

Soft g^* -Closed Sets in Soft Minimal Spaces

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Abstract: In this paper, we define soft g^* -closed and soft g^* -open sets in soft minimal spaces which are defined over an initial universe with a fixed set of parameters. Further, we discuss basic properties of their sets.

Keywords: soft sets, soft minimal space, sg^*m -closed sets, sg^*m -open sets.

AMS Subject Classification: 54A10, 54C08

1. Introduction:

The concept of soft set theory as a new mathematical tool was initiated by Molodtsov [9]. M. Shabir and M. Naz [13] introduced soft topological spaces and the notions of soft closed sets, soft open sets, soft closure, soft interior and soft neighborhood of a point. Levine [8] initiated the study of generalized closed and open sets in topological spaces. Noiri [11] introduced the notion of generalized m -closed (briefly gm -closed) sets and tried to unify certain types of modifications of g -closed sets. Boonpok [1] introduced biminimal structure spaces and studied $m^1_x m^2_x$ -closed and $m^1_x m^2_x$ -open sets in biminimal structure spaces. Recently, in 2015, Gowri and Vembu [4] introduced and studied soft minimal and soft biminimal spaces.

In this present study, we discuss soft g^* -closed and soft g^* -open sets in soft minimal spaces and obtain the basic results and properties.

2. Preliminaries:

Definition 2.1 [9]: Let U be an initial universe and E is a set of parameters. Let $P(U)$ denotes the power set of U and A is a nonempty subset of E . A pair (F, A) is called a soft set over U , where F is a mapping given by $F: A \rightarrow P(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U . For $\mathcal{E} \in A$, $F(\mathcal{E})$ may be considered as the set of \mathcal{E} - approximate elements of the soft set (F, A) .

Definition 2.2 [4]: Let X be an initial universe set, E be the set of parameters and A be the soft subset of E ($A \subseteq E$). Let F_A be a nonempty soft set over X and $\tilde{p}(F_A)$ is the soft

power set of F_A . A subfamily \tilde{m} of \tilde{p} (F_A) is called a soft minimal over X if $F_\Phi \in \tilde{m}$ and $F_A \in \tilde{m}$.

(F_A, \tilde{m}) or (X, \tilde{m}, E) is called a soft minimal space over X . Each member of \tilde{m} is said to be \tilde{m} -soft open set and the complement of a \tilde{m} -soft open set is said to be \tilde{m} -soft closed set over X .

Example 2.3:

Let $U = \{u_1, u_2\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$ and

$F_A = \{(x_1\{u_1, u_2\}), (x_2\{u_1, u_2\})\}$. Then

$F_{A1} = \{(x_1\{u_1\})\}$, $F_{A2} = \{(x_1\{u_2\})\}$, $F_{A3} = \{(x_1\{u_1, u_2\})\}$,

$F_{A4} = \{(x_2\{u_1\})\}$, $F_{A5} = \{(x_2\{u_2\})\}$, $F_{A6} = \{(x_2\{u_1, u_2\})\}$,

$F_{A7} = \{(x_1\{u_1\}), (x_2\{u_1\})\}$, $F_{A8} = \{(x_1\{u_1\}), (x_2\{u_2\})\}$

$F_{A9} = \{(x_1\{u_1\}), (x_2\{u_1, u_2\})\}$, $F_{A10} = \{(x_1\{u_2\}), (x_2\{u_1\})\}$

$F_{A11} = \{(x_1\{u_2\}), (x_2\{u_2\})\}$, $F_{A12} = \{(x_1\{u_2\}), (x_2\{u_1, u_2\})\}$

$F_{A13} = \{(x_1\{u_1, u_2\}), (x_2\{u_1\})\}$, $F_{A14} = \{(x_1\{u_1, u_2\}), (x_2\{u_2\})\}$

$F_{A15} = F_A$, $F_{A16} = F_\Phi$ are all soft subset of F_A ,

Soft minimal $(\tilde{m}) = \{F_\Phi, F_{A1}, F_{A3}, F_{A7}, F_{A11}, F_{A13}, F_{A15}\}$.

Definition 2.4 [4]: Let (F_A, \tilde{m}) be a soft minimal space with nonempty soft set F_A is said to have property B if the union of any family of soft subsets belonging to \tilde{m} belongs to \tilde{m}

Definition 2.5 [4]: Let (F_A, \tilde{m}) be a soft minimal space over X . For a soft subset F_B of F_A the \tilde{m} -soft closure of F_B and \tilde{m} -soft interior of F_B are defined as follows:

$$(1) \tilde{m}Cl(F_B) = \bigcap \{F_\alpha : F_B \subseteq F_\alpha, F_A - F_\alpha \in \tilde{m}\},$$

$$(2) \tilde{m}Int(F_B) = \bigcup \{F_\beta : F_\beta \subseteq F_B, F_\beta \in \tilde{m}\}.$$

Lemma 2.6 [4]:

Let (F_A, \tilde{m}) be a soft minimal space over X . For a soft subset F_B and F_C of F_A , the following properties hold:

$$(1) \tilde{m}Cl(F_A - F_B) = F_A - \tilde{m}Int(F_B) \text{ and } \tilde{m}Int(F_A - F_B) = F_A - \tilde{m}Cl(F_B),$$

$$(2) \text{ If } (F_A - F_B) \in \tilde{m}, \text{ then } \tilde{m}Cl(F_B) = F_B \text{ and if } F_B \in \tilde{m}, \text{ then } \tilde{m}Int(F_B) = F_B,$$

$$(3) \tilde{m}Cl(F_\Phi) = F_\Phi, \tilde{m}Cl(F_A) = F_A, \tilde{m}Int(F_\Phi) = F_\Phi \text{ and } \tilde{m}Int(F_A) = F_A,$$

$$(4) \text{ If } F_B \subseteq F_C \text{ then } \tilde{m}Cl(F_B) \subseteq \tilde{m}Cl(F_C) \text{ and } \tilde{m}Int(F_B) \subseteq \tilde{m}Int(F_C),$$

$$(5) F_B \subseteq \tilde{m}Cl(F_B) \text{ and } \tilde{m}Int(F_B) \subseteq F_B,$$

$$(6) \tilde{m}Cl(\tilde{m}Cl(F_B)) = \tilde{m}Cl(F_B) \text{ and } \tilde{m}Int(\tilde{m}Int(F_B)) = \tilde{m}Int(F_B).$$

Lemma 2.7 [4]: Let F_A be a nonempty soft set with a soft minimal \tilde{m} on F_A satisfying property B. For a soft subset F_B of F_A the following properties hold:

- (1) $F_B \in \tilde{m}$ if and only if $\tilde{m}Int(F_B) = F_B$
- (2) If F_B is soft \tilde{m} -closed if and only if $\tilde{m}Cl(F_B) = F_B$
- (3) $\tilde{m}Int(F_B) \in \tilde{m}$ and $\tilde{m}Cl(F_B)$ is soft \tilde{m} -closed.

Definition 2.8 [5]: A soft subset F_B of a soft minimal space (F_A, \tilde{m}) is said to be soft generalized \tilde{m} -closed (briefly $sg\tilde{m}$ -closed) if $\tilde{m}Cl(F_B) \subseteq U_B$ whenever $F_B \subseteq U_B$ and U_B is soft \tilde{m} -open. The complement of soft generalized \tilde{m} -closed set is said to be soft generalized \tilde{m} open (briefly $sg\tilde{m}$ -open)

3. Soft g^* -closed sets in soft minimal space

Definition 3.1: A soft subset F_B of a soft minimal space (F_A, \tilde{m}) is said to be soft $g^*\tilde{m}$ -closed set (briefly $sg^*\tilde{m}$ -closed set) if $\tilde{m}Cl(F_B) \subseteq U_B$ whenever $F_B \subseteq U_B$ and U_B is $sg\tilde{m}$ -open.

Example 3.2: Let us consider the soft subsets of F_A that are given in Example 2.3. Let (F_A, \tilde{m}) be a soft minimal space where $U = \{u_1, u_2\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1\{u_1, u_2\}), (x_2\{u_1, u_2\})\}$. Then $\tilde{m} = \{F_\Phi, F_{A_6}, F_{A_8}, F_{A_{11}}, F_A\}$, $\tilde{m}^C = \{F_\Phi, F_{A_3}, F_{A_{10}}, F_{A_7}, F_A\}$,

$sg\tilde{m}$ - closed sets are $F_\Phi, F_{A_1}, F_{A_2}, F_{A_3}, F_{A_4}, F_{A_7}, F_{A_9}, F_{A_{10}}, F_{A_{12}}, F_{A_{13}}, F_{A_{14}}, F_A$ and

$sg^*\tilde{m}$ -closed sets are $F_\Phi, F_{A_1}, F_{A_2}, F_{A_3}, F_{A_4}, F_{A_7}, F_{A_{10}}, F_{A_{13}}, F_A$.

Theorem 3.3: Every soft \tilde{m} -closed set is $sg^*\tilde{m}$ -closed set.

Proof: Let F_B be soft \tilde{m} -closed set in (F_A, \tilde{m}) and U_B is $sg\tilde{m}$ -open such that $F_B \subseteq U_B$. Then $\tilde{m}Cl(F_B) = F_B$ and so $\tilde{m}Cl(F_B) \subseteq U_B$. Hence F_B be $sg^*\tilde{m}$ -closed set.

Theorem 3.4: Every $sg^*\tilde{m}$ -closed set is $sg\tilde{m}$ -closed set.

Proof: Let F_B be sg^* -closed set in (F_A, \tilde{m}) . Let $F_B \subseteq U_B$ where U_B is soft \tilde{m} -open. Since every soft \tilde{m} -open set is $sg\tilde{m}$ -open set. Since F_B is $sg^*\tilde{m}$ -closed set then we have $\tilde{m}Cl(F_B) \subseteq U_B$. Hence F_B is $sg\tilde{m}$ -closed.

Remark 3.5: From the above observation we get the following implication

$$\text{Soft } \tilde{m}\text{-closed set} \rightarrow \text{sg}^*\tilde{m}\text{-closed set} \rightarrow \text{sg}\tilde{m}\text{-closed set}$$

The reverse implications are not true from the following example

Example 3.6: Let us consider the soft subsets of F_A that are given in Example 2.3. Let (F_A, \tilde{m}) be a soft minimal space where $U = \{u_1, u_2\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1\{u_1, u_2\}), (x_2\{u_1, u_2\})\}$. Then $\tilde{m} = \{F_\Phi, F_{A_6}, F_{A_8}, F_{A_{11}}, F_A\}$. See Example 3.2. F_{A_2} is $sg^*\tilde{m}$ -closed but not soft \tilde{m} -closed and $F_{A_{12}}$ is $sg\tilde{m}$ -closed but not $sg^*\tilde{m}$ -closed.

Theorem 3.7: If F_B and G_B are $sg^*\tilde{m}$ -closed sets in (F_A, \tilde{m}) then their union $F_B \cup G_B$ is also $sg^*\tilde{m}$ -closed sets in (F_A, \tilde{m})

Proof: Suppose F_B and G_B are $sg^*\tilde{m}$ -closed sets in (F_A, \tilde{m}) . Let U_B $sg\tilde{m}$ -open of (F_A, \tilde{m}) such that $F_B \cup G_B \subseteq U_B$. Since $F_B \cup G_B \subseteq U_B$, we have $F_B \subseteq U_B$ and $G_B \subseteq U_B$. Since F_B and G_B are $sg^*\tilde{m}$ -closed, we have $\tilde{m}Cl(F_B) \subseteq U_B$ and $\tilde{m}Cl(G_B) \subseteq U_B$. Therefore $\tilde{m}Cl(F_B \cup G_B) \subseteq \tilde{m}Cl(F_B) \cup \tilde{m}Cl(G_B) \subseteq U_B$. Hence $F_B \cup G_B$ is also $sg^*\tilde{m}$ -closed set.

Remark 3.8: The intersection of two $sg^*\tilde{m}$ -closed sets are need not be $sg^*\tilde{m}$ -closed as seen from the following example.

Example 3.9: Let us consider the soft subsets of F_A that are given in Example 2.3. Let (F_A, \tilde{m}) be a soft minimal space where $U = \{u_1, u_2\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1\{u_1, u_2\}), (x_2\{u_1, u_2\})\}$. Then $\tilde{m} = \{F_\Phi, F_{A_1}, F_{A_2}, F_{A_3}, F_{A_7}, F_{A_6}, F_{A_{12}}, F_A\}$. Then $F_B = F_{A_6}$ and $G_B = F_{A_8}$ are $sg^*\tilde{m}$ -closed but $F_B \cap G_B = F_{A_5}$ is not $sg^*\tilde{m}$ -closed set.

Theorem 3.10: For each element $(x, u) \in (F_A, \tilde{m})$ then singleton $\{(x, u)\}$ is sgm -closed or $\{(x, u)\}^c$ is $sg^*\tilde{m}$ -closed set.

Proof: Let $(x, u) \in (F_A, \tilde{m})$ and the singleton $\{(x, u)\}$ is not $sg\tilde{m}$ -closed. Then $\{(x, u)\}^c$ is not $sg\tilde{m}$ -open set and F_A is the only $sg\tilde{m}$ -open set which contains $\{(x, u)\}^c$ and $\{(x, u)\}^c$ is $sg^*\tilde{m}$ -closed set.

Theorem 3.11: Let (F_A, \tilde{m}) be soft minimal space. If F_B is $sg^*\tilde{m}$ -closed, then $\tilde{m}Cl(F_B) - F_B$ contains no nonempty $sg\tilde{m}$ -closed.

Proof: Let F_B is $sg^*\tilde{m}$ -closed set. Let H_B is a $sg\tilde{m}$ -closed subset of $\tilde{m}Cl(F_B) - F_B$. That is $H_B \subseteq \tilde{m}Cl(F_B) - F_B$. Then $F_B \subseteq H_B^c$, H_B^c is $sg\tilde{m}$ open and hence, $\tilde{m}Cl(F_B) \subseteq H_B^c$. Therefore, we have $H_B \subseteq [\tilde{m}Cl(F_B)]^c$ and hence $H_B \subseteq \tilde{m}Cl(F_B) \cap [\tilde{m}Cl(F_B)]^c = F_\Phi$. Hence $H_B = F_\Phi$. Therefore $\tilde{m}Cl(F_B) - F_B$ contains no nonempty $sg\tilde{m}$ -closed.

Corollary 3.12: A soft minimal \tilde{m} on a nonempty soft set F_A satisfying property B. If F_B is $sg^*\tilde{m}$ -closed in (F_A, \tilde{m}) , then F_B is soft \tilde{m} - closed if and only if $\tilde{m}Cl(F_B) - F_B$ is $sg\tilde{m}$ -closed.

Proof: If F_B is soft \tilde{m} -closed, then $\tilde{m}Cl(F_B) = F_B$. That is $\tilde{m}Cl(F_B) - F_B = F\Phi$ and hence $\tilde{m}Cl(F_B) - F_B$ is $sg\tilde{m}$ -closed.

Conversely, if $\tilde{m}Cl(F_B) - F_B$ is $sg\tilde{m}$ -closed, then by Theorem 3.11, $\tilde{m}Cl(F_B) - F_B = F\Phi$. Since F_B is $sg^*\tilde{m}$ -closed. Therefore, F_B is soft \tilde{m} - closed.

Theorem 3.13: If F_B is $sg^*\tilde{m}$ -closed in (F_A, \tilde{m}) and $F_B \tilde{\subseteq} G_B \tilde{\subseteq} \tilde{m}Cl(F_B)$, then G_B is $sg^*\tilde{m}$ -closed.

Proof: Suppose that F_B is $sg^*\tilde{m}$ -closed set in (F_A, \tilde{m}) and $F_B \tilde{\subseteq} G_B \tilde{\subseteq} \tilde{m}Cl(F_B)$. Let $G_B \tilde{\subseteq} U_B$ and U_B is $sg\tilde{m}$ -open in (F_A, \tilde{m}) . Since $F_B \tilde{\subseteq} G_B$ and $G_B \tilde{\subseteq} U_B$, we have $F_B \tilde{\subseteq} U_B$. Since F_B is $sg^*\tilde{m}$ -closed, we have $\tilde{m}Cl(F_B) \tilde{\subseteq} U_B$. Since $G_B \tilde{\subseteq} \tilde{m}Cl(F_B)$, we have $\tilde{m}Cl(G_B) \tilde{\subseteq} \tilde{m}Cl(F_B) \tilde{\subseteq} U_B$. Therefore G_B is $sg^*\tilde{m}$ -closed set.

Definition 3.14: A soft subset F_B of a soft minimal space (F_A, \tilde{m}) is said to be soft $g^*\tilde{m}$ -open (briefly $sg^*\tilde{m}$ -open) if its complement is $sg^*\tilde{m}$ -closed.

Theorem 3.15: A soft subset F_B of a soft minimal space (F_A, \tilde{m}) is $sg^*\tilde{m}$ -open if and only if $H_B \tilde{\subseteq} \tilde{m}Int(F_B)$ whenever $H_B \tilde{\subseteq} F_B$ and H_B is $sg\tilde{m}$ -closed.

Proof: Suppose that F_B is $sg^*\tilde{m}$ -open. Let H_B be a $sg\tilde{m}$ -closed set such that $H_B \tilde{\subseteq} F_B$. Then $F_B^c \tilde{\subseteq} H_B^c$ and H_B^c is $sg\tilde{m}$ -open, we have F_B^c is $sg^*\tilde{m}$ -closed. Hence, $\tilde{m}Cl(F_B)^c \tilde{\subseteq} H_B^c$. Consequently, $[\tilde{m}Int(F_B)]^c \tilde{\subseteq} H_B^c$. Therefore, $H_B \tilde{\subseteq} \tilde{m}Int(F_B)$.

Conversely, assume that $H_B \tilde{\subseteq} \tilde{m}Int(F_B)$ whenever $H_B \tilde{\subseteq} F_B$ and H_B is $sg\tilde{m}$ -closed. Let $F_B^c \tilde{\subseteq} U_B$ and U_B is $sg\tilde{m}$ -open. Then $U_B^c \tilde{\subseteq} F_B$ and U_B^c is $sg\tilde{m}$ -closed. By our assumption, we have $U_B^c \tilde{\subseteq} \tilde{m}Int(F_B)$. Hence, $[\tilde{m}Int(F_B)]^c \tilde{\subseteq} U_B$. Therefore, $\tilde{m}Cl(F_B)^c \tilde{\subseteq} U_B$. Consequently, F_B^c is $sg^*\tilde{m}$ -closed. Hence, F_B is $sg^*\tilde{m}$ -open.

Theorem 3.16: If F_B and G_B are $sg^*\tilde{m}$ -open sets then $F_B \cap G_B$ is also $sg^*\tilde{m}$ -open set.

Proof: Suppose that F_B and G_B are $sg^*\tilde{m}$ -open sets. Let H_B is $sg\tilde{m}$ -closed set and $H_B \tilde{\subseteq} F_B \cap G_B$. Since $H_B \tilde{\subseteq} F_B \cap G_B$, we have $H_B \tilde{\subseteq} F_B$ and $H_B \tilde{\subseteq} G_B$. Since F_B and G_B are $sg^*\tilde{m}$ -open sets, we have $H_B \tilde{\subseteq} \tilde{m}Int(F_B)$ and $H_B \tilde{\subseteq} \tilde{m}Int(G_B)$. Therefore $H_B \tilde{\subseteq} \tilde{m}Int(F_B) \cap \tilde{m}Int(G_B) \tilde{\subseteq} \tilde{m}Int(F_B \cap G_B)$. Hence $F_B \cap G_B$ is $sg^*\tilde{m}$ -open set.

Remark 3.17: The union of two $sg^*\tilde{m}$ -open sets need not be $sg^*\tilde{m}$ -open set as can be seen from the following example.

Example 3.18: Let us consider the soft subsets of F_A that are given in Example 2.3. Let (F_A, \tilde{m}) be a soft minimal space where $U = \{u_1, u_2\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \tilde{\subseteq} E$ and $F_A = \{(x_1\{u_1, u_2\}), (x_2\{u_1, u_2\})\}$. Then $\tilde{m} = \{F_\Phi, F_{A_2}, F_{A_3}, F_{A_8}, F_{A_{12}}, F_A\}$. Then $\text{sg}\tilde{m}$ -closed sets are $F_\Phi, F_{A_1}, F_{A_4}, F_{A_6}, F_{A_7}, F_{A_7}, F_{A_9}, F_{A_{10}}, F_{A_{13}}, F_{A_{14}}, F_A$ and $\text{sg}^*\tilde{m}$ -closed sets are $F_\Phi, F_{A_1}, F_{A_6}, F_{A_7}, F_{A_9}, F_{A_{10}}, F_{A_{13}}, F_A$. $\text{sg}^*\tilde{m}$ -open sets are $F_\Phi, F_{A_{12}}, F_{A_3}, F_{A_{11}}, F_{A_2}, F_{A_8}, F_{A_5}, F_A$. Here, F_{A_2} and F_{A_8} are $\text{sg}^*\tilde{m}$ -open sets but $F_{A_2} \cup F_{A_8} = F_{A_{14}}$ is not $\text{sg}^*\tilde{m}$ -open.

Theorem 3.19: Let F_B and G_B be soft subsets of a soft minimal space (F_A, \tilde{m}) such that $\tilde{m}\text{Int}(F_B) \tilde{\subseteq} G_B \tilde{\subseteq} F_B$. If F_B is $\text{sg}^*\tilde{m}$ -open, then G_B is $\text{sg}^*\tilde{m}$ -open in (F_A, \tilde{m}) .

Proof: Suppose that F_B and G_B be soft subsets of (F_A, \tilde{m}) . Let F_B is $\text{sg}^*\tilde{m}$ -open set in (F_A, \tilde{m}) such that $\tilde{m}\text{Int}(F_B) \tilde{\subseteq} G_B \tilde{\subseteq} F_B$. Let H_B be a $\text{sg}\tilde{m}$ -closed such that $H_B \tilde{\subseteq} G_B$. Since $H_B \tilde{\subseteq} G_B$, and $G_B \tilde{\subseteq} F_B$, we have $H_B \tilde{\subseteq} F_B$. Since F_B is $\text{sg}^*\tilde{m}$ -open set, we have $H_B \tilde{\subseteq} \tilde{m}\text{Int}(F_B)$. Since, $\tilde{m}\text{Int}(F_B) \tilde{\subseteq} G_B$, we have $\tilde{m}\text{Int}(\tilde{m}\text{Int}(F_B)) \tilde{\subseteq} \tilde{m}\text{Int}(G_B)$. Therefore, $\tilde{m}\text{Int}(F_B) \tilde{\subseteq} \tilde{m}\text{Int}(G_B)$. Consequently, $H_B \tilde{\subseteq} \tilde{m}\text{Int}(F_B)$. Hence G_B is $\text{sg}^*\tilde{m}$ -open.

Theorem 3.20: If a soft subset F_B of a soft minimal space (F_A, \tilde{m}) is $\text{sg}^*\tilde{m}$ -closed, then $\tilde{m}\text{Cl}(F_B) - F_B$ is $\text{sg}^*\tilde{m}$ -open.

Proof: Suppose that F_B is $\text{sg}^*\tilde{m}$ -closed in (F_A, \tilde{m}) . Let $H_B \tilde{\subseteq} \tilde{m}\text{Cl}(F_B) - F_B$ and H_B is $\text{sg}\tilde{m}$ -closed. Since F_B is $\text{sg}^*\tilde{m}$ -closed, we have $\tilde{m}\text{Cl}(F_B) - F_B$ does not contain nonempty $\text{sg}\tilde{m}$ -closed by Theorem 3.11. Consequently, $H_B = F_\Phi$. Therefore $F_\Phi \tilde{\subseteq} \tilde{m}\text{Cl}(F_B) - F_B$, $F_\Phi \tilde{\subseteq} \tilde{m}\text{Int}(\tilde{m}\text{Cl}(F_B) - F_B)$, we obtain $H_B \tilde{\subseteq} \tilde{m}\text{Int}(\tilde{m}\text{Cl}(F_B) - F_B)$. Hence $\tilde{m}\text{Cl}(F_B) - F_B$ is $\text{sg}^*\tilde{m}$ -open.

Theorem 3.21: If a soft subset F_B is $\text{sg}^*\tilde{m}$ -open in a soft minimal space (F_A, \tilde{m}) , then $G_B = F_A$ whenever G_B is $\text{sg}\tilde{m}$ -open and $\tilde{m}\text{Int}(F_B) \cup F_B \tilde{\subseteq} G_B$.

Proof: Suppose that F_B is $\text{sg}^*\tilde{m}$ -open in a soft minimal space (F_A, \tilde{m}) and G_B is $\text{sg}\tilde{m}$ -open and $\tilde{m}\text{Int}(F_B) \cup F_B \tilde{\subseteq} G_B$ implies $G_B^c \tilde{\subseteq} \tilde{m}\text{Cl}(F_B^c) - F_B^c$. Since F_B^c is $\text{sg}^*\tilde{m}$ -closed and G_B^c is $\text{sg}\tilde{m}$ -closed. Therefore, $\tilde{m}\text{Cl}(F_B^c) - F_B^c$ contains no nonempty $\text{sg}\tilde{m}$ -closed set (By Theorem 3.11). Consequently, $G_B^c = F_\Phi$ and hence $G_B = F_A$.

Remark 3.22: The converse of the above theorem 3.23 is not true in general as can be seen from the following example.

Example 3.23: In Example 3.20, if we take $F_B = F_{A_9}$, then $\tilde{m}\text{Int}(F_B) \cup F_B \tilde{\subseteq} F_A$, F_A is $\text{sg}\tilde{m}$ -open, but $F_B = F_{A_9}$ is not $\text{sg}^*\tilde{m}$ -open.

4. Conclusion:

In the present work, we have introduced soft g^* -closed and open sets in soft minimal spaces which are defined over an initial universe with a fixed set of parameters. We have explored some basic properties of these concepts. Also, we have established several interesting results and presented its fundamental properties with the help of some examples. In future, these findings may be extended to new types of soft generalized closed and open sets in soft minimal spaces.

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