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Eccentric Domination and Restrained Eccentric Domination in Circulant Graphs

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Abstract: *A subset D of the vertex set V(G) of a graph G is said to be a dominating set if every vertex not in D is adjacent to at least one vertex in D. A dominating set D is said to be an eccentric dominating set if for every* $v \in V-D$ *, there exists at least one eccentric vertex of v in D. Let p* ≥ 4 *be a positive integer. The circulant graph* $C_p(1, 2)$ *is the graph with vertex set* $\{v_0, v_1, v_2, ..., v_{p-1}\}$ and edge *set* $\{ \{v_i, v_{i+j} \}: i \in \{0, 1, 2, \ldots, p-1 \}$ and $j \in \{1, 2\}$. In this paper, we initiate the study of domination *number, eccentric domination number and restrained eccentric domination number in the circulant graphs* C_p $\langle 1, 2 \rangle$ *.*

Keywords: *domination, eccentric domination, restrained eccentric domination, circulant graphs. Mathematics Subject Classification: 05C12, 05C69.*

1. Introduction

 $\overline{}$

Let G be a finite, simple, undirected (p, q) graph with vertex set $V(G)$ and edge set E(G). For graph theoretic terminology refer to Harary [5], Buckley and Harary [3].

The concept of domination in graphs is originated from the chess games theory and that paved the way to the development of the study of various domination parameters and its relation to various other graph parameters. For details on domination theory, refer to Haynes, Hedetniemi and Slater [8]. Janakiraman, Bhanumathi and Muthammai [6] introduced Eccentric domination in Graphs. Bhanumathi, John Flavia and Kavitha [1] introduced and studied the concept of Restrained Eccentric domination in Graphs.

Definition 1.1: Let $p \ge 4$ be a positive integer. The **circulant graph** $C_p(1, 2)$ is the graph with vertex set $\{v_0, v_1, v_2, ..., v_{p-1}\}$ and edge set $\{\{v_i, v_{i+j}\}: i \in \{0, 1, 2, ..., p-1\}$ and $j \in \{1, 2\}\}.$

Definition 1.2:Let G be a connected graph and v be a vertex of G. The **eccentricity** e(v) of v is the distance to a vertex farthest from v. Thus, $e(v) = \max\{d(u, v) : u \in V\}$. The radius $r(G)$ is the minimum eccentricity of the vertices, whereas the diameter diam(G) = $d(G)$ is the maximum eccentricity. For any connected graph G, $r(G) \leq diam(G) \leq 2r(G)$. The vertex v is a central vertex if $e(v) = r(G)$. The **center** $C(G)$ is the set of all central vertices.

For a vertex v, each vertex at a distance $e(v)$ from v is an eccentric vertex of v. Eccentric set of a vertex v is defined as $E(v) = \{u \in V(G) / d(u, v) = e(v)\}.$

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Definition 1.3: A graph G is called a **m-eccentric point graph** if each point of G has exactly $m \geq 1$ eccentric points.

Definition 1.4 [4, 8]: A set $D \subseteq V$ is said to be a **dominating set** in G, if every vertex in $V-D$ is adjacent to some vertex in D. The minimum cardinality of a dominating set is called the **domination number** and is denoted by $\gamma(G)$.

Definition 1.5 [5]: A set $D \subseteq V(G)$ is a **restrained dominating set** if every vertex not in D is adjacent to a vertex in D and to a vertex in $V-D$. The cardinality of minimum restrained dominating set is called the **restrained domination number** and is denoted by $\gamma_r(G)$.

Definition 1.6 [6]: A set $D \subseteq V(G)$ is an **eccentric dominating set** if D is a dominating set of G and for every $v\in V-D$, there exists at least one eccentric vertex of v in D. The minimum cardinality of an eccentric dominating set is called the **eccentric domination number** and is denoted by $\gamma_{\text{ed}}(G)$.

Definition 1.7 [1]: A subset D of V(G) is a **restrained eccentric dominating set** if D is a restrained dominating set of G and for every $v \in V-D$, there exists at least one eccentric vertex of v in D. The minimum of the cardinalities of the restrained eccentric dominating set of G is called the **restrained eccentric domination number** of G and is denoted by $\gamma_{\text{red}}(G)$.

Theorem 1.1 [8]: For any graph $G, \lceil p/(1+\Delta(G)) \rceil \le \gamma(G) \le p-\Delta(G)$.

2. Domination, Eccentric Domination, Restrained Domination and Restrained Eccentric Domination.

In this section, we determine the eccentric domination and the restrained eccentric domination number of circulant graphs $G = C_n(1, 2)$, for any integer $p \ge 4$. Cleary, G is a 4-regular graph on p vertices.

It is obvious that $C_4(1, 2) = K_4$ and $C_5(1, 2) = K_5$. So $\gamma(C_4(1, 2)) = \gamma_{\text{ed}}(C_4(1, 2))$ $\langle 2 \rangle$) = $\gamma_r(C_4\langle 1, 2 \rangle)$ = $\gamma_{\text{red}}(C_4\langle 1, 2 \rangle)$ = 1 and $\gamma(C_5\langle 1, 2 \rangle)$ = $\gamma_{\text{ed}}(C_5\langle 1, 2 \rangle)$ = $\gamma_r(C_5\langle 1, 2 \rangle)$ = $\gamma_{\text{red}}(C_5\langle 1, 2 \rangle) = 1$. For $p \ge 6$ we have the following results:

Example: 2.1

Figure 2.1

In Figure 2.1,

 $D_1 = {v_0, v_{5, v_{10}}}$ is a dominating set of G and is also a restrained dominating set of G. Therefore, $\gamma(G) = \gamma_r(G) = 3$. $D_2 = \{v_0, v_3, v_6, v_9, v_{12}\}\$ is an eccentric dominating set of G and is also a restrained eccentric

dominating set of G. Therefore, $\gamma_{ed}(G) = \gamma_{red}(G) = 5.$

Lemma 2.1: Let $G = C_p(1, 2)$ be a connected graph then for any integer $p \ge 6$, $\gamma(G)$ $\gamma_{r}(G) = \lceil p/5 \rceil.$

Proof: Let $G = C_p\langle 1, 2 \rangle$ and let $v_0, v_1, v_2, ..., v_{p-1}$ be the vertices of G. From Theorem 1.1, $\lceil p/(1+\Delta(G)) \rceil \leq \gamma(G).$ That is, $\left[\rho/5\right] \le \gamma(G)$. ….. (1)

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Case (i): $p = 5k+1, k \ge 1$ **.**

In this case, $S = \{v_0, v_5, v_{10}, ..., v_{p-9}, v_{p-4}\}$ is a dominating set of G. Hence, $\gamma(G) \leq \lceil p/5 \rceil$.

Case (ii): $n = 5k+2, k \ge 1$. In this case, $S = \{v_0, v_5, v_{10}, ..., v_{p-10}, v_{p-5}\}$ is a dominating set of G. Hence, $\gamma(G) \leq \lceil p/5 \rceil$.

Case (iii): $n = 5k+3, k \ge 1$. In this case, $S = \{v_0, v_5, v_{10}, ..., v_{p-6}, v_{p-1}\}$ is a dominating set of G. Hence, $\gamma(G) \leq \lceil p/5 \rceil$.

Case (iv): $n = 5k+4, k \ge 1$. In this case, $S = \{v_0, v_5, v_{10}, ..., v_{p-7}, v_{p-2}\}$ is a dominating set of G. Hence, $\gamma(G) \leq \lceil p/5 \rceil$.

Case (v):
$$
n = 5k+5, k \ge 1
$$
.

In this case, $S = \{v_0, v_5, v_{10}, ..., v_{p-8}, v_{p-3}\}$ is a dominating set of G. Hence, $\gamma(G) \leq \lceil p/5 \rceil$. So in all the cases, $\gamma(G) \le \lceil p/5 \rceil$. ….. (2) From (1) and (2), $\gamma(G) = \lceil p/5 \rceil$.

In all the above cases, S is also a restrained dominating set of G. Therefore, $\gamma_r(G) = \gamma(G)$.

Lemma 2.2: Let G be a connected graph. Let $u \in V(G)$ is eccentric to atmost m vertices, then $\lceil p/(1+m) \rceil \leq \gamma_{\text{ed}}(G)$.

Proof: Let S be a γ_{ed} -set of G. A vertex in G is eccentric to atmost m vertices. Hence, $\lceil p/(1+m) \rceil \leq \gamma_{\text{ed}}(G)$.

Theorem 2.1: Let G = $C_p(1, 2)$ be a connected graph then for any integer $p \ge 6$,

$$
\begin{bmatrix}\n\frac{p}{4} \text{ or } \frac{p}{4} + 1 & \text{ if } & p = 12k & k \ge 1 \\
\left\lceil \frac{p}{5} \right\rceil & \text{ if } & p = 12k + 1 & k \ge 1 \\
\frac{p}{2} & \text{ if } & p = 12k + 2 & k \ge 1 \\
\frac{p}{3} & \text{ if } & p = 12k + 3 & k \ge 1 \\
\frac{p}{4} \text{ or } \frac{p}{4} + 1 & \text{ if } & p = 12k + 4 & k \ge 1 \\
\frac{p}{5} \text{ or } \frac{p}{5} + 1 & \text{ if } & p = 12k + 5 & k \ge 1 \\
\frac{p}{3} & \text{ if } & p = 12k + 6 & k \ge 0 \\
\frac{p}{3} & \text{ if } & p = 12k + 7 & k \ge 0 \\
\frac{p}{4} \text{ or } \frac{p}{4} + 1 & \text{ if } & p = 12k + 8 & k \ge 0 \\
\frac{p}{5} & \text{ if } & p = 12k + 9 & k \ge 1 \\
\frac{p}{3} & \text{ if } & p = 12k + 10 & k \ge 0 \\
\frac{p}{3} & \text{ if } & p = 12k + 11 & k \ge 0\n\end{bmatrix}
$$

 $\gamma_{\rm ed}(C_9\langle 1, 2 \rangle) = 3 = \gamma_{\rm red}(C_9\langle 1, 2 \rangle).$

Proof: Let $G = C_p\langle 1, 2 \rangle$ and let $v_0, v_1, v_2, ..., v_{p-1}$ be the vertices of G.

When G = $C_9(1, 2)$. G is a 2-self centered graph. The vertices $V_{\frac{p-3}{2}+i}$, $V_{\frac{p-1}{2}+i}$, $V_{\frac{p+1}{2}+i}$, $V_{\frac{p+3}{2}+i}$ v_{p-1} , v_{p+1} , v_{p+3} are the eccentric vertices of $v_i(i = 0, 1, 2, ..., p-1)$. S = { v_0 , v_4 , v_8 } is a minimum eccentric dominating set of G. S is also a minimum restrained eccentric dominating set of G. Thus, $\gamma_{\text{ed}}(G) = \gamma_{\text{red}}(G) = 3$.

Case (i): $p = 12k, k \ge 1$.

In this case, G is a $(\frac{p}{4})$ -self centered graph. The vertices $v_{\frac{p-2}{2}+i, \frac{p}{2}+i, \frac{p+2}{2}+i}$ 2 v_{p} v_{p+2} are the

eccentric vertices of v_i (i = 0, 1, 2, ..., p-1). Therefore, G is a 3-eccentric point graph.

Hence, by Lemma 2.2, $\frac{p}{A} \leq \gamma_{\text{ed}}(G)$. *p* p_1 . (1)

 $S =$ J $\left\{ \right.$ \vert $\overline{\mathcal{L}}$ ₹ \int v_0 , v_4 , v_8 , ..., v_m , $v_{\frac{p+4}{2}}$, $v_{\frac{p+4}{2}+4}$, ..., v_n 4 2 $\{0, V_4, V_8, \ldots, V_m, V_{p+4}, V_{p+4}, \ldots, V_t\}$ is an eccentric dominating set of G (where m is the multiple of 4 such that m is the largest integer less than or equal p/2 and t is of the

form $\frac{p+4}{2} + 4n$ such that $t \le p-1$). $|S| = \frac{p}{4}$ if $\frac{p+4}{2}$ is a multiple of 4, otherwise $|S| = \frac{p}{4}$ +1.

Thus,
$$
\gamma_{\text{ed}}(G) = \frac{p}{4}
$$
, otherwise $\gamma_{\text{ed}}(G) = \frac{p}{4} + 1$ (2)

From (1) and (2), $\gamma_{\text{ed}}(G) = \frac{P}{4}$ or $\frac{P}{4} + 1$ $\frac{p}{4}$ *or* $\frac{p}{4}$ +1.

Case (ii): $p = 12k+1, k \ge 1$.

In this case, G is a $(\frac{p-1}{4})$ -self centered graph.

The vertices
$$
v_{p-3+i, y_{p-1+i, y_{p+1}} v_{p+3+i}} v_{p+3+i}
$$
 are the eccentric vertices of

 v_i (i = 0, 1, 2, ..., p-1). Therefore, G is a 4-eccentric point graph.

Hence, by Lemma 2.2, $\lceil p/5 \rceil \leq \gamma_{\text{ed}}(G)$. ….. (3) $S =$ J $\left\{ \right.$ \vert $\overline{\mathcal{L}}$ ₹ \int v_0 , v_5 , v_{10} , ..., v_m , $v_{\frac{p+5}{2}}$, $v_{\frac{p+5}{2}+5}$, ... v_i 5 2 $\{0, V_5, V_{10}, \ldots, V_m, V_{p+5}, V_{p+5}, \ldots, V_t\}$ is an eccentric dominating set of G (where m is

the multiple of 5 such that m is the largest integer less than or equal p/2 and t is of the form $\frac{p+5}{2}$ + 5*n* such that t≤p-1). $|S| = |p/5|$.

Thus, γed(G) ≤ p/5. ….. (4) From (3) and (4), γed(G) = p/5.

Case (iii): $p = 12k+2, k \ge 1$.

In this case, G is a $(\frac{p+2}{4})$ -self centered graph.

The vertex $v_{\underline{p}_{+i}}$ is the eccentric vertex of v_i (i = 0, 1, 2, ..., p-1). Therefore, G is a 2

self centered unique eccentric point graph.

Hence,
$$
\gamma_{ed}(G) \ge \frac{p}{2}
$$
 (5)
\n $S = \{v_0, v_2, v_4, ..., v_{p-6}, v_{p-4}, v_{p-2}\}$ is an eccentric dominating set of G.
\nThus, $\gamma_{ed}(G) \le \frac{p}{2}$ (6)

From (5) and (6), $\gamma_{\text{ed}}(G) = \frac{p}{2}$.

Case (iv): $p = 12k+3, k \ge 1$.

In this case, G is a ($\frac{p+1}{4}$)-self centered graph. The vertices $v_{\frac{p-1}{2}+i,\frac{p+1}{2}+i}$ v_{p+1} are the

eccentric vertices of v_i (i = 0, 1, 2, ..., p-1). Therefore, G is a 2-eccentric point graph.

Hence, by Lemma 2.2,
$$
\frac{p}{3} \le \gamma_{\text{ed}}(G)
$$
. (7)

 $S = \{v_0, v_3, v_6, ..., v_{p-9}, v_{p-6}, v_{p-3}\}\$ is an eccentric dominating set of G.

Thus, γed(G) ≤ ³ *^p* . ….. (8)

From (7) and (8), $\gamma_{\text{ed}}(G) = \frac{P}{3}$. *p*

Case (v): $p = 12k+4, k \ge 1$.

In this case, G is a $(\frac{p}{4})$ -self centered graph. The vertices $v_{\frac{p-2+i}{2}+i,\frac{p}{2}+i,\frac{p+2}{2}+i}$ v_{p} v_{p+2} are the

eccentric vertices of v_i (i = 0, 1, 2, ..., p-1). Therefore, G is a 3-eccentric point graph.

Hence, by Lemma 2.2, $\frac{p}{4} \leq \gamma_{\text{ed}}(G)$. ….. (9) $\left| \right|$

 $S =$ J $\left\{ \right.$ $\overline{\mathcal{L}}$ ⇃ \int v_0 , v_4 , v_8 , ..., v_m , $v_{\frac{p+4}{2}}$, $v_{\frac{p+4}{2}+4}$, ..., v_n 4 2 $\{0, V_4, V_8, \ldots, V_m, V_{p+4}, V_{p+4}, \ldots, V_t\}$ is an eccentric dominating set of G (where m is the multiple of 4 such that m is the largest integer less than or equal p/2 and t is of the

form
$$
\frac{p+4}{2} + 4n
$$
 such that $t \le p-1$). $|S| = \frac{p}{4}$ if $\frac{p+4}{2}$ is a multiple of 4, otherwise $|S| = \frac{p}{4} + 1$.
\nThus, $\gamma_{ed}(G) = \frac{p}{4}$, otherwise $\gamma_{ed}(G) = \frac{p}{4} + 1$(10)

From (9) and (10), $\gamma_{\text{ed}}(G) = \frac{P}{4} \omega \frac{P}{4} + 1$ $\frac{p}{4}$ *or* $\frac{p}{4}$ +1.

Case (vi): $p = 12k+5, k \ge 1$ **.**

In this case, G is a $(\frac{p-1}{4})$ -self centered graph. The vertices $v_{\frac{p-3}{2}+i}$, $v_{\frac{p-1}{2}+i}$, $v_{\frac{p+1}{2}+i}$, $v_{\frac{p+3}{2}+i}$ v_{p-1} v_{p+1} v_{p+3} are the eccentric vertices of v_i (i = 0,

1, 2, ..., $p-1$). Therefore, G is a 4-eccentric point graph.

Hence, by Lemma 2.2, $\lceil p/5 \rceil \leq \gamma_{\text{ed}}(G)$. ….. (11) $S =$ J $\left\{ \right.$ \vert $\overline{\mathcal{L}}$ ┤ \int v_0 , v_5 , v_{10} , ..., v_m , $v_{\frac{p+5}{2}}$, $v_{\frac{p+5}{2}+5}$, ..., v_n 5 2 $\{v_0, v_5, v_{10}, \ldots, v_m, v_{p+5}, v_{p+5}, \ldots, v_t\}$ is an eccentric dominating set of G (where m

is the multiple of 5 such that m is the largest integer less than or equal p/2 and t is of the

form $\frac{p+5}{2}$ + 5*n* such that t \leq p-1). $|S| = |p/5|$ if $\frac{p+5}{2}$ is a multiple of 5, otherwise \vert S = \vert p/5 +1.

Thus,
$$
\gamma_{\text{ed}}(G) = \lceil p/5 \rceil
$$
, otherwise $\gamma_{\text{ed}}(G) = \lceil p/5 \rceil + 1$ (12)
From (11) and (12), $\gamma_{\text{ed}}(G) = \lceil p/5 \rceil$ or $\lceil p/5 \rceil + 1$.

Case (vii): $p = 12k+6, k \ge 0$ **.**

In this case, G is a ($\frac{p+2}{4}$)-self centered graph. The vertex $v_{\frac{p}{2}+i}$ is the eccentric

vertex of v_i (i = 0, 1, 2, ..., p-1). Therefore, G is a self centered unique eccentric point graph.

Hence, γed(G) ≥ ² *^p* . ….. (13)

 $S = \{v_0, v_2, v_4, ..., v_{p-6}, v_{p-4}, v_{p-2}\}\$ is an eccentric dominating set of G.

Thus, γed(G) ≤ ² *^p* . ….. (14)

From (13) and (14), $\gamma_{\text{ed}}(G) = \frac{p}{2}$.

Case (viii): $p = 12k+7, k \ge 0$ **.**

In this case, G is a ($\frac{p+1}{4}$)-self centered graph. The vertices $v_{\frac{p-1}{2}+i,\frac{p+1}{2}+i}$ v_{p+1} are the

eccentric vertices of v_i (i = 0, 1, 2, ..., p-1). Therefore, G is a 2-eccentric point graph.

Hence, by Lemma 2.2, $\lceil p/3 \rceil \leq \gamma_{\text{ed}}(G)$. ….. (15)

 $S =$ J $\left\{ \right.$ \vert $\overline{\mathcal{L}}$ ⇃ $\left($ v_0 , v_3 , v_6 , ..., v_m , $v_{\frac{p+3}{2}}$, $v_{\frac{p+3}{2}+3}$, ... v_i 3 2 $\{0, V_3, V_6, \ldots, V_m, V_{p+3}, V_{p+3}, \ldots, V_t\}$ is an eccentric dominating set of G (where m is

the multiple of 3 such that m is the largest integer less than or equal to p/2 and t is of the form $\frac{p+3}{2}$ + 3*n* such that t \leq p-1). $|S| = |p/3|$.

Thus,
$$
\gamma_{\text{ed}}(G) \le \lceil p/3 \rceil
$$
. \n
$$
\dots \text{ (16)}
$$
\nFrom (15) and (16), $\gamma_{\text{ed}}(G) = \lceil p/3 \rceil$.

Case (ix): $p = 12k+8, k \ge 0$.

In this case, G is a $(\frac{p}{4})$ -self centered graph. The vertices $v_{\frac{p-2+i}{2}+i,\frac{p}{2}+i,\frac{p+2}{2}+i}$ v_{p} v_{p+2} are the

eccentric vertices of v_i (i = 0, 1, 2, ..., p-1). Therefore, G is a 3-eccentric point graph.

Hence, by Lemma 2.2, $\frac{p}{A} \leq \gamma_{\text{ed}}(G)$. *p p* p = p (17) $S =$ J $\left\{ \right.$ \vert $\overline{\mathcal{L}}$ ⇃ \int v_0 , v_4 , v_8 , ..., v_m , $v_{\frac{p+4}{2}}$, $v_{\frac{p+4}{2}+4}$, ..., v_t 4 2 $\{0, V_4, V_8, \ldots, V_m, V_{p+4}, V_{p+4}, \ldots, V_t\}$ is an eccentric dominating set of G (where m is the multiple of 4 such that m is the largest integer less than or equal p/2 and t is of the form $\frac{p+4}{2} + 4n$ such that $t \le p-1$). $|S| = \frac{p}{4}$ if $\frac{p+4}{2}$ is a multiple of 4, otherwise $|S| = \frac{p}{4} + 1$. Thus, $\gamma_{\text{ed}}(G) = \frac{p}{4}$, otherwise $\gamma_{\text{ed}}(G) = \frac{p}{4} + 1$ (18) From (17) and (18), $\gamma_{\text{ed}}(G) = \frac{p}{4}$ or $\frac{p}{4} + 1$ $\frac{p}{4}$ *or* $\frac{p}{4}$ +1. Case (x): $p = 12k+9, k \ge 1$. In this case, G is a $(\frac{p-1}{4})$ -self centered graph. The vertices $v_{\frac{p-3}{2}+i, \frac{v_{p-1}}{2}+i, \frac{v_{p+1}}{2}+i, \frac{v_{p+3}}{2}+i}$ v_{p-1} , v_{p+1} , v_{p+3} are the eccentric vertices of v_i (i = 0, 1, 2, ..., p-1). Therefore, G is a 4-eccentric point graph. Hence, by Lemma 2.2, $\lceil p/5 \rceil \leq \gamma_{\text{ed}}(G)$. ….. (19) \vert \int

 $S =$ J $\left\{ \right.$ $\overline{\mathcal{L}}$ ⇃ v_0 , v_5 , v_{10} , ..., v_m , $v_{\frac{p+5}{2}}$, $v_{\frac{p+5}{2}+5}$, ... v_i 5 2 $\{0, V_5, V_{10}, \ldots, V_m, V_{p+5}, V_{p+5}, \ldots, V_t\}$ is an eccentric dominating set of G (where m is

the multiple of 5 such that m is the largest integer less than or equal to p/2 and t is of the form $\frac{p+5}{2} + 5n$ such that $t \le p-1$). $|S| = |p/5|$.

Thus,
$$
\gamma_{\text{ed}}(G) \le \lceil p/5 \rceil
$$
. \n
$$
\dots \text{ (20)}
$$
\nFrom (19) and (20), $\gamma_{\text{ed}}(G) = \lceil p/5 \rceil$.

Case (xi): $p = 12k+10, k \ge 0$.

In this case, G is a $(\frac{p+2}{4})$ -self centered graph.

The vertex $v_{\underline{p}_{+i}}$ is the eccentric vertex of v_i (i = 0, 1, 2, ..., p-1). Therefore, G is a 2

self centered unique eccentric point graph.

Hence,
$$
\gamma_{\text{ed}}(G) \ge \frac{p}{2}
$$
. (21)

 $S = \{v_0, v_2, v_4, ..., v_{p-6}, v_{p-4}, v_{p-2}\}\$ is an eccentric dominating set of G.

Thus, γed(G) ≤ ² *^p* . ….. (22)

From (21) and (22), $\gamma_{\text{ed}}(G) = \frac{p}{2}$.

Case (xii): $p = 12k+11, k \ge 0$

In this case, G is a $(\frac{p+1}{4})$ -self centered graph.

The vertices $v_{\frac{p-1}{2}+i, \frac{v_{p+1}}{2}+i}$ v_{p+1} are the eccentric vertices of v_i (i = 0, 1, 2, ..., p-1).

Therefore, G is a 2-eccentric point graph.

Hence, by Lemma 2.2, $\lceil p/3 \rceil \leq \gamma_{\text{ed}}(G)$. ….. (23) $S =$ J $\left\{ \right\}$ \vert $\overline{\mathcal{L}}$ ⇃ \int v_0 , v_3 , v_6 , ..., v_m , $v_{\frac{p+3}{2}}$, $v_{\frac{p+3}{2}+3}$, ... v_i 3 2 $\{0, V_3, V_6, \ldots, V_m, V_{p+3}, V_{p+3}, \ldots, V_t\}$ is an eccentric dominating set of G (where m is the multiple of 3 such that m is the largest integer less than or equal p/2 and t is of the

form $\frac{p+3}{2}$ + 3*n* such that t \leq p-1). $|S| = |p/3|$.

Thus, $\gamma_{ed}(G) \le \begin{bmatrix} p/3 \end{bmatrix}$. ….. (24) From (23) and (24), $\gamma_{\rm ed}(G) = |p/3|$.

In all the above cases, S is also a restrained eccentric dominating set of G. Therefore, $\gamma_{\text{red}}(G) = \gamma_{\text{ed}}(G)$.

Conclusion:

Here we have studied eccentric domination and restrained eccentric domination in circulant graph $C_p(1, 2)$, and also studied a bound for the eccentric domination number of a connected graph.

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