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Separation axioms in Soft Multi Topological Spaces

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Abstract: *The purpose of this paper is to study the concept of topological structure formed by soft multi sets. The notions of soft multi sets, soft multi open and closed set, neighbourhoods, interior and closure of a soft multi set etc., are to be studied. In addition, separation axioms such as T_0, T_1 and T_2 in soft multi topological spaces are introduced and their properties are analysed.*

Keywords: *soft sets, soft multi sets, soft multi topological spaces, soft multi open sets, soft multi closed sets, soft multi T_i-space (i=0, 1, 2).*
AMS Subject Classification: *54A40, 03E70*

1. Introduction

In 1999, Molodtsov [7] initiated soft set theory as a completely generic Mathematical tool for modelling vague concepts. In soft set theory there is no limited condition to the description of objects, so researchers can choose the form of parameters as they need, which greatly simplify the decision making process and make the process more efficient in the absence of partial information. In 2011, Alkhazaleh et al. [1] as a generalization of Molodtsov's soft set, the definition of a soft multiset, its basic operations such as complement, union and intersection etc., are introduced.

In 2011, Cagman et al. [3] introduced soft topology. Topological structure of soft set was introduced by Sabir and Naz [8]. They defined the soft topological spaces which were defined over a initial universe with a fixed set of parameter. Tokat and Osmanoglu [10] introduced soft multi topology. In 2013, Anjan Mukherjee et al. [2] introduced topological structure formed by soft multi sets and soft multi compact space. Recently Tantawy et al. [9] introduced separation axioms on soft topological spaces (X, τ, E) and study some of their properties. Separation axioms studied in some researchers [5] [6].

In this paper, we introduce separation axioms T_i (i = 0, 1, 2) on a soft multi topological space ((U,E), τ) and study their properties.

2. Preliminaries

Definition 2.1 [1]: Let U be an initial universe and E be a set of parameters. Let P(U) denote the power set of U and A be a nonempty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by F : A $\rightarrow$ P(U).

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In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in$ A, F(ε) may be considered as the set of ε - approximate elements of the soft set (F,A).

Definition 2.2 [2]: Let {U_i: i ∈ I} be a collection of universes and such that $\bigcap_{i\in I} U_i = \emptyset$ and let {E_{U_i} : i∈I} be a collection of sets of parameters. Let U = $\prod_{i\in I} P(U_i)$ where P(U_i) denotes the power set of U_i, E = $\prod_{i\in I} E_{U_i}$ and A ⊆ E. A pair (F, A) is called a soft multiset over U (briefly SMS(U,E)), where F is a mapping given by F : A ⟶ U.

Definition 2.3 [6]: A soft multi set (F, A) ∈ SM S(U, E) is called a soft multi point in (U, E), denoted by $e_{(F,A)}$, if for the element e ∈ A, F (e) ≠ ∅ and ∀e′∈ A − {e}, F (e′) = ∅.

Definition 2.4 [6]: A sub family τ of SMS(U, E) is called soft multi topology on (U, E), if the following axioms are satisfied:

(O_1): $\widetilde{\emptyset}$, $\widetilde{E}$ ∈ τ,

(O_2): The union of any number of soft multisets in τ belongs to τ , ie. for any {(F_k , A_k) : k ∈ K} ⊆ τ ⇒ $\widetilde{U}_{k\epsilon k}$ (F_k , A_k) ∈ τ ,

(O_3): If (F, A), (G, B) ∈ τ, then (F, A)$\widetilde{\cap}$ (G,B) ∈ τ.

Example 2.5: Let us consider there are three universes U_1, U_2 and U_3. Let U_1 = {h_1, h_2, h_3, h_4}, U_2 = {c_1, c_2, c_3} and U_3 = {v_1, v_2}. Let { E_{U_1}, E_{U_2}, E_{U_3} } be a collection of sets of decision parameters related to the above universes, where E_{U_1} = {$e_{u_1,1}$ = expensive, $e_{u_1,2}$ = cheap, $e_{u_1,3}$ = wooden, $e_{u_1,4}$ = in green surroundings}, E_{U_2} = { $e_{u_2,1}$ = expensive, $e_{u_2,2}$ = cheap, $e_{u_2,3}$ = sporty}, E_{U_3} = { $e_{u_3,1}$ = expensive, $e_{u_3,2}$ = cheap},

Let U = $\prod_{i=1}^{3} P(U_i)$, E = $\prod_{i=1}^{3} E_{U_i}$, A_1 = {e_1 = ($e_{u_1,1}$, $e_{u_2,1}$, $e_{u_3,1}$), e_2 = ($e_{u_1,1}$, $e_{u_2,2}$, $e_{u_3,1}$)} and A_2 = {e_1 = (($e_{u_1,1}$, $e_{u_2,1}$, $e_{u_3,1}$), e_3 = ($e_{u_1,2}$, $e_{u_2,3}$, $e_{u_3,1}$)}

Suppose that

(F_1, A_1) = {(e_1, ({h_1, h_2 }, {c_1, c_2 }, {v_1 })), (e_2, ({h_3, h_4 }, {c_1, c_3 }, {v_2 }))},

(F_2, A_2) = {(e_1 , ({h_1, h_3 }, {c_2, c_3 }, {v_1, v_2 })), (e_3, ({h_2, h_4 }, {c_1, c_2 }, {v_2 }))},

(F_3, A_3) = (F_1, A_1) $\widetilde{\cup}$ (F_2, A_2)

= {(e_1 , ({h_1, h_2, h_3}, {c_1, c_2, c_3}, {v_1, v_2})), (e_2, ({h_3, h_4)}, {c_1, c_3}, {v_2 })) ,

(e_3, ({h_2, h_4 }, {c_1, c_2}, {v_2 }))},

(F_4, A_4) = (F_1, A_1) $\widetilde{\cap}$ (F_2, A_2)

= {(e_1 , ({h_1 }, {c_2}, {v_1 })), (e_2 , ({h_3, h_4 }, {c_1, c_3}, {v_2 })) , (e_3, ({h_2, h_4}, {c_1, c_2 }, {v_2}))},

Where A_3 = A_4 = A_1 ∪ A_2 = { e_1 = ($e_{u_1,1}$, $e_{u_2,1}$, $e_{u_3,1}$), e_2 = ($e_{u_1,1}$, $e_{u_2,2}$, $e_{u_3,1}$), e_3 = ($e_{u_1,2}$, $e_{u_2,3}$, $e_{u_3,1}$)}

Then we observe that the subfamily τ_1 = { $\widetilde{\emptyset}$, $\widetilde{E}$, (F_1, A_1), (F_2, A_2), (F_3, A_3), (F_4, A_4)} of

SMS(U,E) is a soft multi topology on (U, E), since it satisfies the necessary three axioms (O_1), (O_2) and (O_3) and ((U, E), τ_1) is a soft multi topological space.

Definition 2.6 [6]: As every soft multi topology on (U, E) must contain the sets $\tilde{\emptyset}$ *and* $\tilde{E}$, so the family I = {$\tilde{\emptyset}$, $\tilde{E}$} forms a soft multi topology on (U, E). This soft multi topology is called indiscrete soft multi topology and the pair ((U, E), I) is called an indiscrete soft multi topological space.

Definition 2.7 [6]: Let D denote family of all the soft multi subsets of (U, E). Then we observe that D satisfies all the axioms for soft multi topology on (U, E). This soft multi topology is called discrete soft multi topology and the pair ((U,E), D) is called a discrete soft multi topological space.

Definition 2.8 [6]: Let ((U, E), τ_1) and ((U, E), τ_2) be two soft multi topological spaces. If each (F, A) $\in \tau_1 \Rightarrow$ (F, A) $\in \tau_2$, then τ_2 is called soft multi finer (stronger) topology than τ_1 and τ_1 is called soft multi coarser (or weaker) topology than τ_2 and denoted by $\tau_1 \overrightarrow{\sqsubseteq} \tau_2$.

Definition 2.9 [6]: Let ((U, E), τ) be a soft multi topological space over (U, E). A soft multi subset (F, A) of (U, E) is called soft multi closed if its relative complement $(F, A)'$ is a member of τ.

Definition 2.10 [6]: Let τ be the soft multi topology on (U, E). A soft multiset (F, A) in SMS(U,E) is a neighbourhood of a soft multi set (G,B) if and only if there exists an τ - open soft multi set (H,C) i.e. (H, C) $\in \tau$ such that (G, B) $\sqsubseteq$ (H, C) $\sqsubseteq$ (F, A).

Definition 2.11 [6]: Let ((U, E), τ) be a soft multi topological space on (U, E) and (F,A) be a soft multiset in SMS(U,E). Then the union of all softmulti open sets contained in (F,A) is called the interior of (F,A) and is denoted by int(F,A) and defined by

int(F, A) = $\tilde{\cup}$ {(G, B) : (G, B) is a soft multi open set contained in (F,A)}.

Definition 2.12 [6]: Let ((U, E), τ) be a soft multi topological space on (U,E) and (F,A) be a soft multiset in SMS(U,E). Then the intersection of all soft multi closed set containing (F,A) is called the closure of (F,A) and is denoted by cl(F,A) and defined by

cl(F, A) = $\tilde{\cap}$ {(G, B) : (G, B) is a soft multi closed set containing (F,A)}

Observe first that cl(F,A) is a soft multi closed set, since it is the intersection of soft multi closed sets. Furthermore, cl(F,A) is the smallest soft multi closed set containing (F,A).

Definition 2.13 [6]; Let ((U, E), τ) be an soft multi topological space on (U,E) and (F,A) be an soft multi set in SMS(U,E). Then the soft multi topology

$$\tau_{(F,A)} = \{(F, A) \tilde{\cap} (G, B) : (G, B) \in \tau \}$$ is called soft multi subspace topology and ((F, A), $\tau_{(F,A)}$) is called soft multi topological subspace of ((U, E), τ).

3. Separation Axioms in Soft Multi Topological Spaces

Definition 3.1: A soft multi topological space ((U, E), τ) is said to be soft multi T_0 -space if for every two soft multi points $e_{1_{(f_1,a_1)}}$, $e_{1_{(f_2,a_2)}}$ (Where (f_1, a_1) $\in$ (F_1, A_1) and (f_2, a_2) $\in$ (F_2, A_2)) such that (f_1, a_1) $\neq$ (f_2, a_2) there exists $G_E \in \tau$ such that $e_{1_{(f_1,a_1)}} \in G_E$, $e_{1_{(f_2,a_2)}} \notin G_E$ or there exists $H_E \in \tau$ such that $e_{1_{(f_2,a_2)}} \in H_E$, $e_{1_{(f_1,a_1)}} \notin H_E$.

Definition 3.2: A soft multi topological space ((U, E), τ) is said to be soft multi T_1-space if for every two soft multi points $e_{1_{(f_1,a_1)}}$, $e_{1_{(f_2,a_2)}}$ (Where (f_1, a_1) $\in$ (F_1, A_1) and (f_2, a_2) $\in$ (F_2, A_2)) such that (f_1, a_1) $\neq$ (f_2, a_2) there exists G_E, $H_E \in \tau$ such that $e_{1_{(f_1,a_1)}} \in G_E$, $e_{1_{(f_2,a_2)}} \notin G_E$ and $e_{1_{(f_2,a_2)}} \in H_E$, $e_{1_{(f_1,a_1)}} \notin H_E$.

Example 3.3: Let us consider there are three universes U_1 , U_2 and U_3. Let U_1 = {h_1, h_2, h_3, h_4}, U_2 = {c_1, c_2, c_3} and U_3 = {v_1, v_2}.
Let { E_{U_1}, E_{U_2}, E_{U_3} } be a collection of sets of decision parameters related to the above universes, Where E_{U_1} = { $e_{u_1,1}$ = expensive, $e_{u_1,2}$ = cheap, $e_{u_1,3}$ = wooden, $e_{u_1,4}$ = in green surroundings}, E_{U_2} = { $e_{u_2,1}$ = expensive, $e_{u_2,2}$ = cheap, $e_{u_2,3}$ = sporty }, E_{U_3} = { $e_{u_3,1}$ = expensive, $e_{u_3,2}$ = cheap },
Let U = $\prod_{i=1}^{3} P(U_i)$, E = $\prod_{i=1}^{3} E_{U_i}$, and A = {e_1 = ($e_{u_1,1}$, $e_{u_2,1}$, $e_{u_3,1}$), e_2 = ($e_{u_1,1}$, $e_{u_2,2}$, $e_{u_3,1}$)}

$$\tau = \{ \tilde{\emptyset} , \tilde{E}, (F_1, A), (F_2, A), (F_3, A), (F_4, A)\}$$

Suppose that

(F_1, A) = {(e_1, ({h_1, h_2}, {c_1, c_2}, {v_1})), (e_2, ({h_3, h_4}, {c_1, c_3}, {v_2}))},
(F_2, A) = {(e_1, ({h_1, h_3}, {c_2, c_3}, {v_1, v_2})), (e_2, ({h_2, h_4}, {c_1, c_2}, {v_2}))},
(F_3, A) = (F_1, A) $\tilde{\cup}$ (F_2, A)
= {(e_1, ({h_1, h_2, h_3}, {c_1, c_2, c_3}, {v_1, v_2})), (e_2, ({h_2, h_3, h_4}, {c_1, c_2, c_3}, {v_2}))},
(F_4, A) = (F_1, A) $\tilde{\cap}$ (F_2, A)
= {(e_1, ({h_1}, {c_2}, {v_1})), (e_2, ({h_4}, {c_1}, {v_2 }))},

Then ((U, E), τ) is a soft multi T_0 space because for the soft multi points $e_{1_{(f_1,a_1)}}$, $e_{1_{(f_2,a_2)}}$ (Where (f_1, a_1) $\in$ (F_1, A) and (f_2, a_2) $\in$ (F_1, A)) such that (f , a_1) $\neq$ (f_2, a_2) there exists (F_1, A) $\in \tau$ such that $e_{1_{(f_1,a_1)}} \in$ (F_1, A) but $e_{1_{(f_2,a_2)}} \notin$ (F_1, A) and for the soft multi points $e_{2_{(f_3,a_3)}}$, $e_{2_{(f_4,a_4)}}$ (Where (f_3, a_3) $\in$ (F_2, A) and

$(f_4, a_4) \in (F_2, A))$ such that $(f_3, a_3) \neq (f_4, a_4)$ there exists $(F_2, A) \in \tau$ such that $e_{2_{(f_3, a_3)}} \in (F_2, A)$, but $e_{2_{(f_4, a_4)}} \notin (F_2, A)$.

Theorem 3.4: A soft multi subspace topology of a soft multi T_0 space is soft multi T_0 space.

Proof: Let $((U, E), \tau)$ be a soft multi T_0 space and $((F, A), \tau_{(F,A)})$ be the soft multi subspace of $((U, E), \tau)$. Let $e_{1_{(f_1, a_1)}}$ and $e_{1_{(f_2, a_2)}}$ (Where $(f_1, a_1) \in (F_1, A)$ and $(f_2, a_2) \in (F_2, A))$ are two distinct points in (F,A). Since $((F, A), \tau_{(F,A)}) \subseteq ((U, E), \tau)$, then there exists $(G_1, B_1) \in \tau_{(F,A)}$, such that $e_{1_{(f_1, a_1)}} \in (G_1, B_1)$, $e_{1_{(f_2, a_2)}} \notin (G_1, B_1)$ or there exists $(G_2, B_2) \in \tau_{(F,A)}$, such that $e_{1_{(f_1, a_1)}} \notin (G_2, B_2)$, $e_{1_{(f_2, a_2)}} \in (G_2, B_2)$. Hence, $((F, A), \tau_{(F,A)})$ is a soft multi T_0 space.

Theorem 3.5: A soft multi subspace topology of a soft multi T_1 space is soft multi T_1 space.

Proof: The proof is similar to the proof of Theorem 3.4.

Theorem 3.6: If the soft multi point $e_{1_{(f,a)}}$ is a closed soft multi set $\forall\, e_i \in E$, then $((U, E), \tau)$ is a soft multi T_1 space.

Proof: Let $e_{1_{(f_1, a_1)}}$ and $e_{1_{(f_2, a_2)}}$ (Where $(f_1, a_1) \in (F_1, A)$ and $(f_2, a_2) \in (F_2, A))$ be two soft multi points over (U, E) such that $(f_1, a_1) \neq (f_2, a_2)$. From hypothesis $e_{1_{(f_1, a_1)}}$ and $e_{1_{(f_2, a_2)}}$ are closed soft multi sets. So $e_{1_{(f_1, a_1)}} \cap e_{1_{(f_2, a_2)}} = \emptyset$. Then $e_{1_{(f_1, a_1)}} \in (e_{1_{(f_2, a_2)}})^c$ and $e_{1_{(f_2, a_2)}} \in (e_{1_{(f_1, a_1)}})^c$. Since $e_{1_{(f_1, a_1)}}$ and $e_{1_{(f_2, a_2)}}$ are two closed soft multi sets. Then $(e_{1_{(f_1, a_1)}})^c$, $(e_{1_{(f_2, a_2)}})^c$ are two open soft multi sets.

Now $e_{1_{(f_1, a_1)}} \subseteq (e_{1_{(f_2, a_2)}})^c$, $e_{1_{(f_2, a_2)}} \nsubseteq (e_{1_{(f_2, a_2)}})^c$ and $e_{1_{(f_2, a_2)}} \subseteq (e_{1_{(f_1, a_1)}})^c$, $e_{1_{(f_1, a_1)}} \nsubseteq (e_{1_{(f_1, a_1)}})^c$. Hence $((U, E), \tau)$ is a soft multi T_1 space.

Remark 3.7: The converse of the above Theorem is not true.

Example 3.8: Let us consider the soft multi topological space, $\tau = \{\ \widetilde{\emptyset}\ , \widetilde{E}, F_E, G_E\ \}$ where F_E , G_E : $E_{U_i} \longrightarrow P(U_i)$ such that

$F_E(e_1) = \{\{h_1, h_2\}, \{c_1\}, \emptyset\}$

$F_E(e_2) = \{\{h_1, h_3\}, \{c_2, c_3\}, \{v_2\}$ and

$G_E(e_1) = \{\{h_1, h_3\}, \{c_2, c_3\}, \{v_2\}$

$G_E(e_2) = \{\{h_1, h_2\}, \{c_1\}, \emptyset\}$

Now $e_{1_{(f_1, a_1)}}$, $e_{1_{(f_2, a_2)}}$, $e_{2_{(f_1, a_1)}}$, $e_{2_{(f_2, a_2)}}$ are soft multi points.

For the soft multi points $e_{1_{(f_1, a_1)}}$, $e_{1_{(f_2, a_2)}}$, We have $e_{1_{(f_1, a_1)}} \in F_E$, $e_{1_{(f_2, a_2)}} \notin F_E$ and $e_{1_{(f_2, a_2)}} \in G_E$, $e_{1_{(f_1, a_1)}} \notin G_E$.

For the soft multi points $e_{2_{(f_1,a_1)}}$, $e_{2_{(f_2,a_2)}}$, We have $e_{2_{(f_1,a_1)}} \in G_E$, $e_{2_{(f_2,a_2)}} \notin G_E$ and $e_{2_{(f_2,a_2)}} \in F_E$, $e_{2_{(f_1,a_1)}} \notin F_E$. Then ((U, E), τ) is a soft multi T_1 space but $e_{1_{(f,\ a)}}$ is not soft multi closed set.

Theorem 3.9: A soft multi topological space ((U, E), τ) is a soft multi T_1 space if and only if $e_{1_{(f_1,a_1)}} = \bigcap\{G_E\,(e_i) : G_E \in \tau, e_{i_{(f,\ a)}} \in G_E\}$ for every soft multi point $e_{1_{(f,\ a)}}$ in (U,E).

Proof: Let ((U, E), τ) is a soft multi T_1 space and let $e_{1_{(f,\ a)}}$ be a soft multi point.

Then $e_{1_{(f_1,a_1)}} \subseteq \bigcap\{ G_E\,(e_i) : G_E \in \tau, e_{i_{(f,\ a)}} \in G_E\}$

we prove that $\bigcap\{G_E\,(e_i) : G_E \in \tau, e_{i_{(f,\ a)}} \in G_E\} \subseteq \{ e_{1_{(f_1,a_1)}}\}$

Indeed, let (h, a) $\in \bigcap\{G_E\,(e_i) : G_E \in \tau, e_{i_{(f,\ a)}} \in G_E\}$ and (f, a) $\neq$ (h, a).

Since ((U, E), τ) is a soft multi T_1 space there exists $H_E \in \tau$ such that $e_{1_{(f,\ a)}} \in H_E$ and $e_{1_{(f,\ a)}} \notin H_E$.

Hence (h, a) $\notin H(e_1)$ and therefore (h, a) $\notin \bigcap\{G_E\,(e_i) : G_E \in \tau, e_{i_{(f,\ a)}} \in G_E\}$, which is a contradiction.

Conversely, let $e_{1_{(f_1,a_1)}}$ and $e_{1_{(f_2,a_2)}}$ be two soft multi points such that $(f_1, a_1) \neq (f_2, a_2)$. Suppose that for every $G_E \in \tau$ with $e_{1_{(f_1,a_1)}} \in G_E$ we have $e_{1_{(f_2,a_2)}} \in G_E$.

Then $e_{1_{(f_2,a_2)}} \in \bigcap\{G_E(e_i) : G_E \in \tau, e_{i_{(f,\ a)}} \in G_E\} = \{e_{1_{(f_1,a_1)}}\}$ which is a contradiction. Therefore, there exists $G_E \in \tau$ such that $e_{1_{(f_1,a_1)}} \in G_E$ and $e_{1_{(f_2,a_2)}} \notin G_E$.

Theorem 3.10: Every soft multi T_1 space is a soft multi T_0 space.

Proof: The proof is straightforward.

Remark 3.11: The following example shows that the soft multi T_0 space may not be a soft multi T_1 space.

Example 3.12: Let us consider the soft multi topological space $\tau = \{\widetilde{\emptyset}\ , \widetilde{E}, (F_1, A_1)\}$ as in example 3.3, where, $(F_1, A_1) = \{(e_1, (\{h_1, h_2\}, \{c_1\}, \{v_1\}))\}$

Then ((U, E), τ) is a soft multi T_0 space but not soft multi T_1 space. Since $e_{1_{(f_1,a_1)}}$, $e_{1_{(f_2,a_2)}}$ (Where $(f_1, a_1) \in (F_1, A_1)$ and $(f_2, a_2) \in (F_2, A_2)$) are two distinct soft multi points, and the only open soft multi set which containing $e_{1_{(f_2,a_2)}}$ is (U,E) also contains $e_{1_{(f_1,a_1)}}$.

Hence ((U, E), τ) is not a soft multi T_1 space. On the other hand it is a soft multi T_0 space. Since for each two soft multi points $e_{1_{(f_1,a_1)}}$, $e_{1_{(f_2,a_2)}}$ and open soft multi set (F_1, A_1) ($e_{1_{(f_1,a_1)}} \in (F_1, A_1)$ but $e_{1_{(f_1,a_1)}} \notin (F_2, A_2)$).

Theorem 3.13: If ((U, E), τ) is a soft multi T_1 space, $\tau \le \tau^*$ (τ coarser than τ^*) then ((U, E), τ^*) is a soft multi T_1 space

Proof: The proof is straightforward.

Definition 3.14: A soft multi topological space ((U, E), τ) is a soft multi T_2 space if for every two soft multi points $e_{1_{(f_1, a_1)}}$, $e_{1_{(f_2, a_2)}}$ (Where (f_1, a_1) $\in$ (F_1, A_1) and (f_2, a_2) $\in$ (F_2, A_2)) such that (f_1, a_1) $\neq$ (f_2, a_2) there exists G_E, $H_E \in \tau$ such that $e_{1_{(f_1, a_1)}} \in G_E$, $e_{1_{(f_2, a_2)}} \in H_E$ and $G_E \cap H_E = \emptyset$.

Theorem 3.15: Every soft multi T_2 space is a soft multi T_1 space.

Proof: The proof is immediately follows from the definitions of 3.2 and 3.14.

Theorem 3.16: A soft multi topological space ((U, E), τ) is a soft multi T_2 space if and only if $e_{1_{(f_1, a_1)}} = \cap\{F_E(e_i) : F_E$ is soft multi closed neighbourhood of $e_{i_{(f,\ a)}}\}$.

Proof: Let ((U, E), τ) is a soft multi T_2 space and let $e_{1_{(f,\ a)}}$ be a soft multi point.

Then $e_{1_{(f,\ a)}} \subseteq \cap\{F_E(e_i) : F_E$ is soft multi closed neighbourhood of $e_{i_{(f,\ a)}}\}$. We prove that $\cap\{F_E(e_i) : F_E$ is soft multi closed neighbourhood of $e_{i_{(f,\ a)}}\} \subseteq \{e_{1_{(f,\ a)}}\}$.

Indeed let (f_2, a_2) $\in \cap\{F_E(e_i) : F_E$ is soft multi closed neighbourhood of $e_{1_{(f,\ a)}}\}$ and (f_1, a_1) $\neq$ (f_2, a_2). Since ((U, E), τ) is a soft multi T_2 space, there exists G_E, $H_E \in \tau$ such that $e_{1_{(f_1, a_1)}} \in G_E$, $e_{1_{(f_2, a_2)}} \in H_E$ and $G_E \cap H_E = \emptyset$. Then there exists $H_E^c \in \tau^c$ such that $e_{1_{(f_1, a_1)}} \in H_E^c$ and $e_{1_{(f_2, a_2)}} \in H_E^c$. Hence ($f_2$, a_2) $\in H_E^c(e_1)$ and therefore (f_2, a_2) $\in \cap\{F_E(e_i) : F_E$ is soft multi closed neighbourhood of $e_{i_{(f,\ a)}}\}$. This is a contradiction.

Let $e_{1_{(f_1, a_1)}}$, $e_{1_{(f_2, a_2)}}$ (Where (f_1, a_1) $\in$ (F_1, A_1) and (f_2, a_2) $\in$ (F_2, A_2)) be two soft multi points such that (f_1, a_1) $\neq$ (f_2, a_2). Then $e_{1_{(f_1, a_1)}} = \cap\{F_E(e_i) : F_E$ is a soft multi closed neighbourhood of $e_{i_{(f,\ a)}}\}$. It follows that, there exists a soft multi closed set F_E^* such that $e_{1_{(f_2, a_2)}} \notin F_E^*(e_1)$, $e_{1_{(f_1, a_1)}} \in F_E^*$. Also, $e_{1_{(f_2, a_2)}} \in (F_E^*)^c$ (1)

Since F_E is a soft multi closed neighbourhood of $e_{1_{(f_1, a_1)}}$, then by definition of soft multi neighbourhood there exists $G_E \in \tau$ such that $e_{1_{(f_1, a_1)}} \in G_E \subseteq F_E$ (2)

From (1) and (2) we get G_E, $(F_E^*)^c \in \tau$ such that $e_{1_{(f_1, a_1)}} \in G_E$, $e_{1_{(f_2, a_2)}} \in (F_E^*)^c$ and $G_E \cap (F_E^*)^c = \emptyset$. Hence ((U, E), τ) is a soft multi T_2 space.

Theorem 3.17: Every finite point set in a soft multi T_2 space is soft multi closed set.

Proof: It suffices to show that everyone soft multi point $\{e_{1_{(f_1, a_1)}}\}$ is closed. If $\{e_{1_{(f_2, a_2)}}\}$ is a soft multi point of ((U, E), τ) different from $\{e_{1_{(f_1, a_1)}}\}$ then $\{e_{1_{(f_1, a_1)}}\}$ and

$\{e_{1_{(f_2,a_2)}}\}$ have disjoint neighbourhoods of F_E and G_E respectively. Since G_E does not intersect the soft multi point $\{e_{1_{(f_1,a_1)}}\}$, the soft multi point $\{e_{1_{(f_2,a_2)}}\}$ cannot belong to the closure of the soft multi point set $\{e_{1_{(f_1,a_1)}}\}$. Then by definition of soft multi closure space, the closure of the soft multi point set $\{e_{1_{(f_1,a_1)}}\}$ is $\{e_{1_{(f_1,a_1)}}\}$ itself. So that it is soft multi closed set.

Conclusion:

Recently, many scientists have studied the soft multi set theory which is initiated by Alkzhaleh et al and easily applied to many problems having uncertainties from social life. In the present work, we investigate more properties of separation axioms on soft multi topological spaces. In the end, we must say that, this paper is just a beginning of a new structure and we have studied a few ideas only, it will be necessary to carry out more theoretical research to establish a general framework for the practical applications.

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