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Root Cube Mean Labeling of Graphs

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Abstract: *A graph $G = (V,E)$ with p vertices and q edges is said to be a Root Cube Mean graph if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1,2,\ldots,q+1$ in such a way that when each edge $e=uv$ is labeled with $f(e=uv) = \left\lceil \sqrt{\frac{f(u)^3+f(v)^3}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^3+f(v)^3}{2}} \right\rfloor$, then the resulting edge labels are distinct. Here f is called a Root Cube Mean labeling of G. In this paper we prove that Path P_n, Cycle C_n, Comb, Ladder, and Triangular Snake T_n are Root Cube Mean graphs.*

Keywords: *Mean Labeling of graphs, Root Cube Mean Labeling of graphs.*

AMS Subject Classification (2010):05C78

1. Introduction

Graceful and Harmonious Labelings, Variations of Graceful Labelings are studied by Gallian[1]. The concept of Mean Labeling has been introduced by Somasundaram and Ponraj [3]. Root Square Mean Labeling of Graphs has been introduced by Sandhya, Somasundaram and Anusa[4]. Motivated by the above works we introduced a new type of labeling called Root Cube Mean Labeling. The standard terminology and notations we follow Harary [2]. In this paper we investigate the Root Cube Mean Labeling of Path, Cycle, Comb, Ladder, and Triangular Snake Graphs.

2. Preliminaries

Definition 2.1: [4]

A walk in which vertices are distinct is called a path. A path on n vertices is denoted by p_n .

Definition 2.2: [4]

A closed path is called a cycle. A cycle on n vertices is denoted by C_n.

Definition 2.3: [4]

The graph obtained by joining a single pendent edge to each vertex of a path is called a Comb graph.

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Definition 2.4: [4]

The product graph $P_2 \text{ X } P_n$ is called a Ladder and it is denoted by L_n.

Definition 2.5: [4]

A Triangular Snake T_n is obtained from a path $u_1, u_2, ..., u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \le i \le n-1$. That is every edge of a path is replaced by a triangle C_3.

Definition 2.6:

Let G be simple Graphs with vertex set $V(G)$ and the Edge set $E(G)$ respectively. Vertex set $V(G)$ are labeled arbitrary by positive integers and let $E(e)$ denoted the edge label such that it is the sum of labels of vertices incident with edge e.

3. Root Cube Mean Labeling of Graphs

Definition 3.1:

A graph $G = (V, E)$ with p vertices and q edges is said to be a Root Cube Mean Graph if it is possible to label the vertices $x \in v$ with distinct elements $f(x)$ from $1,2,...,q+1$ in such a way that when each edge $e = uv$ is labeled with $f(e = uv) = \left\lceil \sqrt{\dfrac{f(u)^3 + f(v)^3}{2}} \right\rceil$ or $\left\lfloor \sqrt{\dfrac{f(u)^3 + f(v)^3}{2}} \right\rfloor$, than the resulting edge labels are distinct. Here f is called a Root Cube Mean Labeling of G.

Theorem 3.2: Any path p_n is a Root Cube Mean Labeling Graph.

Proof:

Let $G = p_n$ be the graph with vertices $u_1, u_2, ..., u_n$ and the edges $e_1, e_2, ... e_q$.

Define the function $f : V(p_n) \rightarrow (1,2,...,q+1)$ as follows $f(u_i) = i$, for $1 \le i \le n$.

and the induced edge labeling function $f^* : E(G) \rightarrow N$ is defined by

$$f^*(e = uv) = \left\lceil \sqrt{\frac{f(u)^3 + f(v)^3}{2}} \right\rceil$$

Now, $$f^*(u_i u_{i+1}) = \left\lceil \sqrt{\frac{f(u_i)^3 + f(u_{i+1})^3}{2}} \right\rceil$$

$$= \left\lceil \sqrt{\frac{2i^3 + 3i^2 + 3i + 1}{2}} \right\rceil \quad \text{for} \quad 1 \le i \le n-1$$

Clearly $f^*(u_{n-1}u_n) = \left\lceil \sqrt{\frac{2n^3 - 3n^2 + 3n - 1}{2}} \right\rceil$

Hence the edge labels are distinct.

Thus path graph admits a Root Cube Mean Labeling of Graphs.

Example 3.3:

The following example shows that p_5 is Root Cube Mean Labeling Graph.

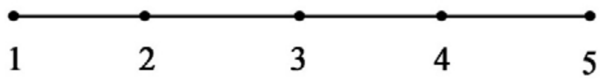

Figure 3.1

Here, $f(u_1) = 1$, $f(u_2) = 2$, $f(u_3) = 3$, $f(u_4) = 4$, $f(u_5) = 5$.

Now, $f^*(u_1u_2) = \left\lceil \sqrt{\frac{1^3 + 2^3}{2}} \right\rceil = \left\lceil \sqrt{\frac{9}{2}} \right\rceil = 2$, similarly $f^*(u_2u_3) = 4$, $f^*(u_3u_4) = 6$,

$f^*(u_4u_5) = 9$. Hence Edge Labeling of p_5 are distinct.

Theorem 3.4: Any Cyclic C_n is a Root Cube Mean Labeling Graph.

Proof:

Let $G = C_n$ be the graph with vertices $u_1, u_2, \ldots, u_n$ and the edges $e_1, e_2, \ldots e_q$.

Define the function $f : V(C_n) \to (1,2,\ldots,q+1)$ as follows $f(u_i) = i$ for $1 \le i \le n$.

and the induced edge labeling function $f^* : E(G) \to N$ defined by

$$f^*(e = uv) = \left\lceil \sqrt{\frac{f(u)^3 + f(v)^3}{2}} \right\rceil$$

Now, $f^*(u_iu_{i+1}) = \left\lceil \sqrt{\frac{2i^3 + 3i^2 + 3i + 1}{2}} \right\rceil$ for $1 \le i \le n-1$

Clearly, $f^*(u_{n-1}u_n) = \left\lceil \sqrt{\frac{2n^3 - 3n^2 + 3n - 1}{2}} \right\rceil$

$$f^*(u_n u_1) = \left\lceil \sqrt{\frac{n^3+1}{2}} \right\rceil$$

Thus the edge labels are distinct.

Hence the cycle graph C_n admits a Root Cube Mean Labeling of Graphs.

Example 3.5:

The following is an example for C_5 is a Root Cube Mean Labeling Graphs.

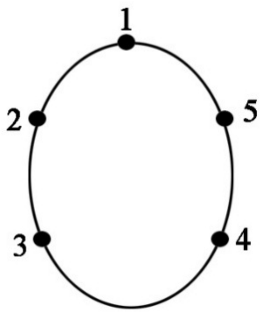

Figure 3.2

Theorem 3.6: The Ladder L_n is a Root Cube Mean Labeling Graph.

Proof:

Let G be the Ladder graph with vertices $u_1, u_2, ..., u_n$ and the edges $e_1, e_2, ...e_q$.

Define the function $f : V(L_n) \to (1,2,...,q+1)$ as follows $f(u_i) = 2i-1,\ f(v_i) = 2i$ for $1 \le i \le n$. And the induced edge labeling function $f^* : E(G) \to N$ defined by

$$f^*(e = uv) = \left\lceil \sqrt{\frac{f(u)^3 + f(v)^3}{2}} \right\rceil \text{ (or) } \left\lfloor \sqrt{\frac{f(u)^3 + f(v)^3}{2}} \right\rfloor$$

Now, $f^{\cdot}(u_i u_{i+1}) = \left\lceil \sqrt{8i^3 + 6i} \right\rceil$ (or) $\left\lfloor \sqrt{8i^3 + 6i} \right\rfloor$ for $1 \le i \le n-1$

Clearly $f^*(u_{n-1} u_n) = \left\lceil \sqrt{8n^3 - 24n^2 + 30n - 14} \right\rceil$ (or) $\left\lfloor \sqrt{8n^3 - 24n^2 + 30n - 14} \right\rfloor$

$f^*(v_i v_{i+1}) = \left\lceil \sqrt{8i^3 + 12i^2 + 12i + 4} \right\rceil$ (or) $\left\lfloor \sqrt{8i^3 + 12i^2 + 12i + 4} \right\rfloor$ for $1 \le i \le n-1$

$$f^*(v_{n-1} v_n) = \left\lceil \sqrt{8n^3 - 12n^2 + 12n - 4} \right\rceil \text{ (or) } \left\lfloor \sqrt{8n^3 - 12n^2 + 12n - 4} \right\rfloor$$

$f^*(u_i v_i) = \left\lceil \sqrt{\frac{16i^3 - 12i^2 + 6i - 1}{2}} \right\rceil$ (or) $\left\lfloor \sqrt{\frac{16i^3 - 12i^2 + 6i - 1}{2}} \right\rfloor$ for $1 \le i \le n-1$

$$f^{*}(u_n v_n) = \left\lceil \sqrt{\frac{16n^3 - 12n^2 + 6n - 1}{2}} \right\rceil \text{(or)} \left\lfloor \sqrt{\frac{16n^3 - 12n^2 + 6n - 1}{2}} \right\rfloor$$

Here the edge labels are distinct.

Hence the Ladder graph admits a Root Cube Mean Labeling of Graphs.

Example 3.7:

The Root Cube Mean labeling of L_6 is given below.

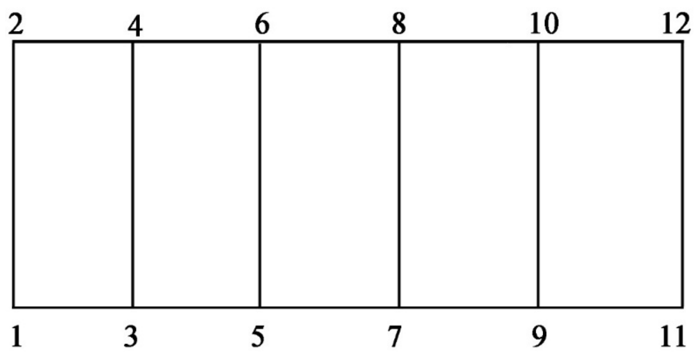

Figure 3.3

Here $f(u_1) = 1,\ f(u_2) = 3, f(u_3) = 5,..., f(u_n) = 11$ and

$f(v_1) = 2, f(v_2) = 4, f(v_3) = 6,..., f(v_n) = 12.$

And the edge labels are $f^{*}(u_1u_2) = 3, f^{*}(u_2u_3) = 8, f^{*}(u_3u_4) = 15,..., f^{*}(u_5u_6) = 32$ and

$f^{*}(v_1v_2) = 6, f^{*}(v_2v_3) = 11,.., f^{*}(v_5v_6) = 36, f^{*}(u_1v_1) = 2, f^{*}(u_2v_2) = 7,.., f^{*}(u_6v_6) = 39$

all are distinct.

Theorem: 3.8: Comb graph is Root Cube Mean Labeling Graphs.

Proof:

Let G be a Comb graph with vertices $u_1, u_2,..., u_n$ and the edges $e_1, e_2,...e_q$.

Let p_n be the path $u_1, u_2,..., u_n$ in G and join a vertex v_i to u_i, for $1 \le i \le n$.

Define the function $f : V(C_n) \to (1,2,...,q+1)$ as follows $f(u_i) = i$ for $1 \le i \le n$.

and the induced edge labeling function $f^{*} : E(G) \to N$ defined by

$$f^{*}(e = uv) = \left\lceil \sqrt{\frac{f(u)^3 + f(v)^3}{2}} \right\rceil$$

Now, $f^{\cdot}(u_i u_{i+1}) = \left\lceil \sqrt{8i^3 + 6i} \right\rceil$ for $1 \le i \le n-1$

Clearly $f^*(u_{n-1}u_n) = \left\lceil \sqrt{8n^3 - 24n^2 + 30n - 14} \right\rceil$

$$f^*(u_i v_i) = \left\lceil \sqrt{\frac{16i^3 - 12i^2 + 6i - 1}{2}} \right\rceil \quad \text{for } 1 \le i \le n-1$$

$$f^*(u_n v_n) = \left\lceil \sqrt{\frac{16n^3 - 12n^2 + 6n - 1}{2}} \right\rceil$$

Thus the edge labels are distinct.

Hence the Comb graph admits a Root Cube Mean Labeling of Graphs.

Example 3.9:

Root Cube Mean Labeling of Comb graph obtained from P_6 is given below.

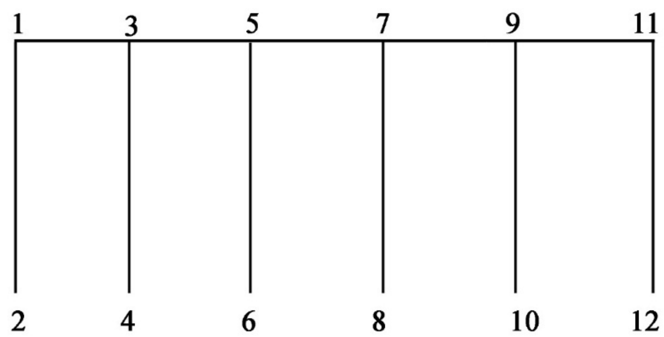

Figure 3.4

Here the vertex labels $f(u_1) = 1, f(u_2) = 3, f(u_3) = 5, \ldots, f(u_n) = 11$ and $f(v_1) = 2, f(v_2) = 4, f(v_3) = 6, \ldots, f(v_n) = 12$, and the edge labels are

$f^*(u_1u_2) = 3, f^*(u_2u_3) = 8, f^*(u_3u_4) = 15, \ldots, f^*(u_5u_6) = 32, f^*(u_1v_1) = 2,$

$f^*(u_2v_2) = 6, \ldots, f^*(u_6v_6) = 39$

Here the edge labels are distinct.

Hence Comb graph obtained from P_6 is Root Cube Mean Labeling of Graphs.

Theorem 3.10: Triangular Snake T_n is a Root Cube Mean Labeling Graph.

Proof:

Let T_n be a triangular snake. Define a function $f : V(T_n) \to (1, 2, \ldots, q+1)$ as follows

$f(u_i) = 2i - 1,\ 1 \le i \le n,\ f(v_i) = 2i,\ 1 \le i \le n-1$

and the induced edge labeling function $f^* : E(G) \to N$ is defined by

$$f^*(e=uv)=\left\lceil \sqrt{\frac{f(u)^3+f(v)^3}{2}} \right\rceil$$

Now, $f^*(u_i u_{i+1})=\left\lceil \sqrt{\frac{f(u_i)^3+f(v_i)^3}{2}} \right\rceil$

$$f^*(u_i u_{i+1})=\left\lceil \sqrt{8i^3+6i} \right\rceil \text{ for } 1 \le i \le n-1$$

Clearly $f^*(u_{n-1}u_n)=\left\lceil \sqrt{8n^3-24n^2+30n-14} \right\rceil$

$$f^*(u_i v_i)=\left\lceil \sqrt{\frac{16i^3-12i^2+6i-1}{2}} \right\rceil \quad \text{for } 1 \le i \le n-1$$

$$f^*(u_n v_n)=\left\lceil \sqrt{\frac{16n^3-12n^2+6n-1}{2}} \right\rceil$$

$$f^*(u_{i+1} v_i)=\left\lceil \sqrt{\frac{16i^3+12i^2+6i+1}{2}} \right\rceil \quad \text{for } 1 \le i \le n-1$$

$$f^*(u_n v_{n-1})=\left\lceil \sqrt{\frac{16n^3-36n^2+1}{2}} \right\rceil$$

Hence the edge labels are distinct.

Thus the Triangular Snake T_n is a Root Cube Mean Labeling of Graphs.

Example: 3.11:

This example shows that T_4 is Root cube Mean Labeling Graphs.

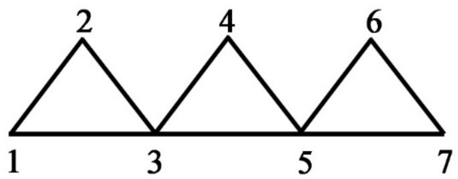

Figure 3.5

Conclusion:

It is very interesting to investigate graphs which admit Root Cube Mean Labeling. In this paper we proved that path, Cycle, Comb, Ladder, Triangular Snake is Root Cube Mean Labeling Graphs. It is possible to investigate similar results for several other graphs.

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