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Ratios of Polygonal Numbers as Continued Fractions

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Abstract: *Representations of rational numbers as continued fraction always exist. In number theory study of polygonal numbers has various approaches. Here in this paper we identify the patterns of continued fractions of ratios of polygonal numbers of consecutive order.*

Keywords: *Continued fractions, Simple continued fraction, Euclidean algorithm, Square numbers, Hexagonal numbers, Centered Hexagonal numbers, Star numbers.*

Notations:

1. $\langle a_0, a_1, a_2, a_3, \cdots a_n \rangle$: Continued fraction expansion.
2. $\left[\frac{n}{2}\right]$: Integer part of the rational number $n/2$.
3. $T_n = \frac{n(n+1)}{2}$: n^{th} Triangular number
4. $S_n = n^2$: n^{th} Square number
5. $H_n = 3n(n-1)+1$: n^{th} Centered Hexagonal number
6. $S_n^* = 6n(n-1)+1$: n^{th} Star number

1. Introduction

The Indian mathematician Aryabhata used a continued fraction to solve a linear indeterminate equation. For more than a thousand years, any work that used continued fractions was restricted to specific examples. Throughout Greek and Arab mathematical writing, we can find examples and traces of continued fractions. Euler showed that every rational can be expressed as a terminating simple continued fraction. He also provided an expression for e in continued fraction form. He used this expression to show that e and e^2 are irrational [3].

Polygonal numbers have graphical representation. Golden ratio which is an irrational also has a graphical representation. This idea motivated me to create a set of rational numbers using polygonal numbers and represent them in terms of continued

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fractions. First we give different representations of a rational number as a continued fraction [2, 3, 4, 5].

An expression of the form

$$\frac{p}{q} = a_0 + \cfrac{b_0}{a_1 + \cfrac{b_1}{a_2 + \cfrac{b_2}{a_3 + \cfrac{b_3}{\ddots}}}}$$

Where a_i, b_i are real or complex numbers is called a continued fraction.

An expression of the form

$$\frac{p}{q} = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{\ddots}}}}$$

Where $b_i = 1 \ \forall i$, and $a_0, a_1, a_2, \cdots$ are each positive integers also represents a simple continued fraction.

The continued fraction is commonly expressed as

$$\frac{p}{q} = a_0 + \frac{1}{a_1 +}\frac{1}{a_2 +}\frac{1}{a_3 +}\cdots \text{ or simply as } \langle a_0, a_1, a_2, a_3, \cdots \rangle.$$

The elements $a_0, a_1, a_2, a_3, \cdots$ are called the partial quotients. If there are finite number of partial quotients, we call it finite simple continued fraction, otherwise it is infinite. We have to use either Euclidean algorithm[1] or continued fraction algorithm to find such partial quotients. The finite simple continued fraction is denoted by $\langle a_0, a_1, a_2, a_3, \cdots a_n \rangle$ and it has an alternate form $\langle a_0, a_1, a_2, a_3, \cdots a_n - 1, 1 \rangle$.

1.1 The Continued Fraction Algorithm

Suppose we wish to find continued fraction expansion of $x \in R$.

Let $x_0 = x$ *and set* $a_0 = [x_0]$. Define $x_1 = \dfrac{1}{x_0 - [x_0]}$ *and set* $a_1 = [x_1]$,

$$\text{and } x_2 = \frac{1}{x_1 - [x_1]} \Rightarrow a_2 = [x_2], \ldots\ldots, x_k = \frac{1}{x_{k-1} - [x_{k-1}]} \Rightarrow a_k = [x_k], \ldots.$$

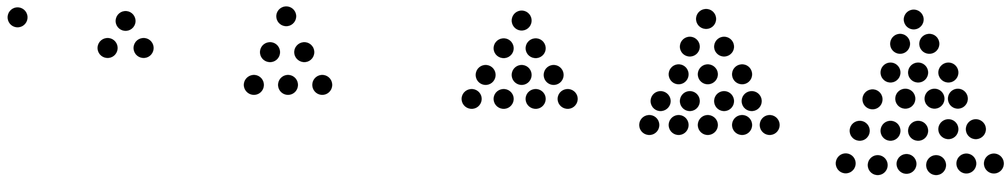
This process is continued infinitely or to some finite stage till an $x_i \in N$ exists such that $a_i = [x_i]$.

2. Triangular Numbers

Definition 2.1[5, 6]: Triangular Numbers

The numbers 1, 3, 6, 10, 15, 21, 28, 36, 45,..., $T_n = \frac{n(n+1)}{2}$,... are called triangular numbers, since the nth number counts the number of dots in an equilateral triangular array with n dots to the side.

$T_1 = 1$ $T_2 = 3$ $T_3 = 6$ $T_4 = 10$ $T_5 = 15$ $T_6 = 21$



Theorem: 2.1:

For $n \geq 3$,
$$\frac{T_n}{T_{n+1}} = \begin{cases} \left\langle 0; 1, \left[\frac{n}{2}\right], 2 \right\rangle & \text{when } n \text{ is odd.} \\ \left\langle 0;\ 1,\ \frac{n}{2} \right\rangle & \text{when } n \text{ is even.} \end{cases}$$

Proof:

Using algorithm 1.1 the proof follows

Since $T_n = \frac{n(n+1)}{2}$ and $T_{n+1} = \frac{(n+1)(n+2)}{2}$,

$$\frac{T_n}{T_{n+1}} = \frac{n}{n+2}$$

When n is even,
$$\frac{T_n}{T_{n+1}} = \frac{n}{n+2} = 0 + \frac{1}{\frac{n+2}{n}} = 0 + \frac{1}{1+\frac{2}{n}} = 0 + \frac{1}{1+\frac{1}{\frac{n}{2}}}.$$

When n is odd,
$$\frac{T_n}{T_{n+1}} = \frac{n}{n+2} = 0 + \frac{1}{\frac{n+2}{n}} = 0 + \frac{1}{1+\frac{2}{n}} = 0 + \frac{1}{1+\frac{1}{\left[\frac{n}{2}\right]+\frac{1}{2}}}.$$

Hence For each $n \geq 3$, the continued fraction expansion of $\frac{T_n}{T_{n+1}}$ is $\left\langle 0;\ 1, \frac{n}{2} \right\rangle$ when n is even and is $\left\langle 0; 1, \left[\frac{n}{2}\right], 2 \right\rangle$ when n is odd.

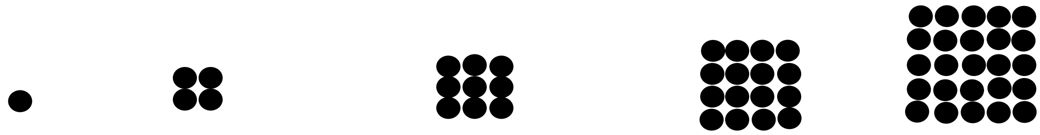
3. Square Numbers

Definition 3.1[5, 6]: Square Numbers

A square number or perfect square is an integer that is the square of an integer or it is the product of some integer with itself.

In other wards the numbers 1,4,9,16,25,36,49,64,81,..., $S_n = n^2$,... are called square numbers, since the nth number counts the number of dots in a square array with n dots to each side.

$S_1 = 1$ $S_2 = 4$ $S_3 = 9$ $S_4 = 16$ $S_5 = 25$



Theorem: 3 .1:

For $n \geq 3$,

$$\frac{S_n}{S_{n+1}} = \begin{cases} \left\langle 0; 1, \left[\frac{n}{2}\right], 3, 1, \left[\frac{n}{2}\right] \right\rangle & when\, n\, is\, odd. \\ \left\langle 0;\ 1, \frac{n}{2} - 1, 1, 3, \frac{n}{2} \right\rangle & when\, n\, is\, even. \end{cases}$$

Proof:

Using algorithm 1.1 the proof follows

Case (i): n is odd

Taking $n = 3$, $\frac{S_3}{S_4} = \frac{9}{16}$

Let $x_0 = \frac{9}{16}, so\, a_0 = 0.$

Then $x_1 = \frac{1}{x_0 - [x_0]} = \frac{16}{9} = 1 + \frac{7}{9} \Rightarrow a_1 = 1.$

$$x_2 = \frac{1}{x_1 - [x_1]} = \frac{9}{7} = 1 + \frac{2}{7} \Rightarrow a_2 = 1.$$

$$x_3 = \frac{1}{x_2 - [x_2]} = \frac{7}{2} = 3 + \frac{1}{2} \Rightarrow a_3 = 3.$$

$$x_4 = \frac{1}{x_3 - [x_3]} = \frac{2}{1} = 2 \Rightarrow a_3 = 2.$$

Therefore $\frac{9}{16} = \langle 0; 1, 1, 3, 2 \rangle$ or $\frac{9}{16} = \langle 0; 1, 1, 3, 1, 1 \rangle.$

Hence $\frac{S_3}{S_4}=\langle 0;1,1,3,1,1\rangle$. The result is true when $n=3$. The result is true for $n=2k-1,\ where\ 3\le k\le n.$

Then, $\frac{S_{2k-1}}{S_{2k}}=\langle 0;1,k-1,3,1,k-1\rangle$

We prove the result for $n=2k+1$.

Here we have to find the continued fraction of $\frac{S_{2k+1}}{S_{2k+2}}=\frac{(2k+1)^2}{(2k+2)^2}$.

Let $x_0=\frac{(2k+1)^2}{(2k+2)^2}=\frac{4k^2+4k+1}{4k^2+8k+4}$, so $a_0=0.$

Then
$$x_1=\frac{1}{x_0-[x_0]}=\frac{4k^2+8k+4}{4k^2+4k+1}=1+\frac{4k+3}{4k^2+4k+1}\Rightarrow a_1=1.$$

$$x_2=\frac{1}{x_1-[x_1]}=\frac{4k^2+4k+1}{4k+3}=k+\frac{k+1}{4k+3}\Rightarrow a_2=k.$$

$$x_3=\frac{1}{x_2-[x_2]}=\frac{4k+3}{k+1}=3+\frac{k}{k+1}\Rightarrow a_3=3.$$

$$x_4=\frac{1}{x_3-[x_3]}=\frac{k+1}{k}=1+\frac{1}{k}\Rightarrow a_4=1.$$

$$x_5=\frac{1}{x_4-[x_4]}=k\Rightarrow a_5=k.$$

Since $x_5=k$, $\Rightarrow\frac{S_{2k+1}}{S_{2k+2}}=\langle 0;1,k,3,1,k\rangle.$

Hence by induction the result is true for all values of n where n is odd.

Case(ii): n is even

Taking $n=4$, $\frac{S_4}{S_5}=\frac{16}{25}$

Let $x_0=\frac{16}{25}$, so $a_0=0.$

Then
$$x_1=\frac{1}{x_0-[x_0]}=\frac{25}{16}=1+\frac{9}{16}\Rightarrow a_1=1.$$

$$x_2=\frac{1}{x_1-[x_1]}=\frac{16}{9}=1+\frac{7}{9}\Rightarrow a_2=1.$$

$$x_3 = \frac{1}{x_2 - [x_2]} = \frac{9}{7} = 1 + \frac{2}{7} \Rightarrow a_3 = 1.$$

$$x_4 = \frac{1}{x_3 - [x_3]} = \frac{7}{2} = 3 + \frac{1}{2} \Rightarrow a_4 = 3.$$

$$x_5 = \frac{1}{x_4 - [x_4]} = 2 = \Rightarrow a_5 = 2.$$

Since $x_5 = 2, \quad \Rightarrow \frac{S_4}{S_5} = \langle 0; 1, 1, 1, 3, 2 \rangle.$

The result is true when $n = 4$. The result is true for $n = 2k - 2$, $where\, 3 \leq k \leq n.$

Then, $\frac{S_{2k-2}}{S_{2k-1}} = \langle 0; 1, k-2, 1, 3, k-1 \rangle.$

We prove the result for $n = 2k$.

Here we have to find the continued fraction of $\frac{S_{2k}}{S_{2k+1}} = \frac{(2k)^2}{(2k+1)^2}.$

Let $x_0 = \frac{(2k)^2}{(2k+1)^2} = \frac{4k^2}{4k^2 + 4k + 1}$, $so\ a_0 = 0.$

Then $x_1 = \frac{1}{x_0 - [x_0]} = \frac{4k^2 + 4k + 1}{4k^2} = 1 + \frac{4k+1}{4k^2} \Rightarrow a_1 = 1.$

$$x_2 = \frac{1}{x_1 - [x_1]} = \frac{4k^2}{4k+1} = k - 1 + \frac{3k+1}{4k+1} \Rightarrow a_2 = k - 1.$$

$$x_3 = \frac{1}{x_2 - [x_2]} = \frac{4k+1}{3k+1} = 1 + \frac{k}{3k+1} \Rightarrow a_3 = 1.$$

$$x_4 = \frac{1}{x_3 - [x_3]} = \frac{3k+1}{k} = 3 + \frac{1}{k} \Rightarrow a_4 = 3.$$

$$x_5 = \frac{1}{x_4 - [x_4]} = k \Rightarrow a_5 = k.$$

Since $x_5 = k, \quad \Rightarrow \frac{S_{2k}}{S_{2k+1}} = \langle 0; 1, k-1, 1, 3, k \rangle.$

Hence by induction the result is true for all values of n where n is even. Hence from case(i) and (ii) we have For each $n \geq 3$, the continued fraction expansion of

$\frac{S_n}{S_{n+1}}$ is $\left\langle 0; 1, \left[\frac{n}{2}\right], 3, 1, \left[\frac{n}{2}\right] \right\rangle$ when n is odd and is $\left\langle 0; 1, \frac{n}{2} - 1, 1, 3, \frac{n}{2} \right\rangle$ when n is even.

4. Centered Hexagonal Number

Definition 4.1[7]: Centered Hexagonal Number

A centered hexagonal number, or hex number, is a centered figurate number that represents a hexagon with a dot in the center and all other dots surrounding the center dot in a hexagonal lattice.

The nth centered hexagonal number is given by the formula

$$H_n = 3n(n-1) + 1$$

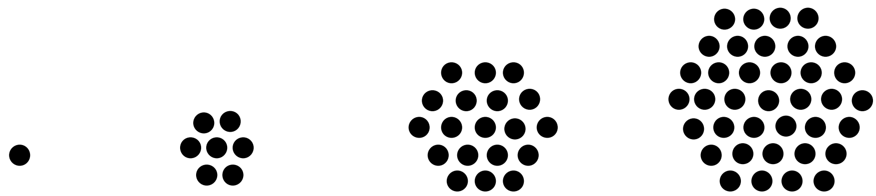
Another way of expressing the hex number as

$$H_n = 6\left[\frac{1}{2}n(n-1)\right] + 1$$

Shows that the centered hexagonal number for n is 1 more than 6 times the $(n - 1)^{th}$ triangular number. The first few centered hexagonal numbers are

1, 7, 19, 37, 61, 91, 127, 169, 217, 271, 331, 397, 469, 547, 631, 721, 817, 919

$H_1 = 1$ $H_2 = 7$ $H_3 = 19$ $H_4 = 37$



Theorem: 4.1:

For $n \geq 3$,

$$\frac{H_n}{H_{n+1}} = \begin{cases} \left\langle 0; 1, \left[\frac{n}{2}\right], 6n \right\rangle & when\, n\, is\, odd. \\ \left\langle 0; 1, \frac{n}{2} - 1, 1, 1, \frac{3n}{2} - 1, 2 \right\rangle & when\, n\, is\, even. \end{cases}$$

Proof:

Using algorithm 1.1 the proof follows

Case (i): n is odd

Taking $n = 3$, $\frac{H_3}{H_4} = \frac{19}{37}$

Let $x_0 = \frac{19}{37}$, so $a_0 = 0$.

Then $x_1 = \frac{1}{x_0 - [x_0]} = \frac{37}{19} = 1 + \frac{18}{19} \Rightarrow a_1 = 1.$

$x_2 = \frac{1}{x_1 - [x_1]} = \frac{19}{18} = 1 + \frac{1}{18} \Rightarrow a_2 = 1.$

$x_3 = \frac{1}{x_2 - [x_2]} = 18 \Rightarrow a_3 = 18.$

Therefore $\frac{19}{37} = \langle 0; 1, 1, 18 \rangle$

Hence $\frac{H_3}{H_4} = \langle 0; 1, 1, 18 \rangle.$

The result is true when $n = 3$. The result is true for $n = 2k - 1$, *where* $3 \leq k \leq n.$

Then, $\frac{H_{2k-1}}{H_{2k}} = \langle 0; 1, k-1, 6(2k-1) \rangle.$

We prove the result for $n = 2k + 1.$

Here we have to find the continued fraction of $\frac{H_{2k+1}}{H_{2k+2}} = \frac{12k^2 + 6k + 1}{12k^2 + 18k + 7}.$

Let $x_0 = \frac{(2k+1)^2}{(2k+2)^2} = \frac{12k^2 + 6k + 1}{12k^2 + 18k + 7}$, *so* $a_0 = 0.$

Then $x_1 = \frac{1}{x_0 - [x_0]} = \frac{12k^2 + 18k + 7}{12k^2 + 6k + 1} = 1 + \frac{12k + 6}{12k^2 + 6k + 1} \Rightarrow a_1 = 1.$

$x_2 = \frac{1}{x_1 - [x_1]} = \frac{12k^2 + 6k + 1}{12k + 6} = k + \frac{1}{12k + 6} \Rightarrow a_2 = k.$

$x_3 = \frac{1}{x_2 - [x_2]} = 12k + 6 \Rightarrow a_3 = 12k + 6.$

Since $x_3 = 12k + 6, \quad \Rightarrow \frac{H_{2k+1}}{H_{2k+2}} = \langle 0; 1, k, 6(2k+1) \rangle.$

Hence by induction the result is true for all values of n where n is odd.

Case(ii): n is even. Taking $n = 4$ then $\frac{H_4}{H_5} = \frac{37}{61}$

Let $x_0 = \frac{37}{61}$, *so* $a_0 = 0.$

Then
$$x_1=\frac{1}{x_0-[x_0]}=\frac{61}{37}=1+\frac{24}{37}\Rightarrow a_1=1.$$
$$x_2=\frac{1}{x_1-[x_1]}=\frac{37}{24}=1+\frac{13}{24}\Rightarrow a_2=1.$$
$$x_3=\frac{1}{x_2-[x_2]}=\frac{24}{13}=1+\frac{11}{13}\Rightarrow a_3=1.$$
$$x_4=\frac{1}{x_3-[x_3]}=\frac{13}{11}=1+\frac{2}{11}\Rightarrow a_4=1.$$
$$x_5=\frac{1}{x_4-[x_4]}=\frac{11}{2}=5+\frac{1}{2}\Rightarrow a_5=5.$$
$$x_6=\frac{1}{x_5-[x_5]}=2\Rightarrow a_6=2.$$

Since $x_2=2, \quad \Rightarrow \frac{H_4}{H_5}=\langle 0;1,1,1,1,5,2\rangle.$

The result is true when $n=4.$. The result is true for $n=2k-2,\ where\ 3\leq k\leq n.$

Then, $\frac{H_{2k-2}}{H_{2k-1}}=\langle 0;1,k-2,1,1,3k-4,2\rangle.$

We prove the result for $n=2k.$

Here we have to find the continued fraction of $\frac{H_{2k}}{H_{2k+1}}=\frac{12k^2-6k+1}{12k^2+6k+1}.$

Let $x_0=\frac{(2k)^2}{(2k+1)^2}=\frac{12k^2-6k+1}{12k^2+6k+1},\ so\ a_0=0.$

Then
$$x_1=\frac{1}{x_0-[x_0]}=\frac{12k^2+6k+1}{12k^2-6k+1}=1+\frac{12k}{12k^2-6k+1}\Rightarrow a_1=1.$$
$$x_2=\frac{1}{x_1-[x_1]}=\frac{12k^2-6k+1}{12k}=k-1+\frac{6k+1}{12k}\Rightarrow a_2=k-1.$$
$$x_3=\frac{1}{x_2-[x_2]}=\frac{12k}{6k+1}=1+\frac{6k-1}{6k+1}\Rightarrow a_3=1.$$
$$x_4=\frac{1}{x_3-[x_3]}=\frac{6k+1}{6k-1}=1+\frac{2}{6k-1}\Rightarrow a_4=1.$$
$$x_5=\frac{1}{x_4-[x_4]}=\frac{6k-1}{2}=3k-1+\frac{1}{2}\Rightarrow a_5=3k-1.$$

$$x_6 = \frac{1}{x_5 - [x_5]} = 2 \Rightarrow a_6 = 2.$$

Since $x_6 = 2, \quad \Rightarrow \frac{H_{2k}}{H_{2k+1}} = \langle 0; 1, k-1, 1, 1, 3k-1, 2 \rangle.$

Hence by induction the result is true for all values of n where n is even.

Hence from case(i) and (ii) we have For each $n \geq 3$, the continued fraction expansion of $\frac{H_n}{H_{n+1}}$ is $\langle 0; 1, \left[\frac{n}{2}\right], 6n \rangle$ when n is odd and is $\langle 0; 1, \frac{n}{2} - 1, 1, 1, \frac{3n}{2} - 1, 2 \rangle$ when n is even.

5. Star Numbers

Definition 5.1[7]: Star Numbers

A star number is a centered figurate number that represents a centered hexagram (six-pointed star), such as the one that Chinese checkers is played on.

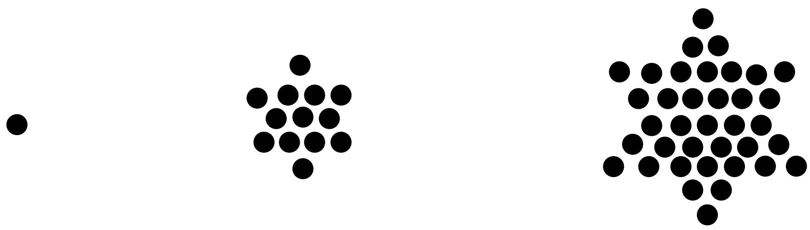
The *n*th star number is given by the formula

$$S_n^* = 6n(n-1) + 1$$

The first few star numbers are 1, 13, 37, 73, 121, 181, 253, 337, 433, 541

The digital root of a star number is always 1 or 4, and progresses in the sequence 1, 4, 1.

$S_1^* = 1$ $\qquad$ $S_2^* = 13$ $\qquad$ $S_3^* = 37$



Theorem: 5.1:

For $n \geq 3$,
$$\frac{S_n^*}{S_{n+1}^*} = \begin{cases} \langle 0; 1, \left[\frac{n}{2}\right], 12n \rangle & when\, n\, is\, odd. \\ \langle 0; 1, \frac{n}{2} - 1, 1, 1, 3n - 1, 2 \rangle & when\, n\, is\, even. \end{cases}$$

Proof : Similar to the proof of theorem 2

6. The following table summarizes the continued fraction expansion of consecutive fraction of some polygonal numbers.

Types of Numbers	Consecutive Fraction of Numbers	Continued Fractions	Nature of n
Triangular number	$\frac{T_n}{T_{n+1}}$	$\left\langle 0;1,\left[\frac{n}{2}\right],2\right\rangle$	Odd
Triangular number	$\frac{T_n}{T_{n+1}}$	$\left\langle 0;1,\frac{n}{2}\right\rangle$	Even
Square number	$\frac{S_n}{S_{n+1}}$	$\left\langle 0;1,\left[\frac{n}{2}\right],3,1,\left[\frac{n}{2}\right]\right\rangle$	Odd
Square number	$\frac{S_n}{S_{n+1}}$	$\left\langle 0;\ 1,\ \frac{n}{2}-1,1,3,\ \frac{n}{2}\right\rangle$	Even
Centered Hexagonal Number	$\frac{H_n}{H_{n+1}}$	$\left\langle 0;1,\left[\frac{n}{2}\right],6n\right\rangle$	Odd
Centered Hexagonal Number	$\frac{H_n}{H_{n+1}}$	$\left\langle 0;\ 1,\ \frac{n}{2}-1,1,1,\frac{3n}{2}-1,\ 2\right\rangle$	Even
Star Number	$\frac{S^*_n}{S^*_{n+1}}$	$\left\langle 0;1,\left[\frac{n}{2}\right],12n\right\rangle$	Odd
Star Number	$\frac{S^*_n}{S^*_{n+1}}$	$\left\langle 0;\ 1,\ \frac{n}{2}-1,1,1,3n-1,\ 2\right\rangle$	Even

7. Illustration

The following table gives the patterns of continued fraction of consecutive fractions of some polygonal numbers.

Consecutive fraction of numbers	Continued fraction expansion
T_7/T_8	$\left\langle 0;1,\ 3,2\right\rangle$
T_{94}/T_{93}	$\left\langle 0;1,\ 47\right\rangle$
S_{10}/S_{11}	$\left\langle 0;1,4,1,3,5\right\rangle$

S_{27}/S_{28}	$\langle 0;1,13,3,1,13\rangle$
H_5/H_6	$\langle 0;1,\ 2,\ 30\rangle$
H_{10}/H_{11}	$\langle 0;1,4,1,1,14,2\rangle$
S_9^*/S_{10}^*	$\langle 0;1,\ 4,108\rangle$
S_{18}^*/S_{19}^*	$\langle 0;1,\ 8,1,1,53,2\rangle$

8. Conclusion

In this paper we have identified various patterns of continued fractions of ratios of polygonal numbers of consecutive order and rank. This work may be extended to higher order figurate numbers like pyramidal numbers.

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