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Split Edge Domination in Boolean Function Graph B(G, L(G), NINC) of a Graph

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Abstract: *For any graph G, let V(G) and E(G) denote the vertex set and edge set of G respectively. The Boolean function graph B(G, L(G), NINC) of G is a graph with vertex set V(G) ∪ E(G) and two vertices in B(G, L(G), NINC) are adjacent if and only if they correspond to two adjacent vertices of G, two adjacent edges of G or to a vertex and an edge not incident to it in G. For brevity, this graph is denoted by $B_1(G)$. In this paper, Split edge domination numbers of Boolean Function Graphs of some standard graphs are obtained.*

Keywords: *Boolean Function graph, Edge Domination Number.*

1. Introduction

Graphs discussed in this paper are undirected and simple graphs. For a graph G, let V(G) and E(G) denote its vertex set and edge set respectively. A subset D of V is called a dominating set of G if every vertex not in D is adjacent to some vertex in S. The domination number $\gamma(G)$ of G is the minimum cardinality taken over all dominating sets of G. The open neighborhood N(v) of v in V is the set of vertices adjacent to v, and the set $N[v] = N(v) \cup \{v\}$ is the closed neighborhood of v. An edge e of a graph is said to be incident with the vertex v if v is an end vertex of e. In this case, we also say that v is incident with e.

A subset $F \subseteq E$ is called an edge dominating set of G if every edge not in F is adjacent to some edge in F. The edge domination number $\gamma'(G)$ of G is the minimum cardinality taken over all edge dominating sets of G. An edge dominating set $F \subseteq E$ of a connected graph G is a split edge dominating set, if the induced subgraph $< E(G) - F >$ is disconnected. The split edge domination number $\gamma s'(G)$ of G is the minimum cardinality of a split edge dominating set. The maximum order of a partition of E into edge dominating sets of G is called the edge domatic number of G and is denoted by $d'(G)$. The concept of edge domination was introduced by Mitchell and Hedetniemi [8]. Jayaram [6] studied line (edge) dominating sets and obtained bounds for the line (edge) domination number and obtained Nordhaus-Gaddum results for the line domination number. Arumugam and Velammal [1] have discussed edge domination number and edge domatic number. The complementary edge domination in graphs is studied by Kulli and Soner [7].

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For graph theoretic notations and terminology, we follow Harary [2]. Janakiraman et al., introduced the concept of Boolean function graphs [3 - 5]. For a real x, $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.

Theorem 1.1: [6]. For any (p, q) graph G, $\gamma' \leq \lfloor p/2 \rfloor$

Theorem 1.2: [6]. For any (p, q) graph G, $\gamma' \leq q - \beta_1 + q_0$, where β_1 is the edge independence number and q_0 is the number of isolated edges in G.

Theorem 1.3: [6].For any (p, q) graph G, $\gamma' \leq q - \Delta'$, where Δ' denotes the maximum degree of an edge in G.

Observation. [3].

1.4. G and L(G) are induced subgraphs of $B_1(G)$.

1.5. Number of vertices in $B_1(G)$ is p + q and if $d_i = deg_G(v_i)$, $v_i \in V(G)$, then the number of edges in $B_1(G)$ is $q(p - 2) + ½\sum_{1 \leq i \leq p} d_i^2$.

1.6. The degree of a vertex of G in $B_1(G)$ is q and the degree of a vertex of L(G) e' in $B_1(G)$ is $deg_{L(G)}(e') + p - 2$. Also if $d^*(e')$ is the degree of a vertex e' of L(G) in $B_1(G)$, then $0 \leq d^*(e') \leq p + q - 3$. The lower bound is attained, if $G \cong K_2$ and the upper bound is attained, if $G \cong K_{1,n}$, for $n \geq 2$.

Theorem 1.7: [3] $B_1(G)$ is disconnected if and only if G is one of the following graphs: nK_1, K_2, $2K_2$ and $K_2 \cup nK_1$, for $n \geq 1$.

In this paper, Split edge domination number of Boolean Function Graph B(G, L(G), NINC) of some standard graphs and its bounds are obtained.

2. Main Results

In the following edge domination number of $B_1(P_n)$, $B_1(C_n)$, $B_1(K_{1,n})$ are found.

Theorem 2.1:

For the path P_n on n ($n \geq 6$) vertices, $\gamma_s'(B_1(P_n)) \leq 2(2n - 7)$

Proof: Let $v_1, v_2, \ldots, v_n$ be the vertices and $e_{12}, e_{23}, \ldots, e_{n-1,n}$ be the edges of P_n, where $e_{i,i+1} = (v_i, v_{i+1})$ i = 1, 2, …, n-1. Then $v_1, v_2, \ldots, v_n$ $e_{12}, e_{23}, \ldots, e_{n-1,n} \in V(B_1(P_n))$.

$B_1(P_n)$ has 2n - 1vertices and $n^2 - n - 1$ edges.

Let $F_1 = \{(v_1, e_{n-1,n}), (v_1, e_{23}), (v_3, e_{12}), (v_{n-2}, v_{n-1}), (e_{23}, e_{34})\}$

$F_2 = \bigcup_{i=4}^{n-2}\{(v_i, e_{12}) (v_i, e_{23})\}$,

$F_3 = \bigcup_{i=3}^{n-3}\{(v_{n-1}, e_{i,i+1})\}$ and $F_4 = \bigcup_{i=3}^{n-2}\{(v_n, e_{i,i+1})\}$

and let $D = F_1 \cup F_2 \cup F_3 \cup F_4 \subseteq E(B_1(P_n))$. $|D| = 5 + 2(n - 5) + n - 5 + n - 4 = 2(2n - 7)$. D is an edge dominating set of $B_1(G)$ and $<V((B_1(P_n)) - D>$ is disconnected with two components, one of the components being K_4 induced by the edges (v_{n-1}, e_{12}), (v_n, e_{12}), (v_{n-1}, e_{23}), (v_n, e_{23}). Therefore D is a split edge dominating set of $B_1(P_n)$ and hence $\gamma_s'(B_1(P_n)) \leq |D| = 2(2n - 7)$.

Remark 2.1: $\gamma_s'(B_1(P_3)) = 2$

Let v_1, v_2, v_3, be the vertices and e_{12}, e_{23}, be the edges of P_3. Then $v_1, v_2, v_3, e_{12}, e_{23} \in V(B_1(P_3))$. $D = \{(v_1, e_{23}), (v_3, e_{12})\}$ is a minimum split edge dominating set of $B_1(P_3)$. Therefore, $\gamma_s'(B_1(P_3)) = 4$.

Remark 2.2: $\gamma_s'(B_1(P_4)) = 4$

Let v_1, v_2, v_3, v_4 be the vertices and e_{12}, e_{23}, e_{34} be the edges of P_4. Then $v_1, v_2, v_3, v_4, e_{12}, e_{23}, e_{34} \in V(B_1(P_4))$. $D = \{(v_2, v_3), (v_4, e_{23}), (v_1, e_{34}), (e_{12}, e_{23})\}$is a minimum split edge dominating set of $B_1(P_4)$. Therefore, $\gamma_s'(B_1(P_4)) = 4$.

Remark 2.3: $\gamma_s'(B_1(P_5)) = 6$

Let v_1, v_2, v_3, v_4, v_5 be the vertices and where $e_{i,i+1} = (v_i, v_{i+1})$ $i = 1, 2,3,4,5$ be the edges of P_5. Then $v_1, v_2, v_3, v_4, v_5, e_{12}, e_{23}, e_{34}, e_{45} \in V(B_1(P_5))$. The set $D = \{(v_3, v_4), (v_1, e_{23}), (v_1, e_{45}), (v_3, e_{12}), (v_5, e_{34}), (e_{23}, e_{34})\}$ is a minimum split edge dominating set of $B_1(P_5)$. Therefore, $\gamma_s'(B_1(P_5)) = 6$.

Theorem 2.2:

For the cycle C_n $(n \geq 6)$ on n vertices, $\gamma_s'(B_1(C_n)) \leq 2(2n - 6) = 4(n-3)$.

Proof: Let $v_1, v_2, \ldots, v_n$ be the vertices and $e_{12}, e_{23}, \ldots, e_{n-1,n}$ be the edges of C_n, where $e_{i,i+1} = (v_i, v_{i+1})$ $i = 1, 2, \ldots, n - 1$ and $e_{n1} = (v_n, v_1)$. Then $v_1, v_2, \ldots, v_n$ $e_{12}, e_{23}, \ldots, e_{n-1,n}, e_{n1} \in V(B_1(Cn))$. $B_1(C_n)$ has 2n vertices and n^2 edges.

Let $F_1 = \{(v_1, e_{n-1,n}),(v_1, e_{23}),(v_3, e_{12}), (v_{n-2}, v_{n-1}), (e_{23}, e_{34}),(v_{n-1}, e_{n1}),(e_{12}, e_{n1})\}$

$F_2 = \bigcup_{i=4}^{n-2}\{(v_i, e_{12})\ (v_i, e_{23})\}$,

$F_3 = \bigcup_{i=3}^{n-3}\{(v_{n-1}, e_{i,i+1})\}$ and $F_4 = \bigcup_{i=3}^{n-2}\{(v_n, e_{i,i+1})\}$

and let $D = F_1 \cup F_2 \cup F_3 \cup F_4 \subseteq E(B_1(C_n))$. $|D| = 7 + 2(n - 5) + n - 5 + n - 4 = 4n - 12 = 4(n - 3)$. D is an edge dominating set of $B_1(C_n)$ and $< V(B_1(C_n)) - D >$ is disconnected with two components, one of the components being K_4. Therefore D is a split edge dominating set of $B_1(C_n)$ and hence $\gamma_s'(B_1(C_n)) \leq |D| = 4(n - 3)$.

Remark 2.4: $\gamma_s'(B_1(C_3)) = 4$

Let v_1, v_2, v_3 be the vertices and e_{12}, e_{13}, e_{23} be the edges of C_3. Then $v_1, v_2, v_3, e_{12}, e_{13}, e_{23} \in V(B_1(C_3))$. $D = \{(v_2, v_3), (v_1, e_{23}), (v_2, e_{13}), (e_{12}, e_{23})\}$ is a minimum split edge dominating set of $B_1(C_3)$.Therefore,$\gamma_s'(B_1(C_3) = 4$.

Remark 2.5: $\gamma_s'(B_1(C_4)) = 6$

Let v_1, v_2, v_3, v_4 be the vertices and $e_{12}, e_{14}, e_{23}, e_{34}$ be the edges of C_4. Then $v_1, v_2, v_3, v_4, e_{12}, e_{14}, e_{23}, e_{34} \in V(B_1(C_4))$. $D = \{(v_1, v_4), (v_2, v_3), (v_3, e_{14}), (v_4, e_{23}), (e_{12}, e_{14}), (e_{23}, e_{34})\}$is a minimum split edge dominating set of $B_1(C_4)$. Therefore, $\gamma_s'(B_1(C_4)) = 6$.

Remark 2.6: $\gamma_s'(B_1(C_5)) = 8$

Let v_1, v_2, v_3, v_4, v_5 be the vertices and $e_{12}, e_{23}, e_{34}, e_{45}, e_{15}$ be the edges of C_5. Then $v_1, v_2, v_3, v_4, v_5, e_{12}, e_{23}, e_{34}, e_{45}, e_{15} \in V(B_1(P_5))$. The set $D = \{(v_1, e_{23}), (v_2, v_3), (v_4, v_5), (v_2, e_{34}), (v_4, e_{15}), (v_5, e_{12}), (e_{12}, e_{23}), (e_{34}, e_{45})\}$ is a minimum split edge dominating set of $B_1(C_5)$. Therefore, $\gamma_s'(B_1(C_5)) = 8$.

Theorem 2.3:

For the star $K_{1,n}$ on $(n + 1)$ vertices, $\gamma_s'(B_1(K_{1,n})) \leq n(n-1)$, $n \geq 2$.

Proof: Let $v_1, v_2, \ldots, v_{n+1}$ be the vertices of $K_{1,n}$ with v_1 as the central vertex, where $e_i = (v_1, v_j)$, $i = 2, 3, \ldots, n + 1$. $B_1(K_{1,n})$ has $(2n+1)$ vertices and $n(3n-1)/2$ edges. Then $v_1, v_2, \ldots, v_n\ e_1, e_2, \ldots, e_n \in V(B_1(K_{1,n}))$.

$E(B_1(K_{1,n}) = E(K_{1,n}) \cup E(K_n) \cup \{(v_i, e_j) / 1 \leq I \leq n, 1 \leq j \leq n, i \neq j\}$.

Let $D = \{(v_i, e_j) / 1 \leq i \leq n, 1 \leq j \leq n, i \neq j\}$ and $|D| = n(n-1)$. Then D is an edge dominating set of $B_1(K_{1,n})$. $V(B_1(K_{1,n})) - D$ is disconnected with the components $K_{1,n}$ and K_n. Therefore, D is a split edge dominating set of $B_1(K_{1,n})$. Hence, $\gamma_s'(B_1(K_{1,n})) \leq |D| = n(n-1)$

Theorem 2.4:

For any connected graph G, $\gamma_s'(B_1(G)) \leq (p + q) - (\beta_1(G) + \beta_1'(G) + 1)$, where $\beta_1(G)$ and $\beta_1'(G)$ is the independence numbers of G and L(G) respectively.

Proof: Let D' and D'' be edge independent sets of G and L(G) respectively, such that $|D'| = \beta_1(G)$ and $|D''| = \beta_1'(G)$. Let $e = (u, v) \in D'$ and let $w \in V(G)$ be adjacent to v. Then $(w, e) \in E(B_1(G))$. Let $D''' = \{(w, e)\}$ and $D = (B_1(G)) - (D' \cup D'' \cup D''')$. Then D is an edge dominating set of $B_1(G)$ and <D> is disconnected, one of the components being K_2. Therefore, D is a split edge dominating set of $B_1(G)$ and hence $\gamma_s'(B_1(G)) \leq |D| = (p + q) - (\beta_1(G) + \beta_1'(G) + 1)$.

Equality holds $G \cong P_3$. $\beta_1(P_3) = 1$, $\beta_1'(P_3) = 1$ implies that $\gamma_s'(B_1(P_3)) = (p + q) - 3 = 3 + 2 - 3 = 2$.

Theorem 2.5:

For any connected graph G, $\gamma_s'(B_1(G)) \leq q$.

Proof: Let $v_1, v_2, \ldots, v_p$ be the vertices of G and let $e_i \in E(G)$ be not incident with v_i, $i = 1, 2, \ldots, p$. Let $D = \{(v_i, e_i), i = 1,2,\ldots p\}$. Then $D' = V(B_1(G)) - D$ is an edge dominating set of $B_1(G)$ and $\langle D' \rangle \cong pK_2$. Therefore, D' is a split edge dominating set of $B_1(G)$ and $\gamma_s'(B_1(G)) \leq |D'| = p + q - |D| = p + q - p = q$.

Equality holds if $G \cong K_{1,2}$, P_4.

Theorem 2.6:

Let G be a connected (p, q) graph. Then $\gamma_s'(B_1(G)) \leq 2(q - \delta(G) - 1) + p$

Proof: Let $e \in V(L(G))$ be a vertex of maximum degree in L(G). Then $e \in E(G)$. Let $e = (u, v) \in E(G)$ and let $deg_G(u) = m$ and $deg_G(v) = n$. Then $deg_{L(G)}\, e = m + n - 2$. Let D' and D'' be the sets of edges not incident with u and v respectively. Then $|D'| = q - m$ and $|D''| = q - n$. Let D''' be the set of edges of G adjacent to e. Therefore $|D'''| = deg_{L(G)}e$. Let S be the set of vertices in G adjacent to none of u and v. Let $S' = \{(w, e)/w \in S\}$. $|S'| = p - (deg_G u + deg_G v) \leq p - 2\delta(G)$. If $D = D' \cup D'' \cup D''' \cup S'$, then $D \subseteq E(B_1(G))$ is an edge dominating set of $B_1(G)$ and e is an isolated edge in $\langle V(B_1(G)) - D \rangle$ and hence D is a split edge dominating set of $B_1(G)$ and

$$\gamma_s'(B_1(G)) \leq |D| \leq q - m + q - n + deg_{L(G)}\, e + p - 2\delta(G)$$

$$= 2q - (deg_{L(G)}\, e + 2) + deg_{L(G)}\, e + p - 2\delta(G) = 2q + p - 2\delta(G) - 2.$$

Equality holds if $G \cong C_n$.

3. Conclusion

In this paper, split edge domination numbers of Boolean Function Graph B(G, L(G), NINC) of path, cycle, stars and bounds are obtained.

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