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# On Connected Eccentric Domination in Trees

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Abstract: *A subset D of the vertex set V(G) of a graph G is said to be a dominating set if every vertex not in D is adjacent to at least one vertex in D. A dominating set D is a connected dominating set, if < D > is a connected sub graph of G. For a Connected Graph G, a connected dominating set D is said to be a connected eccentric dominating set if for every $v \in V-D$, there exists at least one eccentric point of v in D. The minimum of the cardinalities of the connected eccentric dominating sets of G is called the connected eccentric domination number $\gamma_{ced}(G)$ of G. In this paper, characterization of trees with $\gamma_{ced}(T) = \gamma_c(T)+2$, $\gamma_{ced}(T) = \gamma_c(T)+1$ are studied and bounds for $\gamma_{ced}(T)$, its exact value for some particular classes of trees are found. Also, we analyze the bounds of connected eccentric domination number of a tree in terms of $\Delta(T)$, where the radius $r(T) \geq 2$.*

Keywords: *Eccentric domination, connected eccentric domination, connected eccentric domination number.*

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## 1. Introduction

Let G be a finite, simple, undirected connected graph on n vertices with vertex set V(G) and edge set E(G). For graph theoretic terminology refer to Harary [8], Buckley and Harary [6].

**Definition 1.1:** Let G be a connected graph and v be a vertex of G. The **eccentricity** e(v) of v is the distance to a vertex farthest from v. Thus, $e(v) = \max \{d(u, v) : u \in V\}$. The **radius** r(G) is the minimum eccentricity of the vertices, whereas the **diameter** diam(G) is the maximum eccentricity. For any connected graph G, $r(G) \leq diam(G) \leq 2r(G)$. v is a central vertex if e(v) = r(G). The **center C(G)** is the set of all central vertices. The central subgraph < C(G) > of a graph G is the subgraph induced by the center. v is a peripheral vertex if e(v) = d(G). The periphery P(G) is the set of all peripheral vertices.

For a vertex v, each vertex at a distance e(v) from v is an **eccentric vertex**. **Eccentric set of a vertex v** is defined as $E(v) = \{u \in V(G) / d(u, v) = e(v)\}$.

**Definition 1.2** [7, 9]: A set $S \subseteq V$ is said to be a **dominating set** in G, if every vertex in $V-S$ is adjacent to some vertex in S. A dominating set D is an **independent dominating set**, if no two vertices in D are adjacent that is D is an independent set. A dominating set D is a **connected dominating set**, if < D > is a connected sub graph of G. A set $D \subseteq V(G)$ is a **global dominating set**, if D is a dominating set in G and $\overline{G}$.

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**Definition: 1.3** [10]: A set D $\subseteq$ V(G) is an **eccentric dominating set** if D is a dominating set of G and for every v $\in$ V – D, there exists at least one eccentric point of v in D. If D is an eccentric dominating set, then every superset $D' \supseteq$ D is also an eccentric dominating set. But $D'' \subseteq$ D is not necessarily an eccentric dominating set. An eccentric dominating set D is a **minimal eccentric dominating set** if no proper subset $D'' \subseteq$ D is an eccentric dominating set.

**Definition: 1.4** [10]: The **eccentric domination number $\gamma_{ed}$(G)** of a graph G equals the minimum cardinality of an eccentric dominating set. That is, $\gamma_{ed}(G) = \min |D|$, where the minimum is taken over D in $\wp$, where $\wp$ is the set of all minimal eccentric dominating sets of G.

The various domination parameters introduced till now find many applications in covering of entire graph by the different sets with each of which has some specified property. These concepts are helpful to find centrally located sets to cover entire graph in which they are defined. The concept of eccentric set of a node has application in the location of farthest set of a vertex of a graph and hence in [10] we have defined new concept named eccentric domination and studied the structural properties of graph using this concept.

**Definition: 1.5** [10]: **Eccentric point set of G:**

Let S $\subseteq$ V(G). Then S is known as an eccentric point set of G if for every v $\in$ V–S, S has at least one vertex u such that u $\in$ E(v).

An eccentric point set S of G is a minimal eccentric point set if no proper subset $S'$ of S is an eccentric point set of G.

S is known as a minimum eccentric point set if S is an eccentric point set with minimum cardinality.

Let e(G) be the cardinality of a minimum eccentric point set of G.

e(G) can be called as **eccentric number of G.**

**Definition: 1.6** [1]: For a Connected Graph G, a connected dominating set D is said to be a **connected eccentric dominating set** if for every v $\in$ V−D, there exists at least one eccentric point of v in D. The minimum of the cardinalities of the connected eccentric dominating sets of G is called the **connected eccentric domination number $\gamma_{ced}$(G)** of G. Connected eccentric domination number is defined for connected graphs only. So, for the rest of this section, assume that G is a connected Graph.

V(G) is a connected eccentric dominating set for any connected graph G. Hence, $\gamma_{ced}(G)$ is an well defined parameter. Obviously, $\gamma(G) \leq \gamma_{ed}(G) \leq \gamma_{ced}(G)$ and $\gamma_c(G) \leq \gamma_{ced}(G) \leq \gamma_c(G) + e(G)$.

In [1], Bhanumathi has given many bounds for connected eccentric domination number of a graph. We need the following theorems for further study.

**Theorem: 1.1** [9]: For any tree T, $\gamma_c(T) \leq n - \Delta(T)$.

**Theorem: 1.2** [9]: For any tree T, $\gamma_c(T) = n - p$, where p is the number of pendant vertices of T.

**Theorem: 1.3** [9]; For any tree T, $\gamma_c(T) = n - \Delta(T)$ if and only if T has atmost one vertex of degree three or more.

**Theorem: 1.4** [1]; If d is the diameter of G, then $\gamma_{ced}(G) \geq d$.

**Theorem: 1.5** [1]: For a graph G, $\gamma_{ced}(G) \leq n - \Delta(G) + e(G)$ and $\gamma_{ced}(G) \leq n - \in_T(G) + e(G)$, where $\in_T(G)$ is the maximum number of pendant edges in any spanning tree of G.

**Theorem: 1.6** [1]: For a tree T, $\gamma_{ced}(T) = n - p + 1$ or $n - p + 2$, where p is the number of pendant vertices of T.

**Theorem: 1.7** [1]: For a tree T, $\gamma_{ced}(T) = n - p + 1$ if and only if there exists a peripheral vertex which is an eccentric vertex of every other pendant vertices, otherwise $\gamma_{ced}(T) = n - p + 2$, where p is the number of pendant vertices of T.

**Corollary: 1.7** [1]: $\gamma_{ced}(P_n) = n - 1$ for all n.

**Theorem: 1.8** [1]: $\gamma_{ced}(C_n) = n - 2$ for all n.

**Theorem 1.9** [1]: For any tree T, $\gamma_{ced}(T) \leq n - \Delta(T) + 2$.

## 2. New results on connected eccentric domination number of trees

Here, we proceed to study characterization of trees with $\gamma_{ced}(T) = \gamma_c(T) + 2$, $\gamma_{ced}(T) = \gamma_c(T) + 1$ and some bounds for $\gamma_{ced}(T)$ and its exact value for some particular classes of trees.

For any tree T if p is the number of pendant vertices of T, we have $\gamma_c(T) = n-p$ and no $\gamma_c$- set is a $\gamma_{ced}$-set. Hence $\gamma_{ced}(T) > \gamma_c(T)$. First, we shall find out the exact value of $\gamma_{ced}(T)$ for some particular classes of trees.

**Theorem: 2.1:** For a bi-central tree with radius 2, $\gamma_{ced}(T) = 3$ or 4 **.**

**Proof:** Let u and v be the central vertices of T, then N[u] and N[v] are connected eccentric dominating sets of T. $V-N(u) \cup \{v\}$, $V-N(v) \cup \{u\}$ are also connected eccentric dominating sets of T. Now, deg u + deg v = n and deg u or deg v is $\Delta(T)$. Hence, $\gamma_{ced}(T) \leq n - \Delta(T) + 1$. All the four vertices of a diametral path also form a connected dominating set. Hence $\gamma_{ced}(T) \leq \min\{n-\Delta(G)+1, 1+\Delta(T), 4\}$. Also $\gamma_{ced}(T) > 2$. Hence $\gamma_{ced}(T) = 3$ or 4.

**Corollary: 2.1:** (i) For a bicentral tree $T \neq P_4$ with radius 2, $\gamma_{ced}(T) = 3 = n - \Delta(T)+1$ if and only if T is a wounded spider having atmost one non wounded leg. In this case, T is a double star $\overline{K_m}+K_1+K_1+\overline{K_n}$ with $m \geq 2$ and $n = 1$.

(ii) For a bicentral tree T with radius 2, $\gamma_{ced}(T) = 4$ if and only if degree of both the central vertices are > 2. In this case, T is a double star $\overline{K_m}+K_1+K_1+\overline{K_n}$ with $m \geq 2$ and $n \geq 2$.

**Theorem: 2.2:** If G is a spider, then, $\gamma_{ced}(G) = 2 + \Delta(G) = n - \Delta(T)+1$.

**Proof:** Let G be a spider, and u be a vertex of maximum degree $\Delta(G)$. $|N[u]|$ vertices form a connected dominating set. $D = N[u] \cup \{v\}$, where v is a pendant vertex, is a minimum connected eccentric dominating set. Hence, $\gamma_{ced}(G) = |N[u]| + 1 = 2 + \Delta(G)$.

**Theorem: 2.3:** If G is a wounded spider, then $\gamma_{ced}(G) = s+2 = n - \Delta(T)+1$, where S is the set of support vertices which are adjacent to non-wounded legs and $s = |S|$.

**Proof:** Let G be a wounded spider. Let u be the vertex of maximum degree $\Delta(G)$, and S be the set of support vertices which are adjacent to non-wounded legs. The set $S \cup \{u\}$ form a connected dominating set. But it is not an eccentric dominating set. If G has more than one non-wounded leg, let v be a vertex of eccentricity four, otherwise let v be a vertex of eccentricity three, then $S \cup \{u\} \cup \{v\}$ form a minimum connected eccentric dominating set. Hence, $\gamma_{ced}(G) = s+2$.

**Theorem: 2.4:** $\gamma_{ced}(T) = n-1$ if and only if T is a path on n vertices.

**Proof:** We have for a tree T, $\gamma_{ced}(T) = n-p+1$ or $n-p+2$, where p is the number of pendant vertices of T.

**Case 1:** $\gamma_{ced}(T) = n-p+1$

$\gamma_{ced}(T) = n-1$ implies, $n-p+1 = n-1$. Hence $p = 2$. Thus T is a tree with exactly two pendant vertices. Therefore, T is a path on n vertices.

**Case 2:** $\gamma_{ced}(T) = n-p+2$

$\gamma_{ced}(T) = n-1$ implies, $n-p+2 = n-1$. Hence $p = 3$. Thus T is a tree with exactly three pendant vertices. But we know that any tree has atleast two peripheral vertices u and v at distance equal to the diameter of T. If w is the third pendant vertex, u or v is the eccentric vertex of w. If u is the eccentric vertex of w, $V-\{v, w\}$ is a connected eccentric dominating set of T. Hence $\gamma_{ced}(T) = n-2$, which is a contradiction to $\gamma_{ced}(T) = n-1$. Thus, $\gamma_{ced}(T) = n-p+2$ is not possible when $\gamma_{ced}(T) = n-1$.

Therefore, $\gamma_{ced}(T) = n-1$ if and only if T is a path on n vertices.

Next theorem gives an upper bound for $\gamma_{ced}(T)$.

**Theorem: 2.5:** For a tree T, $\gamma_{ced}(T) \leq n-\Delta(T)+1$.

**Proof:** For a tree T, we have $\gamma_c(T) \leq n-\Delta(T)$ and $\gamma_c(T) = n-\Delta(T)$ if and only if T has at most one vertex of degree three or more.

Let T be a tree with at most one vertex $v \in V(T)$ of degree three or more. Let u be an eccentric vertex of v. If S is the set of all pendant vertices of T, $(V-S)\cup\{u\}$ is a minimum connected eccentric dominating set of T. Hence, $\gamma_{ced}(T) = n-\Delta(T)+1$.

If T has more than one vertex of degree three or more, then $\gamma_c(T) < n-\Delta(T)$. Let D be a $\gamma_c$- set of T. Then $D\cup\{u, v\}$, where u and v are pendant vertices such that $d(u, v) = diam(T)$ form a connected eccentric dominating set of T. Hence, $\gamma_{ced}(T) < n-\Delta(T)+2$. Thus, we see that $\gamma_{ced}(T) \leq n-\Delta(T)+1$.

Following two theorems characterize trees with $\gamma_{ced}(T) = \gamma_c(T)+2$ and $\gamma_{ced}(T) = \gamma_c(T)+1$.

**Theorem: 2.6:** For a uni-central tree T with $r(T) \geq 3$, $\gamma_{ced}(T) = \gamma_c(T)+1 = n-p+1$ if and only if at least one component of $T-\{v\}$, where v is the central vertex, is a path on r(T) vertices, otherwise $\gamma_{ced}(T) = n-p+2$.

**Proof:** Let T be a uni-central tree of order n, and let S be the set of all pendant vertices of T and let $|S| = p$. Let $v \in V(T)$ be the central vertex. If deg $v = m$, $T - \{v\}$ has m sub trees ($m \geq 2$). If at least one component of $T-\{v\}$ is a path on r(T) vertices, then $D = (V-S) \cup \{x\}$, where x is a peripheral vertex of T and is an end vertex of this path form a connected eccentric dominating set. Hence, $\gamma_{ced}(T) = n-p+1$. If there doesn't exist such a path, then D = (V−S) ∪ {x, y}, where x and y are peripheral vertices of T with d(x, y) = diam(T) form a connected eccentric dominating set. Hence, $\gamma_{ced}(T)$ = n−p+2 = $\gamma_c(T)$+2.

Conversely, suppose $\gamma_{ced}(T)$ = n−p+1, by Theorem 1.7, there exists a peripheral vertex w, which is an eccentric vertex of every other pendant vertices. Thus, for any other pendant vertex s of T, d(s, w) > r(T) (otherwise w cannot be an eccentric vertex of s). This implies that there exists a sub tree of T, which is a component of T−{v} in which w lies is a path. This proves the theorem.

**Theorem: 2.7:** For a bi-central tree T with r(T) ≥ 3, $\gamma_{ced}(T)$ = n−p+1 if and only if at least one component of T−{u} or T−{v} is a path on r(T) vertices, where u and v are the central vertices of T, otherwise $\gamma_{ced}(T)$ = n−p+2.

**Proof:** Let T be a bi-central tree of order n, and let S be the set of all pendant vertices of T and let |S| = p. Let u, v ∈ V(T) be the central vertices of T. If deg v = m, T − {v} has m sub trees (m ≥ 2). If at least one component of T−{u} or T−{v} is a path on r(T) vertices, then D = (V−S) ∪ {x}, where x is a peripheral vertex of T and is an end vertex of this path form a connected eccentric dominating set. Hence, $\gamma_{ced}(T)$ = n−p+1. If there doesn't exist such a path, then D = (V−S) ∪ {x, y}, where x and y are peripheral vertices of T with d(x, y) = diam(T) form a connected eccentric dominating set. Hence, $\gamma_{ced}(T)$ = n−p+2.

Conversely, suppose $\gamma_{ced}(T)$ = n−p+1, by Theorem 1.7, there exists a peripheral vertex w, which is an eccentric vertex of every other pendant vertices. Thus, for any other pendant vertex s of T, d(s, w) > r(T) (otherwise w cannot be an eccentric vertex of s). This implies that there exists a sub tree of T, which is a component of T−{v} in which w lies is a path. This proves the theorem.

Next, we analyze the bounds of connected eccentric domination number of a tree in terms of Δ(T) with r(T) ≥ 2.

**Theorem: 2.8:** For a tree T with r(T) ≥ 2, $\gamma_{ced}(T)$ = n−Δ(T)+1 if and only if T is any one of the following: (i) T has at most one vertex of degree three or more. (ii) T has exactly one vertex u of degree three or more and exactly one vertex v of degree three and u, v lie on different sub trees of T−{x}, where x is a central vertex of T.

**Proof:** If T has no vertex of degree three or more, then T is a path and we have $\gamma_{ced}(T)$ = n−1= n−Δ(T)+1. So, assume that T has exactly **one vertex of degree three or more**.

**Case 1:** Let deg v = Δ(T) and v is a support vertex.

**Subcase 1:** v is a support vertex and all $\Delta(T)-1$ of its neighbors are pendent vertices.

In this case, T has exactly $\Delta(T)$ pendant vertices; $\Delta(T)-1$vertices adjacent to v and one vertex, which is eccentric to v. If S is the set of all pendant vertices adjacent to v, then $V-S$ is a minimum connected eccentric dominating set. Hence, $\gamma_{ced}(T) = n-\Delta(T)+1$.

**Subcase 2:** v is a support vertex and at least two of its neighbors are not pendant vertices.

In this case also T has exactly $p = \Delta(T)$ pendant vertices. Vertices of T are of degree two or one except v, and there is a pendant vertex u which is eccentric to v. Thus, if S is the set of all pendant vertices of T, $V-S\cup\{u\}$ is a minimum connected eccentric dominating set of T. Hence, $\gamma_{ced}(T) = n-\Delta(T)+1$.

**Case 2:** v is not a support vertex.

In this case also T has exactly $p = \Delta(T)$ pendant vertices. Vertices of T are of degree two or one except v, and there is a pendant vertex u, which is eccentric to v. Thus, if S is the set of all pendant vertices of T, $V-S\cup\{u\}$ is a minimum connected eccentric dominating set of T. Hence, $\gamma_{ced}(T) = n-\Delta(T)+1$.

Suppose T has a **vertex u of degree three or more and a vertex v of degree three** and u, v lie on different sub trees of $T-\{x\}$.

In this case, T has $p = \Delta(T)+1$ pendant vertices and since u, v lie on different sub trees of $T-\{x\}$, we need exactly two pendant (peripheral) vertices to dominate T eccentrically. Thus, $\gamma_{ced}(T) = n-(\Delta(T)+1)+2 = n-\Delta(T)+1$.

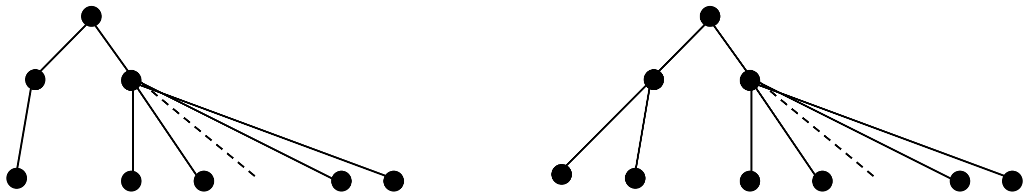
On the other hand, assume that $\gamma_{ced}(T) = n-\Delta(T)+1$.

For a tree T, $\gamma_{ced}(T) = n-p+1$ or $n-p+2$, where p is the number of pendant vertices of T. Therefore, $\gamma_{ced}(T) = n-\Delta(T)+1$ implies either $\Delta(T) = p$ or $\Delta(T)+1= p$.

Assume $\Delta(T) = p$. We have $p \geq 2$. If $p = 2$, $\Delta(T) = 2$. Thus T is a path. If $\Delta(T) > 2$, $\gamma_{ced}(T) = n-p+1$and $\Delta(T) = p$ implies T has at most one vertex of degree three or more (If T has more than one vertex of degree three or more then $p > \Delta(T)$).

Suppose $\Delta(T)+1= p$. $\gamma_{ced}(T) = n-p+2$ implies that T has a vertex u of degree three or more and a vertex v of degree three and u, v lie on different sub trees of $T-\{x\}$.

**Corollary: 2.8:** Let T be a tree with radius 2 and diameter 4. $\gamma_{ced}(T) = n-\Delta(T)+1$ if and only if any one of the following is true: (i) $T = P_5$. (ii) T is a wounded spider having at least two non wounded legs. (iii) T is any one of the following two types of trees.



**Theorem: 2.9:** For a tree T with $r(T) \geq 2$ and $\Delta(T) > 2$, $\gamma_{ced}(T) = n-\Delta(T)$ if and only if any one of the following is true:

(a) T has a vertex u of degree three or more and a vertex v of degree three such that u and v lie on same sub tree of T−{x}, where x is a central vertex of T.

(b) T has a vertex u of degree three or more and a vertex v of degree three such that u = x or v = x, where x is a central vertex of T.

(c) T has exactly one vertex of degree $\Delta(T)$ and exactly two vertices of degree 3 and any two vertices of degree $\geq 3$ must not lie on the same subtree of T−{x}, where x is a central vertex of T.

(d) T has exactly one vertex of degree $\Delta(T)$ and exactly one vertex of degree 4 and any two vertices of degree $\geq 3$ must not lie on the same subtree of T−{x}, where x is a central vertex of T.

**Proof:** Assume $\gamma_{ced}(T) = n-\Delta(T)$. This implies that $\gamma_c(T) < n-\Delta(T)$. Hence, by Theorem 1.3, T has more than one vertex of degree greater than or equal to 3. Also, we have, by Theorem: 1.6, $\gamma_{ced}(T) = n-p+1$ or $n-p+2$, where p is the number of pendent vertices of T. Thus two cases arise:

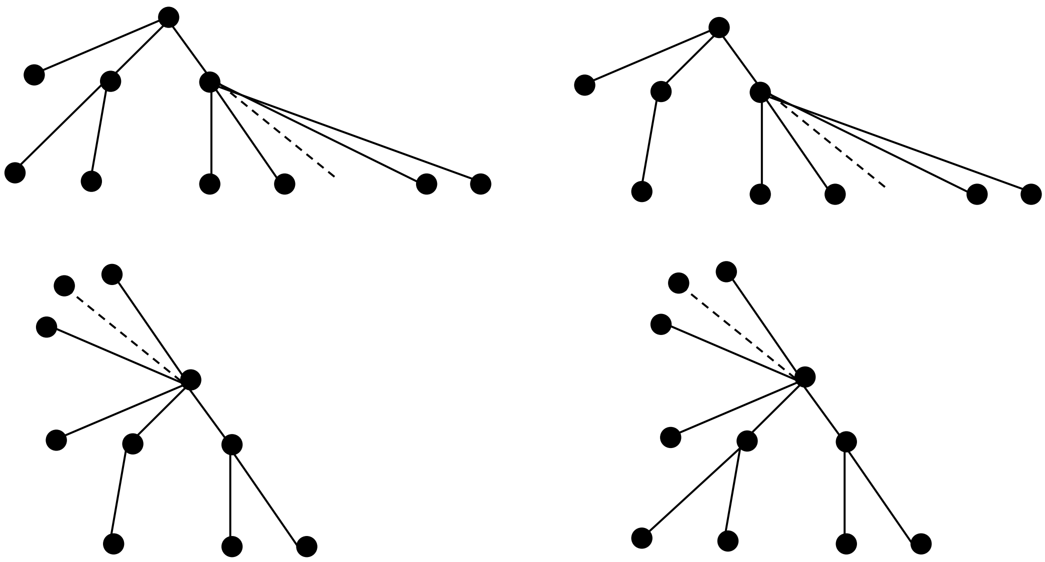
**Case (i):** $\gamma_{ced}(T) = n-p+1 = n-\Delta(T)$.

In this case, $\Delta(T) = p-1$. $p = 1+\Delta(T)$. We have always $p \geq \Delta(T)$, and $p = \Delta(T)$ if T has exactly only one vertex of degree $\Delta(T)$. Therefore, $p = 1+\Delta(T)$ implies that T has exactly one vertex of degree $\Delta(T)$ and another vertex of degree three, all other vertices are of degree $\leq 2$. Hence, by Theorems 2.6 and 2.7, T satisfies (a) or (b).

**Case (ii):** $\gamma_{ced}(T) = n-p+2 = n-\Delta(T)$.

In this case, $\Delta(T) = p-2$. $p = 2+\Delta(T)$. We have always $p \geq \Delta(T)$, and $p = \Delta(T)$ if T has exactly only one vertex of degree $\Delta(T)$. Therefore, $p = 2+\Delta(T)$ implies that T has exactly one vertex of degree $\Delta(T)$ and another vertex of degree four or T has exactly one vertex of degree $\Delta(T)$ and exactly two vertices of degree three, all other vertices are of degree $\leq 2$. Hence, again by Theorems 2.6 and 2.7, T satisfies (c) or (d).

**Corollary: 2.9:** Let T be a tree with radius 2 and diameter 4. $\gamma_{ced}(T) = n - \Delta(T)$ if and only if T is any one of the following four types of trees:



**Theorem: 2.10:** For any connected graph G, $\gamma_{ced}(G) \leq 2m - n+1$. Equality holds if and only if G is a path.

**Proof:** We have $\gamma_{ced}(G) \leq n-1 = 2(n-1)-(n-1) \leq 2m-n+1$, since $m \geq n-1$ for a connected graph G. Second part follows from Theorem 2.4.

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**Kavitha M** was born in India in 1983. She received her B.Sc., M.Sc., and M.Phil degrees in Mathematics from Bharathidasan University, Tiruchirappalli, India in 2004, 2006 and 2007 respectively. She worked as Assistant Professor in Chendhuran College of Engineering and Technology, Pudukkottai and Shanmuganathan Engineering College, Arasampatti, Pudukkottai, India from 2008 to 2013 and from 2016 to 2017 respectively. Her area of interest in research is domination in graphs and graph operations in Graph Theory.