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Computation of Multiplicative Topological Indices of Dutch Windmill Graph

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Abstract: *Among topological descriptors topological indices are very important and they have a prominent role in chemistry. In this paper, we compute the first and second Multiplicative Zagreb indices, the first and second multiplicative hyper-Zagreb indices, first and second Multiplicative modified Zagreb indices, the multiplicative First and second Zagreb polynomial, multiplicative ABC index, multiplicative GA index, multiplicative GA_5, multiplicative ABC_4 and multiplicative Augmented Zagreb (πAZI) index for Dutch Windmill Graph.*

Keywords: *Multiplicative topological indices, molecular graphs and Dutch Windmill Graph.*

1. Introduction

A graph is a collection of points and lines connecting a subset of them. The points and lines of a graph also called vertices and edges of the graph, respectively. If *e* is an edge of *G*, connecting the vertices *u* and *v*, then we write $e = uv$ and say "*u* and *v* are adjacent". A connected graph is a graph such that there is a path between all pairs of vertices. A simple graph is a non-weighted, undirected graph without loops or multiple edges. A molecular graph is a simple graph such that its vertices correspond to the atoms and the edges to the bonds. Note that hydrogen atoms are often omitted.

Molecular descriptors play a significant role in chemistry, pharmacology, etc. Among them, topological indices have a prominent place [1]. One of the best known and widely used is the connectivity index, introduced in 1975 by Milan RandiĆ [2], who has shown this index to reflect molecular branching. Consider the simple graph G, with the set of the following vertices $V = \{v_1, v_2, v_3, \ldots v_n\}$. If $u \in V$, the number of the edges ending in u is defined as the degree of vertex u and is denoted by $\deg(u)$ or simply by $d_G(u)$ or d_u. A large number of such indices depend only on vertex degree of the molecular graph. One of the oldest and well known topological indices is the first and second Zagreb indices, was first introduced by Gutman et al. in 1972 [4], and it is defined as

$$M_1(G) = \sum_{v \in V} d_G(v)^2 = \sum_{uv \in E(G)} [d_G(u) + d_G(v)] \quad (1)$$

$$M_2(G) = \sum_{uv \in E(G)} d_G(u) d_G(v) \quad (2)$$

The first Zagreb Polynomial$M_1(G, X)$ and second Zagreb Polynomial $M_2(G, X)$ are defined as:

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$$M_1(G,X) = \sum_{uv\in E(G)} X^{[d_u+d_v]} \quad (3)$$
$$M_2(G,X) = \sum_{uv\in E(G)} X^{[d_u\times d_v]} \quad (4)$$

The properties of $M_1(G,X)$, $M_2(G,X)$ polynomials for some chemical structures have been studied in [5].The modified first and second Zagreb indices [6] are respectively defined as

$$^{m}M_1(G) = \sum_{u\in V(G)} \frac{1}{d_G(u)^2} \quad (5)$$
$$^{m}M_2(G) = \sum_{uv\in E(G)} \frac{1}{d_G(u)d_G(v)} \quad (6)$$

In [7], Shirdel et al. introduced the first hyper-Zagreb index of a graph G, which is defined as

$$HM_1(G) = \sum_{uv\in E(G)}[d_G(u) + d_G(v)]^2 \quad (7)$$

In [8], the Second hyper-Zagreb index of a graph G is defined as

$$HM_2(G) = \sum_{uv\in E(G)}[d_G(u)d_G(v)]^2 \quad (8)$$

In [9], M. Eliasi introduced the Multiplicative first and second Zagreb Indices defined as

$$\pi_1{}^*(G) = \prod_{uv\in E(G)}[d_G(u) + d_G(v)] \quad (9)$$
$$\pi_2{}^*(G) = \prod_{uv\in E(G)} d_G(u)d_G(v) \quad (10)$$

The Geometric-arithmetic index, $GA(G)$ index of a graph G was introduced by D. Vukicevic et al. [10]. and it is defined as

$$GA = \sum_{uv\in E(G)} \frac{2\sqrt{d_u d_v}}{d_u d_v} \quad (11)$$

The fifth Geometric-arithmetic index, $GA_5(G)$ was introduced by A.Graovac et al. [11] in 2011 which is defined as

$$GA_5(G) = \sum_{uv\in E(G)} \frac{2\sqrt{S_u S_v}}{S_u S_v} \quad (12)$$

where S_u is the sum of the degrees of all neighbors of the vertex u in G, similarly for S_v. The Atom-bond connectivity (ABC) index, proposed by Estrada et al. [12] and is defined as:

$$ABC(G) = \sum_{uv\in E(G)} \sqrt{\frac{d_u+d_v-2}{d_u d_v}} \quad (13)$$

This index provides a good model for the stability of linear and branched alkanes as well as the strain energy of Cyclo alkanes [12,13]. Details about this index can be found in [14-16].

The fourth Atom bond connectivity index, $ABC_4(G)$ index was introduced by M. Ghorbani et al. in 2010.It is defined as

$$ABC_4(G) = \sum_{uv\in E(G)} \sqrt{\frac{S_u+S_v-2}{S_u S_v}} \quad (14)$$

where S_u is the sum of the degrees of all neighbors of the vertex u in G, similarly for S_v. Further studies on $ABC_4(G)$ index can be found in [17].

Inspired by work on the ABC index, Furtula et al. [18] proposed the following modified version of the ABC index and called it as Augmented Zagreb index (AZI):

$$AZI = \sum_{uv\in E(G)} \left(\frac{d_u d_v}{d_u+d_v-2}\right)^3 \quad (15)$$

In [20], M.Eliasi introduced the Multiplicative first and second Zagreb Indices defined as

$$\pi_1{}^*(G) = (G) = \prod_{uv\in E(G)}[d_G(u) + d_G(v)] \quad (16)$$

$$\pi_2{}^*(G) = (G) = \prod_{uv\in E(G)} d_G(u)d_G(v) \quad (17)$$

In [19], V.R. Kulli introduced the multiplicative atom-bond connectivity index and multiplicative Geometric-arithmetic indices which are defined as

$$ABC\pi(G) = \prod_{uv\in E(G)} \sqrt{\frac{d_u+d_v-2}{d_u d_v}} \quad (18)$$

$$GA\pi(G) = \prod_{uv\in E(G)} \frac{2\sqrt{d_u d_v}}{d_u+d_v} \quad (19)$$

Now we introduce ,

(i) The multiplicative first Zagreb polynomial $M_1\pi(G, X)$ and second Zagreb polynomial $M_2\pi(G, X)$ which are defined as

$$M_1\pi(G, X) = \prod_{uv\in E(G)} X^{[d_u+d_v]} \quad (20)$$

$$M_2\pi(G, X) = \prod_{uv\in E(G)} X^{[d_u\times d_v]} \quad (21)$$

(ii) The multiplicative fifth Geometric-arithmetic index $\pi GA_5(G)$ is defined as

$$GA_5\pi(G) = \prod_{uv\in E(G)} \frac{2\sqrt{S_u S_v}}{S_u+S_v} \quad (22)$$

(iii) The multiplicative fourth atom bond connectivity index, $ABC_4(G)$ index is defined as

$$ABC_4\pi(G) = \prod_{uv\in E(G)} \sqrt{\frac{S_u+S_v-2}{S_u S_v}} \quad (23)$$

(iv) The multiplicative Augmented Zagreb index $AZI\pi(G)$ is defined as

$$AZI\pi(G) = \prod_{uv\in E(G)} \left(\frac{d_u d_v}{d_u+d_v-2}\right)^3 \quad (24)$$

2. Computing multiplicative topological indices of some special graphs

In this paper we calculate the first and second Multiplicative Zagreb indices $\left(\pi_1{}^*(G) \text{ and } \pi_2{}^*(G)\right)$, the first and second multiplicative hyper Zagreb indices $HM_1\pi(G)$ and $HM_2\pi(G)$, first and second Multiplicative modified Zagreb indices $\left({}^mM_1(G) \text{and } {}^mM_2(G)\right)$, Multiplicative atom-bond connectivity index $\left(ABC\pi(G)\right)$, Multiplicative Geometric-arithmetic index $\left(GA\pi(G)\right)$, the Multiplicative first Zagreb Polynomial and second Zagreb polynomial $(M_1\pi(G, X)$ and

$M_2\pi(G,X)$, Multiplicative $ABC_4\pi(G)$ index, Multiplicative $GA_5\pi(G)$ index and the Multiplicative Augmented Zagreb index ($AZI\pi(G)$) of Dutch Windmill Graph.

3. Dutch Windmill Graph:

The Dutch windmill graph is denoted by ${D_n}^{(m)}$and it is the graph obtained by taking m copies of the cycle C_n with a vertex in common. The Dutch windmill graph is also called as friendship graph if n = 3 . i.e., friendship graph is the graph obtained by taking m copies of the cycle C_3 with a vertex in common. Dutch windmill graph ${D_n}^{(m)}$contains (n −1)m +1 vertices and mn edges as shown in the Figures 1-3.

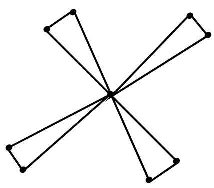

Figure1: ${D_3}^{(4)}$

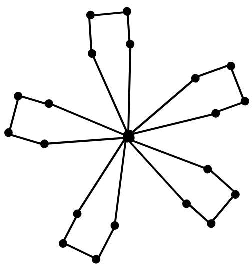

Figure 2: ${D_5}^{(5)}$

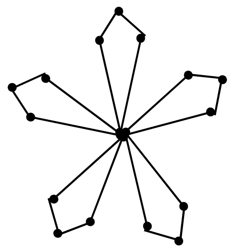

Figure 3: ${D_4}^{(5)}$

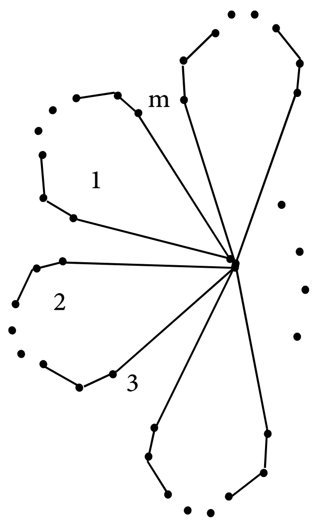

Figure 4: ${D_n}^{(m)}$

Now we shall compute the first and second Multiplicative Zagreb indices(${\pi_1}^*(G)$ and ${\pi_2}^*(G)$) , the first and second multiplicative hyper Zagreb indices ($HM_1\pi(G)$ and $HM_2\pi(G)$), first and second Multiplicative modified Zagreb indices$\left({}^{m}M_1(G) \text{and } {}^{m}M_2(G)\right)$, Multiplicative atom-bond connectivity index

$(ABC\pi(G))$, Multiplicative Geometric-arithmetic index $(GA\pi(G))$, the multiplicative first Zagreb polynomial and second Zagreb polynomial$((M_1\pi(G,X)$ and $M_2\pi(G,X)$, Multiplicative $ABC_4\pi(G)$ index , multiplicative $GA_5\pi(G)$ index and the multiplicative Augmented Zagreb index $(AZI\pi(G))$of Dutch windmill graph $D_n{}^{(m)}$.

Theorem 3.1:

Let us consider the Dutch windmill graph $G = D_n{}^{(m)}$,then

(i) $\pi_1{}^*(D_n{}^{(m)}) = [4]^{m(n-1)} \times [m+1]^{2m}$

$\pi_2{}^*(D_n{}^{(m)}) = 4^{nm} \times m^{2m}$

(ii)$HM_1\pi(D_n{}^{(m)}) = 16^{m(n-1)} \times [m+1]^{4m}$

$HM_2\pi(D_n{}^{(m)}) = 16^{nm} \times m^{4m}$

(iii) ${}^mM_1\left(D_n{}^{(m)}\right) = (n-1)/16m$

${}^mM_2\left(D_n{}^{(m)}\right) = 4^{-mn} \times m^{-2m}$

(iv) $M_1\pi\left(D_n{}^{(m)}, X\right) = [4]^{m(1-n)} \times [m+1]^{-2m}$

$M_2\pi\left(D_n{}^{(m)}, X\right) = 4^{-mn} \times m^{-2m}$

(v) $AZI\pi(D_n{}^{(m)}) = 8^{nm}$

(vi) $ABC\pi\left(D_n{}^{(m)}\right) = 2^{\frac{-nm}{2}}$

(vii)$GA\pi(D_n{}^{(m)}) = 2(m^m)(m+1)^{-2m}$

$$(viii)ABC_4\pi\left(D_n{}^{(m)}\right) = \begin{cases} \left[\frac{3}{8}\right]^{\frac{m(n-4)}{2}} \times \left[\frac{2+m}{4(m+1)}\right]^m \times \left[\frac{3}{4(m+1)}\right]^m, if\ n \geq 4 \\ \left[\frac{(2m+1)}{2(m+1)^2}\right]^{\frac{m}{2}} \times \left[\frac{3}{4(m+1)}\right]^m,\ if\ n = 3 \end{cases}$$

$$(ix)GA_5\pi\left(D_n{}^{(m)}\right) = \begin{cases} 2^{4m} \times \left[\frac{\sqrt{2(m+1)}}{(m+3)}\right]^{2m} \times \left[\frac{\sqrt{2m(m+1)}}{(3m+1)}\right]^{2m}, if\ n \geq 4 \\ \left[\frac{2\sqrt{2m(m+1)}}{(3m+1)}\right]^{2m}, if\ n = 3 \end{cases}$$

Proof:

Consider the following table based on degrees of edges.

Edges of the type $E_{(d_u,d_v)}$	Number of edges
$E_{(2,2)}$	$(n-2)m$
$E_{(2m,2)}$	2m

Table 1: Edge partition based on degrees of end vertices of each edge

For the Dutch windmill graph $D_n{}^{(m)}$, We partition the edges into edges of the type $E_{(d_u,d_v)}$where uv is an edge.In $D_n{}^{(m)}$ we get edges of the type $E_{(2,2)}$ and $E_{(2m,2)}$.

The number of edges of these types are given in the Table 1. Any Dutch windmill graph $D_n{}^{(m)}$contains$(n-1)m+1$ vertices and mn edges. Let $d_u$denote the degree of the vertex u. We partition the edges of $D_n{}^{(m)}$ into edges of the type $E^*{}_{(S_u,S_v)}$ where uv is an edge and S_u is the sum of the degrees of all neighbours of vertex u in G. In other words,$S_u = \sum_{uv\in E(G)} d_v$, Similarly for S_v .

Case (1):

If $n \geq 4$: $In\ D_n{}^{(m)}$ we get edges of the type $E^*{}_{(4,4)}, E^*{}_{(4,2m+2)} and\ E^*{}_{(2m+2,4m)}$. Edges of the type $E^*{}_{(4,4)}, E^*{}_{(4,2m+2)} and\ E^*{}_{(2m+2,4m)}$ are as shown in the figure [1]. The number of edges of these types are given in the following table

Edges of the type	Number of edges
$E^*{}_{(4,4)}$,	$(n-4)m$
$E^*{}_{(4,2m+2)}$	2m
$E^*{}_{(2m+2,4m)}$	2m

Table 2: Edge partition based on degree sum of neighbours of end vertices of each edge.

Case (2):

If $n = 3$, In $D_n{}^{(m)}$ we get edges of the type $E^*{}_{(2m+2\ ,2m+2)}$ $E^*{}_{(2m+2,4m)}$. The number of edges of these types are given in the following Table .

Edges of the type	Number of edges
$E^*{}_{(2m+2\ ,2m+2)}$	m
$E^*{}_{(2m+2,4m)}$	2m

Table 3: Edge partition based on degree sum of neighbours of end vertices of each edge.

We know that, from table 1

$$\text{(i) } \pi_1{}^*(D_n{}^{(m)}) = \prod_{uv\in E(D_n{}^{(m)})}[d_G(u)+d_G(v)]$$
$$= |\ E_{(2,2)}| \prod_{uv\in\ E_{(2,2)}}[d_G(u)+d_G(v)] \times |\ E_{(2m,2)}| \prod_{uv\in\ E_{(2m,2)}}[d_G(u)+d_G(v)]$$
$$= [2+2]^{(n-2)m} \times [2m+2]^{2m} = [4]^{(n-2)m} \times [2(m+1)]^{2m}$$
$$= [4]^{(n-2)m+m} \times [m+1]^{2m} = [4]^{nm-2m+m} \times [m+1]^{2m}$$
$$= [4]^{m(n-1)} \times [m+1]^{2m}$$

$$\pi_2^*\left(D_n^{(m)}\right) = \prod_{uv\in E(D_n^{(m)})} d_G(u)d_G(v)$$
$$= \left| E_{(2,2)}\right| \prod_{uv\in E_{(2,2)}} [d_G(u)d_G(v)] \times \left| E_{(2m,2)}\right| \prod_{uv\in E_{(2m,2)}} [d_G(u)d_G(v)]$$
$$= [2\times 2]^{(n-2)m} \times [2m \times 2]^{2m}$$
$$= [4]^{(n-2)m+2m} \times [m]^{2m} = [4]^{nm-2m+2m} \times [m]^{2m} = 4^{nm} \times m^{2m}$$

(ii) $HM_1\pi(D_n^{(m)}) = \prod_{uv\in E(D_n^{(m)})}[d_G(u) + d_G(v)]^2$
$$= \left| E_{(2,2)}\right| \prod_{uv\in E_{(2,2)}} [d_G(u) + d_G(v)]^2 \times \left| E_{(2m,2)}\right| \prod_{uv\in E_{(2m,2)}} [d_G(u) + d_G(v)]^2$$
$$= [(2+2)^2]^{(n-2)m} \times [(2m+2)^2]^{2m} = [4]^{(n-2)2m+2m} \times [m+1]^{4m}$$
$$= [4]^{2nm-2m} \times [m+1]^{4m} = 16^{m(n-1)} \times [m+1]^{4m}$$

$$HM_2\pi(G) = \prod_{uv\in E(G)}[d_G(u)d_G(v)]^2$$
$$= \left| E_{(2,2)}\right| \prod_{uv\in E_{(2,2)}} [d_G(u)d_G(v)]^2 \times$$
$$\left| E_{(2m,2)}\right| \prod_{uv\in E_{(2m,2)}} [d_G(u)d_G(v)]^2$$
$$= [(2\times 2)^2]^{(n-2)m} \times [(2m\times 2)^2]^{2m}$$
$$= [4]^{(n-2)2m+2m} \times [4m]^{4m} = [4]^{2nm-2m} \times [4m]^{4m}$$
$$= 16^{nm} \times m^{4m}$$

(iii) ${}^{m}M_1\left(D_n^{(m)}\right) = \prod_{u\in V(D_n^{(m)})} \frac{1}{d_G(u)^2} = ((n-1)^m)/4.4m^2$

$=(n-1)/16m$

$${}^{m}M_2(G) = \prod_{uv\in E(G)} \frac{1}{d_G(u)d_G(v)}$$
$$= \left| E_{(2,2)}\right| \prod_{uv\in E_{(2,2)}} \frac{1}{d_G(u)d_G(v)} \times$$
$$\left| E_{(2m,2)}\right| \prod_{uv\in E_{(2m,2)}} \frac{1}{d_G(u)d_G(v)}$$
$$= \left[\frac{1}{2\times 2}\right]^{(n-2)m} \times \left[\frac{1}{2m\times 2}\right]^{2m}$$
$$= [4]^{-(n-2)m-2m} \times [m]^{-2m} = [4]^{-nm+2m-2m} \times [m]^{-2m}$$
$$= 4^{-mn} \times m^{-2m}$$

(iv) $M_1\pi(G,X) = M_1\pi\left(D_n^{(m)},X\right) = \prod_{uv\in E(G)} X^{[d_u+d_v]}$
$$= \left| E_{(2,2)}\right| \prod_{uv\in E_{(2,2)}} X^{[d_u+d_v]} \times$$
$$\left| E_{(2m,2)}\right| \prod_{uv\in E_{(2m,2)}} X^{[d_u+d_v]}$$
$$= \left(X^{[2+2]}\right)^{(n-2)m} \times \left(X^{[2m+2]}\right)^{2m}$$
$$= X^{4m^2+4m} \times X^{4nm-8m} = X^{4m^2+4m+4nm-8m} = X^{4m(m-1+n)}$$

Also $M_2\pi(G,X) = M_2\pi\left(D_n^{(m)},X\right) = \prod_{uv\in E(G)} X^{[d_u\times d_v]}$
$$= \left| E_{(2,2)}\right| \prod_{uv\in E_{(2,2)}} X^{[d_u\times d_v]} \times$$
$$\left| E_{(2m,2)}\right| \prod_{uv\in E_{(2m,2)}} X^{[d_u\times d_v]}$$
$$= \left(X^{[2\times 2]}\right)^{(n-2)m} \times \left(X^{[2m\times 2]}\right)^{2m}$$

$$= X^{8m^2} \times X^{4nm-8m} = X^{8m^2+4nm-8m} = X^{4m(2m-2+n)}$$

$$(v)\ AZI\pi(D_n{}^{(m)}) = \prod_{uv\in E(G)} \left(\frac{d_u d_v}{d_u+d_v-2}\right)^3$$

$$= |\ E_{(2,2)}|\ \prod_{uv\in\ E_{(2,2)}} \left(\frac{d_u d_v}{d_u+d_v-2}\right)^3 \times$$

$$|\ E_{(2m,2)}| \prod_{uv\in\ E_{(2m,2)}} \left(\frac{d_u d_v}{d_u+d_v-2}\right)^3$$

$$= \left[\left(\frac{2\times 2}{2+2-2}\right)^3\right]^{(n-2)m} \times \left[\left(\frac{2m\times 2}{2m+2-2}\right)^3\right]^{2m}$$

$$= \left[\left(\frac{2\times 2}{2+2-2}\right)^3\right]^{(n-2)m} \times \left[\left(\frac{2m\times 2}{2m+2-2}\right)^3\right]^{2m} =$$

$$8^{nm-2n} \times 8^{2m} = 8^{nm}$$

$$(vi)\ ABC\pi\left(D_n{}^{(m)}\right) = \prod_{uv\in E(D_n{}^{(m)})} \sqrt{\frac{d_u+d_v-2}{d_u d_v}}$$

$$= |\ E_{(2,2)}| \prod_{uv\in\ E_{(2,2)}} \sqrt{\frac{d_u+d_v-2}{d_u d_v}}$$

$$\times |\ E_{(2m,2)}|\ \prod_{uv\in\ E_{(2m,2)}} \sqrt{\frac{d_u+d_v-2}{d_u d_v}}$$

$$= \left[\sqrt{\frac{2+2-2}{2\times 2}}\right]^{(n-2)m} \times \left[\sqrt{\frac{2m+2-2}{2m\times 2}}\right]^{2m} =$$

$$\left[\frac{1}{\sqrt{2}}\right]^{\mathrm{nm}-2\mathrm{m}} \times \left[\frac{1}{\sqrt{2}}\right]^{2\mathrm{m}} = 2^{\frac{-nm}{2}+m-m} = 2^{\frac{-nm}{2}}$$

$$(vii) GA\pi(D_n{}^{(m)}) = \prod_{uv\in E(D_n{}^{(m)})} \frac{2\sqrt{d_u d_v}}{d_u+d_v}$$

$$= |\ E_{(2,2)}| \prod_{uv\in\ E_{(2,2)}} \frac{2\sqrt{d_u d_v}}{d_u+d_v} \times$$

$$|\ E_{(2m,2)}| \prod_{uv\in\ E_{(2m,2)}} \frac{2\sqrt{d_u d_v}}{d_u+d_v}$$

$$= \left[\frac{2\sqrt{2\times 2}}{2+2}\right]^{(n-2)m} \times \left[\frac{2\sqrt{2m\times 2}}{2m+2}\right]^{2m} = 1 \times \left[\frac{2\sqrt{4m}}{2(m+1)}\right]^{2m} = 2(\mathrm{m}^{\mathrm{m}})(\mathrm{m}+1)^{-2\mathrm{m}}$$

Now from Table:2 and Table:3 we calculate $GA_5\pi\left(D_n{}^{(m)}\right)$ and $ABC_4\pi\left(D_n{}^{(m)}\right)$ as follows:

$$(iii)\ ABC_4\pi\left(D_n{}^{(m)}\right) = \prod_{uv\in E(D_n{}^{(m)})} \sqrt{\frac{S_u+S_v-2}{S_u S_v}}$$

Case (1):If $n \geq 4$,then

$$ABC_4\pi\left(D_n{}^{(m)}\right) = |E^*{}_{(4,4),}| \prod_{uv\in E^*{}_{(4,4),}} \sqrt{\frac{S_u+S_v-2}{S_u S_v}} \times$$

$$|E^*{}_{(4,2m+2)}| \prod_{uv\in E^*{}_{(4,2m+2)}} \sqrt{\frac{S_u+S_v-2}{S_u S_v}} \times$$

$$|E^*{}_{(2m+2,4m)}| \prod_{uv\in E^*{}_{(2m+2,4m)}} \sqrt{\frac{S_u+S_v-2}{S_u S_v}}$$

$$= \left[\sqrt{\frac{4+4-2}{4\times 4}}\right]^{(n\ -\ 4)m} \times \left[\sqrt{\frac{4+(2m+2)-2}{4\times(2m+2)}}\right]^{2m} \times$$

$$= \left[\sqrt{\frac{3}{8}}\right]^{m(n-4)} \times \left[\sqrt{\frac{4+2m}{8m+8}}\right]^{2m} \times \left[\sqrt{\frac{6m}{8m^2+8}}\right]^{2m} \left[\sqrt{\frac{(2m+2)+4m-2}{(2m+2)\times 4m}}\right]^{2m}$$

$$= \left[\frac{3}{8}\right]^{\frac{m(n-4)}{2}} \times \left[\frac{2+m}{4(m+1)}\right]^{\frac{2m}{2}} \times \left[\frac{3}{4(m+1)}\right]^{\frac{2m}{2}} = \left[\frac{3}{8}\right]^{\frac{m(n-4)}{2}} \times \left[\frac{2+m}{4(m+1)}\right]^{m} \times \left[\frac{3}{4(m+1)}\right]^{m}$$

Case (2): $If \ \ n = 3$, then

$$ABC_4\pi(D_n{}^{(m)}) = \prod_{uv\in E(D_n{}^{(m)})} \sqrt{\frac{S_u+S_v-2}{S_uS_v}}$$

$$= |E^*{}_{(2m+2\ ,2m+2)}| \prod_{uv\in E^*{}_{(2m+2\ ,2m+2)}} \sqrt{\frac{S_u+S_v-2}{S_uS_v}} \times |E^*{}_{(2m+2,4m)}| \prod_{uv\in E^*{}_{(2m+2,4m)}} \sqrt{\frac{S_u+S_v-2}{S_uS_v}}$$

$$= \left[\sqrt{\frac{(2m+2)+(2m+2)-2}{(2m+2)\times(2m+2)}}\right]^{m} \times \left[\sqrt{\frac{(2m+2)+4m-2}{(2m+2)\times 4m}}\right]^{2m}$$

$$= \left[\sqrt{\frac{4m+4}{(2m+2)^2}}\right]^{m} \times \left[\sqrt{\frac{6m}{8\mathrm{m}^2+8m}}\right]^{2m}$$

$$= \left[\frac{2(2m+1)}{4(m+1)^2}\right]^{\frac{m}{2}} \times \left[\frac{3}{4(m+1)}\right]^{\frac{2m}{2}} = \left[\frac{(2m+1)}{2(m+1)^2}\right]^{\frac{m}{2}} \times \left[\frac{3}{4(m+1)}\right]^{m}$$

$$(iv) GA_5\pi(D_n{}^{(m)}) = \prod_{uv\in E(D_n{}^{(m)})} \frac{2\sqrt{S_uS_v}}{S_u+S_v}$$

Case (1):If $n \geq 4$,then

$$GA_5\pi(D_n{}^{(m)}) = |E^*{}_{(4,4),}| \prod_{uv\in E^*{}_{(4,4),}} \frac{2\sqrt{S_uS_v}}{S_u+S_v} \times$$

$$|E^*{}_{(4,2m+2)}| \prod_{uv\in E^*{}_{(4,2m+2)}} \frac{2\sqrt{S_uS_v}}{S_u+S_v} \times |E^*{}_{(2m+2,4m)}| \prod_{uv\in E^*{}_{(2m+2,4m)}} \frac{2\sqrt{S_uS_v}}{S_u+S_v}$$

$$= \left[\frac{2\sqrt{4\times 4}}{4+4}\right]^{(n-4)m} \times \left[\frac{2\sqrt{4\times(2m+2)}}{4+(2m+2)}\right]^{2m} \times \left[\frac{2\sqrt{(2m+2)\times 4m}}{(2m+2)+4m}\right]^{2m}$$

$$= \left[\sqrt{\frac{8}{8}}\right]^{m(n-4)} \times \left[\frac{2\sqrt{8m+8}}{2m+6}\right]^{2m} \times \left[\frac{2\sqrt{8m^2+8}}{6m+2}\right]^{2m}$$

$$= \left[\sqrt{\frac{8}{8}}\right]^{m(n-4)} \times \left[\frac{2\sqrt{8(m+1)}}{2(m+3)}\right]^{2m} \times \left[\frac{2\sqrt{8m(m+1)}}{2(3m+1)}\right]^{2m}$$

$$= 2^{4m} \times \left[\frac{\sqrt{2(m+1)}}{(m+3)}\right]^{2m} \times \left[\frac{\sqrt{2m(m+1)}}{(3m+1)}\right]^{2m}$$

Case (2): $If\ \ n = 3$, then

$$GA_5\pi\left(D_n{}^{(m)}\right) = \prod_{uv\in E(D_n{}^{(m)})} \frac{2\sqrt{S_u S_v}}{S_u+S_v} =$$

$$= \left|E^*{}_{(2m+2\ ,2m+2)}\right| \prod_{uv\in E^*{}_{(2m+2\ ,2m+2)}} \frac{2\sqrt{S_u S_v}}{S_u+S_v} \times \left|E^*{}_{(2m+2,4m)}\right| \prod_{uv\in E^*{}_{(2m+2,4m)}} \frac{2\sqrt{S_u S_v}}{S_u+S_v}$$

$$= \left[\frac{2\sqrt{(2m+2)\times(2m+2)}}{(2m+2)+(2m+2)}\right]^{m} \times \left[\frac{2\sqrt{(2m+2)\times 4m}}{(2m+2)+4m}\right]^{2m}$$

$$= \left[\frac{2\sqrt{(2m+2)^2}}{4m+4}\right]^{m} \times \left[\frac{2\sqrt{8m^2+8m}}{6m+2}\right]^{2m}$$

$$= \left[\frac{4(m+1)}{4(m+1)}\right]^{m} \times \left[\frac{2\sqrt{2m(m+1)}}{(3m+1)}\right]^{2m} = \left[\frac{2\sqrt{2m(m+1)}}{(3m+1)}\right]^{2m}$$

4. Conclusion

We have found the first and second Multiplicative Zagreb indices($\pi_1{}^*(G)$ and $\pi_2{}^*(G)$) , the first and second Multiplicative hyper Zagreb indices ($HM_1\pi(G)$ and $HM_2\pi(G)$), first and second multiplicative modified Zagreb indices$\left({}^{m}M_1(G) \text{and } {}^{m}M_2(G)\right)$, multiplicative Atom-bond connectivity index $(ABC\pi(G))$, multiplicative Geometric-arithmetic index $(GA\pi(G))$, the multiplicative first Zagreb Polynomial and second Zagreb polynomial $((M_1\pi(G,X)$ and $M_2\pi(G,X))$, multiplicative $ABC_4\pi(G)$ index , multiplicative $GA_5\pi(G)$ index and the multiplicative Augmented Zagreb index $(AZI\pi(G))$ for Dutch Windmill Graph.

In future we shall find the comparative study of certain topological indices of Dutch Windmill Graph.

References:

[1] R. Todeschini, V. Consonni, Handbook of Molecular Descriptors, Wiley-VCH, Weinheim, 2000.

[2] M. RandiĆ, Randic Indices of Graphs, J. Am. Chem. Soc.97, 6609 (1975).

[3] M. R. Rajesh Kanna, R. Pradeep Kumar, R. Jagadeesh, Computation of Topological Indices of Dutch Windmill Graph, Open Journal of Discrete Mathematics, 2016, 6, 74-81.

[4] I. Gutman, N. Trinajsti´c, Graph theory and molecular orbitals. Total π-electron energy of alternant hydrocarbons, Chem. Phys. Lett., 17(1972), 535–538.

[5] I.Gutman,K.C.Das,SomeProperties of the Second Zagreb Index, MATCH Commun.Math. Comput. Chem. 50 (2004) 103–112.

[6] S. NikoliĆ, G. KovaĆeviĆ, A. MiliĆeviĆ and N. TrinajstiĆ, The Zagreb indices 30 years after, Croatica Chemica Acta 76(2), (2003) 113-124.

[7] G.H.Shirdel, H. Rezapour and A.M. Sayadi, The hyper-Zagreb index of graph operations, Iranian J. Math. Chem. 4(2) (2013) 213-220.

[8] M.R. Farahani, M.R.R.Kanna and R. Pradeep Kumar, On the hyper-Zagreb indices of nanostructures, Asian Academic Research J. Multidisciplinary, 3(1), (2016).

[9] M.Eliasi, A.Iranmanesh and I. Gutman, Multiplicative versions of first Zagreb index, MATCH Commun. Math. Comput. Chem. 68(2012) 217-230.

[10] Vukicevic, D. and Furtula, B. (2009) Topological Index Based on the Ratios of Geometrical and Arithmetical Means of End-Vertex Degrees of Edges. Journal of Mathematical Chemistry, 46, 1369-1376.

[11] Graovac, A.. Ghorbani, M. and Hosseinzadeh, M.A. (2011) Computing Fifth Geometric-Arithmetic Index for Nanostar Dendrimers. Journal of Mahematical Nanoscience, 1, 33-42.

[12] Estrada, E., Torres, L., Rodrguez, L., Gutman, I.: An atom-bond connectivity index: modelling the enthalpy of formation of alkanes. Indian J Chem Sect. A 37, 849–855 (1998).

[13] Estrada, E.: Atom-bond connectivity and the energetic of branched alkanes. Chem. Phys. Lett. 463, 422–425 (2008).

[14] Chen, J., Liu, J., Guo, X.: Some upper bounds for the atom-bond connectivity index of graphs. Appl. Math. Lett. 25, 1077–1081 (2012).

[15] Das, K.C., Gutman, I., Furtula, B.: On atom-bond connectivity index. Filomat 26, 733–738 (2012).

[16] Furtula, B., Gutman, I., Ivanovi´c, M., Vukiˇcevi´c, D.:Computer search for trees with minimal ABC index. Appl. Math. Comput. 219, 767–772 (2012).

[17] Farahani, M.R. (2013) On the Fourth Atom-Bond Connectivity Index of Armchair Polyhex Nanotube. Proceedings ofthe Romanian Academy—Series B, 15, 3-6.

[18] Furtula, B., Graovac, A., Vukiˇcevi´c, D.: Augmented Zagreb index. J. Math. Chem. 48, 370–380 (2010).

[19] V. R. Kulli , Multiplicative Connectivity Indices of TUC4C8[m, n] and TUC4[m, n] Nanotubes, Journal of Computer and Mathematical Sciences, Vol.7(11), 599-605, November 2016.

[20] M.Eliasi, A.Iranmanesh and I. Gutman, Multiplicative versions of first Zagreb index, MATCH Commun. Math. Comput. Chem. 68(2012) 217-230.

[21] M.Bhanumathi, K.Easu Julia Rani, On some Multiplicative Topological Indices of Graphs, submitted

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