public and private keys and also for authentication. In the following sections, we give our scheme and implementation of procedure in various situations.

### Section 1:

In this section, we give our procedure to share information between users and administrator using matrix transformation in two subsections.

Throughout, we use  $M_i$  to denote either  $m \times n$  or  $n \times n$  matrix,  $K_i$ ,  $G_i$  and  $C_i$  to denote  $n \times 1$  column vectors for  $i = 1, 2, \dots, n$ . Here, the matrices  $M_i$  and  $K_i$  have elements, which are large relatively primes/prime integers for  $i = 1, 2, \dots, n$ .

### 1.1. Generation of public keys between ADMIN and other users

In this subsection, we give the procedure to generate public keys to share the information between ADMIN and other users  $P_i$  for  $i = 1, 2, \dots, n$ . Let  $M_i$  and  $K_i$  be the primary and secondary keys used by the users  $P_i$  for  $i = 1, 2, \dots, n$  respectively.

Let  $M_i K_i = C_i$  be the cipher text sent by  $i^{\text{th}}$  user  $P_i$  to the administrator ADMIN. Let  $G_1, G_2, \dots, G_n$  be  $n$  ADMIN keys, which are used to authenticate the users to share information.

Let the cipher texts  $C_i = (c_1^i, c_2^i, \dots, c_n^i)^T$ , which is public key between  $P_i$  and ADMIN for  $i = 1, 2, \dots, n$ .

Let  $G_i = (g_1^i, g_2^i, \dots, g_n^i)^T$  for  $i = 1, 2, \dots, n$ .

Let  $G_i^* = (c_1^i g_1^i, c_2^i g_2^i, \dots, c_n^i g_n^i)^T$  for  $i = 1, 2, \dots, n$ .

Let  $C_i^* = (c_1^i \alpha^i, c_2^i \alpha^i, \dots, c_n^i \alpha^i)^T$  for  $i = 1, 2, \dots, n$ , where  $\alpha^i$  are some arbitrary constants and let  $\sum_j \alpha^j = \alpha$  for  $i = 1, 2, \dots, n$ . In fact, the  $\alpha^i$  are chosen in such a way that  $\sum_j \alpha^j = \alpha$ , which is not an eigen value of  $C_i^* (G_i^*)^T$ , which is a  $n \times n$  matrix, for  $i = 1, 2, \dots, n$ .

Let  $D_{i,1}^* = C_i^* (G_i^*)^T$  be the ADMIN authentication to the user  $P_i$  for  $i = 1, 2, \dots, n$ . It can be easily verified  $D_{i,1}^*$  is  $n \times n$  matrix and with the property that any  $l^{\text{th}}$  column of it has GCD  $(c_l^i g_l^i)$  and any  $m^{\text{th}}$  row of it has GCD  $(c_m^i g_m^i)$  for  $l, m$  and  $i = 1, 2, \dots, n$ .

Now each of the users  $P_i$  can get the details of  $G_i$  and  $\alpha$  and hence he can compute  $B_i^* = (D_{i,1}^* - \alpha I_n)$ , where  $I_n$  is unit matrix. But for any other intruders, getting  $G_i$  and  $\alpha$  is not easy as it involves factorization of all elements of  $C_i^*$  and  $G_i^*$ , which is not so easy in general and hence the authentication issued by ADMIN could be easily extracted by  $P_i$  well before than intruder gets information and also keys can be randomized.

### 1.2 Symmetric and hybrid symmetric schemes to share messages between ADMIN and other users

Let ADMIN next send the message  $A_i$  ( $m \times n$  matrix with  $m$  need not be  $n$ ) to  $P_i$  encoding it as  $D_{i,2}^*$  as follows for  $i = 1, 2, \dots, n$ :

Let  $A_i B_i^* = D_{i,2}^*$ , for  $i = 1, 2, \dots, n$  (The message sent by ADMIN to  $P_i$ ). Here,  $B_i^*$  is generated such that  $\alpha$  is not eigen value of  $D_{i,1}^*$  and hence  $B_i^*$  is non-singular, which is known to both  $P_i$  and ADMIN for  $i = 1, 2, \dots, n$ . With this,  $P_i$  gets  $A_i$  by computing  $D_{i,2}^* (B_i^*)^{-1}$ , for  $i = 1, 2, \dots, n$ . Thus, the messages  $A_i$  sent by ADMIN are received by  $P_i$  for  $i = 1, 2, \dots, n$  respectively. As the above procedure uses the public single key for encryption and decryption of messages shared between ADMIN and users by  $P_i$  for  $i = 1, 2, \dots, n$  the above procedure implemented is symmetric.

Next, we give a procedure to implement a hybrid symmetric procedure to pass information between ADMIN and other users.

Now let us assume the user  $P_i$  sends a request  $R_i = F_i(B_i^*) PK_i$ , where  $PK_i$  is a private key and  $F_i(B_i^*)$  is a polynomial function on  $B_i^*$  generated by  $P_i$ , whose degree will be at the most  $n-1$ .

In turn, let ADMIN send a secret message  $S_{ADMIN,i}$  ( $m \times n$  matrix with  $m$  need not be  $n$ ) encrypting as  $S_{ADMIN,i}^* = S_{ADMIN,i} (B_i^*)^{-1} R_i$ . Now the user  $P_i$  gets  $S_{ADMIN,i}$  by computing  $S_{ADMIN,i}^* ((B_i^*)^{-1} F_i(B_i^*) PK_i)^{-1}$ . Here, ADMIN has no idea of individual private keys  $PK_i$  and  $F_i(B_i^*)$  of  $P_i$ . But ADMIN has a clue of product of these keys.

Similarly, ADMIN can send a request  $R_{ADMIN,i} = F_{ADMIN,i}(B_i^*) AK_i$  to  $P_i$ , where  $AK_i$  is a private key and  $F_{ADMIN,i}(B_i^*)$  is a polynomial function on  $B_i^*$  generated by ADMIN, whose degree will be at the most  $n-1$ .

In turn, let  $P_i$  send a secret message  $S_{i,ADMIN}$  ( $m \times n$  matrix with  $m$  need not be  $n$ ) encrypting as  $S_{i,ADMIN}^* = (B_i^*)^{-1} R_{ADMIN,i}$ . Now the ADMIN gets  $S_{i,ADMIN}$  by computing the following expression,  $S_{i,ADMIN}^* ((B_i^*)^{-1} F_{ADMIN,i}(B_i^*) AK_i)^{-1}$ . Here, similarly  $P_i$  has no idea of individual private keys  $AK_i$  and  $F_{ADMIN,i}(B_i^*)$  of ADMIN, but has details of product of these keys.

Thus, the above procedure of sharing of information can be treated as hybrid symmetric scheme between ADMIN and the users  $P_i$ , for  $i= 1,2, \dots, n$ . The above procedure adds additional difficulties to any intruder to get information and hence the above sharing is better than the symmetric scheme.

## Section 2:

In this section, we give an asymmetric procedure to share the information between ADMIN and the users  $P_i$ . In this procedure first  $P_i$  sends a request  $R_{i,ADMIN}$  to ADMIN as a cipher text using public keys  $B_i^*$ ,  $PK_i^*$  (transformed  $n \times n$  matrix of a private key  $PK_i$  which is also a  $n \times n$  matrix) with some additional private key  $F_i(B_i^*)$  key. Then ADMIN using that cipher text sends a secret message  $S_{ADMIN,i}$ . After that  $P_i$  decrypts the message sent by ADMIN using his private keys  $PK_i$  and  $F_i(B_i^*)$ . As the private key of  $P_i$  is not known to ADMIN the following method of sharing is asymmetric.

Now, let us see the steps involved in sharing of information through which  $P_i$  gets messages from ADMIN.

**Step 1:** First  $P_i$  needs to share the transformed key  $PK_i^*$  using one of the procedures given in Section 1.2 using  $B_i^*$  with ADMIN.

**Step 2:** Next  $P_i$  sends a request  $(PK_i^*)^{-1} PK_i F_i(B_i^*) PK_i (PK_i^*)^{-1} = R_{i,ADMIN}^*$  (derived public key) to ADMIN, where  $PK_i$  is a private key and  $F_i(B_i^*)$  is a polynomial function on  $B_i^*$  generated by  $P_i$ .

**Step 3:** Let  $S_{ADMIN,i} ((PK_i^*) R_{i,ADMIN}^* (PK_i^*)) = S_{ADMIN,i}^*$  be the encrypted message of secret message  $S_{ADMIN,i}$  ( $m \times n$  matrix with  $m$  need not be  $n$ ) sent by ADMIN to  $P_i$ .

Clearly,  $S_{ADMIN,i}^* = S_{ADMIN,i} PK_i F_i(B_i^*) PK_i$ . As  $P_i$  has information about  $F_i(B_i^*)$  and  $PK_i$ ,  $P_i$  computes  $S_{ADMIN,i}^* (PK_i F_i(B_i^*) PK_i)^{-1}$  and obtains the secret message  $S_{ADMIN,i}$  sent by ADMIN. Thus,  $P_i$  decrypts the message sent by ADMIN using his private keys  $PK_i$  and  $F_i(B_i^*)$ .

Now we give the reverse procedure that ADMIN gets message with the similar steps from  $P_i$ .

Here, ADMIN uses a private keys  $PK_{ADMIN,i}$  and  $F_{ADMIN,i}(B_i^*)$  and public  $PK_{ADMIN,i}^*$  (transformed key of  $PK_{ADMIN,i}$  a  $n \times n$  matrix) gets messages from  $P_i$ .

**Step 1:** Here also, first ADMIN needs to share the transformed key  $PK_{ADMIN,i}^*$  using one of the procedures given in Section 1.2 (using  $B_i^*$ ) with  $P_i$ .

**Step 2:** Secondly, ADMIN makes a request  $(PK_{ADMIN,i}^*)^{-1} PK_{ADMIN,i} F_{ADMIN,i}(B_i^*) PK_{ADMIN,i} (PK_{ADMIN,i}^*)^{-1} = R_{ADMIN,i}^*$  (derived public key) to  $P_i$ , where  $PK_{ADMIN,i}$  is a private key and  $F_{ADMIN,i}(B_i^*)$  is a polynomial function on  $B_i^*$  generated by ADMIN.

**Step 3:** Let  $S_{i,ADMIN} = ((PK_{ADMIN,i}^*) R_{ADMIN,i}^* (PK_{ADMIN,i}^*)) = S_{i,ADMIN}^*$  be the encrypted message of secret message  $S_{i,ADMIN}$  ( $m \times n$  matrix with  $m$  need not be  $n$ ) sent by  $P_i$  to ADMIN.

Clearly,  $S_{i,ADMIN}^* = S_{i,ADMIN} PK_{ADMIN,i} F_{ADMIN,i}(B_i^*) PK_{ADMIN,i}$ . As ADMIN has information about  $F_{ADMIN,i}(B_i^*)$  and  $PK_{ADMIN,i}$ , ADMIN computes  $S_{i,ADMIN}^* (PK_{ADMIN,i} F_{ADMIN,i}(B_i^*) PK_{ADMIN,i})^{-1}$  and obtains the secret message  $S_{i,ADMIN}$  sent by  $P_i$ . Thus, ADMIN decrypts the message sent by  $P_i$  using his private keys  $PK_{ADMIN,i}$  and  $F_{ADMIN,i}(B_i^*)$ . Thus, this sharing of information between ADMIN and  $P_i$  for  $i = 1, 2, \dots, n$  is asymmetric.

### Section 3:

In this section, we give a procedure to share the information between ADMIN and Other multiple users by Sending sequence of messages one embedding on the previous messages as follows.

Let  $PR_i(i) = \prod_{j=1}^i A_j^i$ . (That is the product of the message matrices  $A_1^i, A_2^i, \dots, A_i^i$ , which are already sent by ADMIN to  $P_i$ ) for  $i = 1, 2, \dots, n$ .

If ADMIN wishes to send a new message  $A_{i+1}^i$  to  $P_i$ , then he can combine it with  $PR_i(i)$  and compute  $PR_{i+1}(i)$  and send it using any one of the procedures such as symmetric or hybrid symmetric or asymmetric discussed in section 1 and 2. Now, as  $P_i$  encrypts the message  $Pr_{i+1}(i)$  and obtains that new message  $A_{i+1}^i$  sent by ADMIN.

Thus, the messages between ADMIN and  $P_i$  are shared, for  $i = 1, 2, \dots, n$ .

### Remark:

1. If some of the Matrix messages  $M$  are singular, they can be converted to non singular using the fact that  $(M - \alpha_{ADMIN, P(i)} I_n)$  is non singular for some common value of  $\alpha_{ADMIN, P(i)}$  defined in terms of the components of keys  $G_i$  and  $C_i$ , which are public to

ADMIN and  $P_i$ . After decryption, they can be converted to original matrices as  $\alpha_{\text{ADMIN}, P(i)}$  is known to either side.

2.  $PR_1(i)$  can also be defined as  $\sum_{j=1}^l A_j^i$  and embedded messages can be sent and retrieved similarly.
3. The admin messages  $A_i$  can be  $m \times n$  matrix form for  $i = 1, 2, \dots, n$ .
4. All keys are used with elements relatively prime element to use general terminology of coding theory for high security.
5. The messages sent by  $P_i$  to ADMIN at any instant can be embedded with the messages sent in the all previous instants to have very high security of message sharing.
6. Information between users  $P_i$  and  $P_j$  are passed through ADMIN in general.
7. If  $P_i$  and  $P_j$  want to share information between them, then they can make request to get common authentication  $A_{i,j}$  to share information and share secret information  $S_{i,j}$  among them using the procedures given such as symmetric or hybrid symmetric or asymmetric discussed in section 1 and 2.

## Conclusion:

In this paper, sharing of information Between ADMIN and multiple users are implemented in symmetric, hybrid symmetric and asymmetric procedures. Besides, the embedding of messages and sharing messages are also given among to maintain very good security.

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