International Journal of Engineering Science, Advanced Computing and Bio-Technology
Vol. 8, No. 3, July – September 2017, pp. 119 - 125

Complementary Tree Nil Domination Number of Cubic Graphs

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Abstract:*A set D of a graph G = (V, E) is a dominating set, if every vertex in V(G) - D is adjacent to some vertex in D. The domination number* γ *(G) of G is the minimum cardinality of a dominating set. A dominating set D is called a complementary tree nil dominating set, if V(G) - D is not a dominating set and also the induced subgraph < V(G) - D> is a tree. The minimum cardinality of a complementary tree nil dominating set is called the complementary tree nil domination number of G and is denoted by* γ_{ctnd} *(G). In this paper, some results regarding the complementary tree nil domination number of connected cubic graphs are found.*

Keywords: *domination number, complementary tree nil domination number, cubic graph.*

1.Introduction

Graphs discussed in this paper are finite, undirected and simple graphs. For a graph G, let V(G) and E(G) denote its vertex set and edge set respectively. A graph G with p vertices and q edges is denoted by G(p, q). The concept of domination in graphs was introduced by Ore[4]. A set D $\subseteq$ V(G) is said to be a dominating set of G, if every vertex in V(G) $-$ D is adjacent to some vertex in D. The cardinality of a minimum dominating set in G is called the domination number of G and is denoted by γ (G). Muthammai, Bhanumathi and Vidhya[3] introduced the concept of complementary tree dominating set. A dominating set D $\subseteq$ V(G) is said to be a complementary tree dominating set (ctd-set) if the induced subgraph <V(G) $-$D> is a tree. The minimum cardinality of a ctd-set is called the complementary tree domination number of G and is denoted by γ_{ctd} (G).

The concept of complementary tree nil dominating set is introduced in [2]. A dominating set D $\subseteq$ V(G) is said to be a complementary tree nil dominating set (ctnd-set) if the induced subgraph <V(G) $-$ D> is a tree and V(G) – Dis not a dominating set. The minimum cardinality of a ctnd-set is called the complementary tree nil domination number of G and is denoted by γ_{ctnd} (G). In this paper, some results regarding the complementary tree nil domination number of connected cubic graphs are found.Any undefined terms in this paper may be found in Harary[1].

Received: 24 March, 2017; Revised: 14 June, 2017; Accepted: 24 July, 2017
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2. Prior Result

Theorem 2.1.[2] Let G be a connected graph with p vertices. Then $\gamma_{ctnd}(G) = 2$ if and only if G is a graph obtained by attaching a pendant edge at a vertex of degree p - 2 in T + K_1, where T is a tree on (p - 2) vertices.

3. Main Results

Theorem 3.1:

For any connected cubic graph G with atleast 4 vertices, $\gamma_{ctnd}(G) \geq 4$.

Proof.

By Theorem 2.1, $\gamma_{ctnd}(G) = 2$ if and only if G is a graph obtained by attaching a pendant edge at a vertex of degree (p – 2) in T + K_1, where T is a tree on (p – 2) vertices.

Therefore $\gamma_{ctnd}(G) \geq 3$.

Case 1: $\gamma_{ctnd}(G) = 3$.

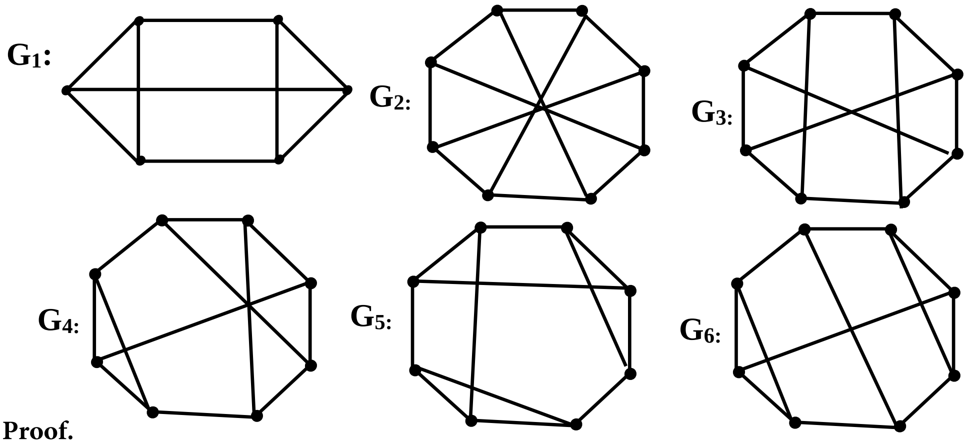
Let D be a ctnd-set of G with |D| = 3. Then <D> is one of the following graphs :C_3, P_3, $K_2 \cup K_1$ and $3K_1$.

Therefore, there exists no vertex $u \in D$ such that $N(u) \subseteq D$, since $\deg_{<D>}^{(u)} \neq 3$.

Therefore $\gamma_{ctnd}(G) \neq 3$ and hence $\gamma_{ctnd}(G) \geq 4$.

Theorem 3.2:

If G be a connected cubic graph with atleast 6 vertices. Then $\gamma_{ctnd}(G) = 4$ if and only if $G \cong K_4, G_1, G_2, G_3, G_4, G_5$ and G_6, where



Proof.

Let D be a ctnd-set of G with |D| = 4.

Since D is a dominating set, <V – D> is a tree and since G is cubic, each vertex in <V – D> is of degree 1 or 2.

Therefore <V – D> is a path.

Since G has an even number of vertices, the number of vertices in <V – D> is even.

Let <V – D> have six vertices. Since G is cubic and |V(G)| = 10, V(G) has 15 edges and each vertex in <D> is of degree atmost 3.

Therefore, <D> has t edges, where $3 \leq t \leq 6$ and <V – D> has 5 edges and hence number of edges between D and V – D is atleast 8. There exists no cubic graph in this case.

Similar is the case, when <V – D> has more than m vertices where m is even and $m \geq 8$.

Case 1: $|V - D| = \phi$

Then $<D> \cong K_4$. Therefore $G \cong K_4$.

Case 2: $|V - D| = 2$

Then $<V - D> \cong K_2$ and the number of edges between D and V – D is 4.

Since G has 6 vertices and 9 edges, number of edges in <D> is $|E(G)| - |E(K_2)| - 4 = 4$.

Since there exists a vertex $u \in D$ such that $N(u) \subseteq D$, <D> is a graph obtained by attaching a pendant edge at a vertex of C_3. Therefore $G \cong G_1$.

Case 2: $|V - D| = 4$.

Then $<V - D> \cong P_4$ and the number of edges between D and V – D is 6.

Since G has 8 vertices and 12 edges, number of edges in <D> is $|E(G)| - |E(P_4)| - 6 = 3$.

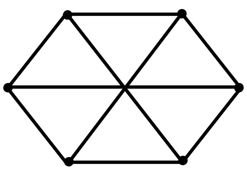
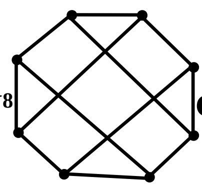
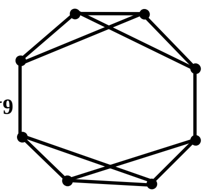
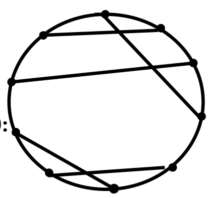
Since there exists a vertex $u \in D$ such that $N(u) \subseteq D$, Therefore $<D> \cong K_{1,3}$.

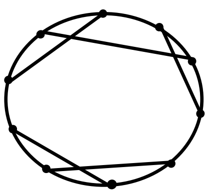
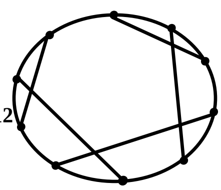
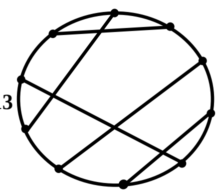
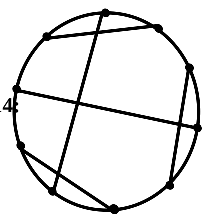
Then G is one of the cubic graphs: G_2, G_3, G_4, G_5 and G_6

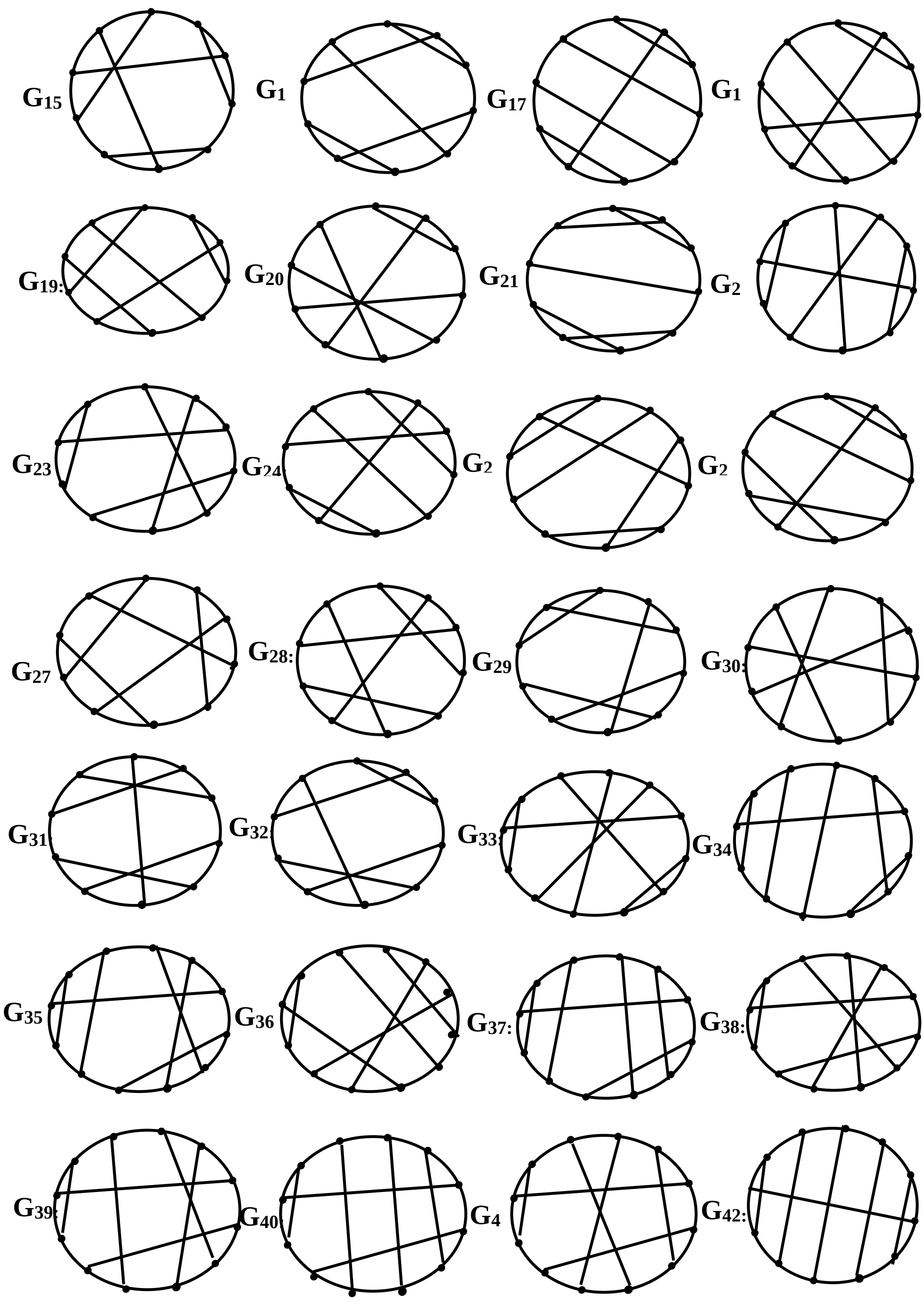
Conversely, if $G \cong K_4, G_1, G_2, G_3, G_4, G_5$ and G_6, then $\gamma_{ctnd}(G) = 4$.

Theorem 3.3:

If G be a connected cubic graph with atleast 6 vertices. Then $\gamma_{ctnd}(G) = 5$ if and only if G is one of the following graphs: G_i, i= 7,8, …, 60, where

G_7

G_8

G_9

G_{10}:


G_1

G_{12}

G_{13}

G_{14}



G_{15}
G_1
G_{17}
G_1
G_{19}:
G_{20}
G_{21}
G_2
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G_{24}
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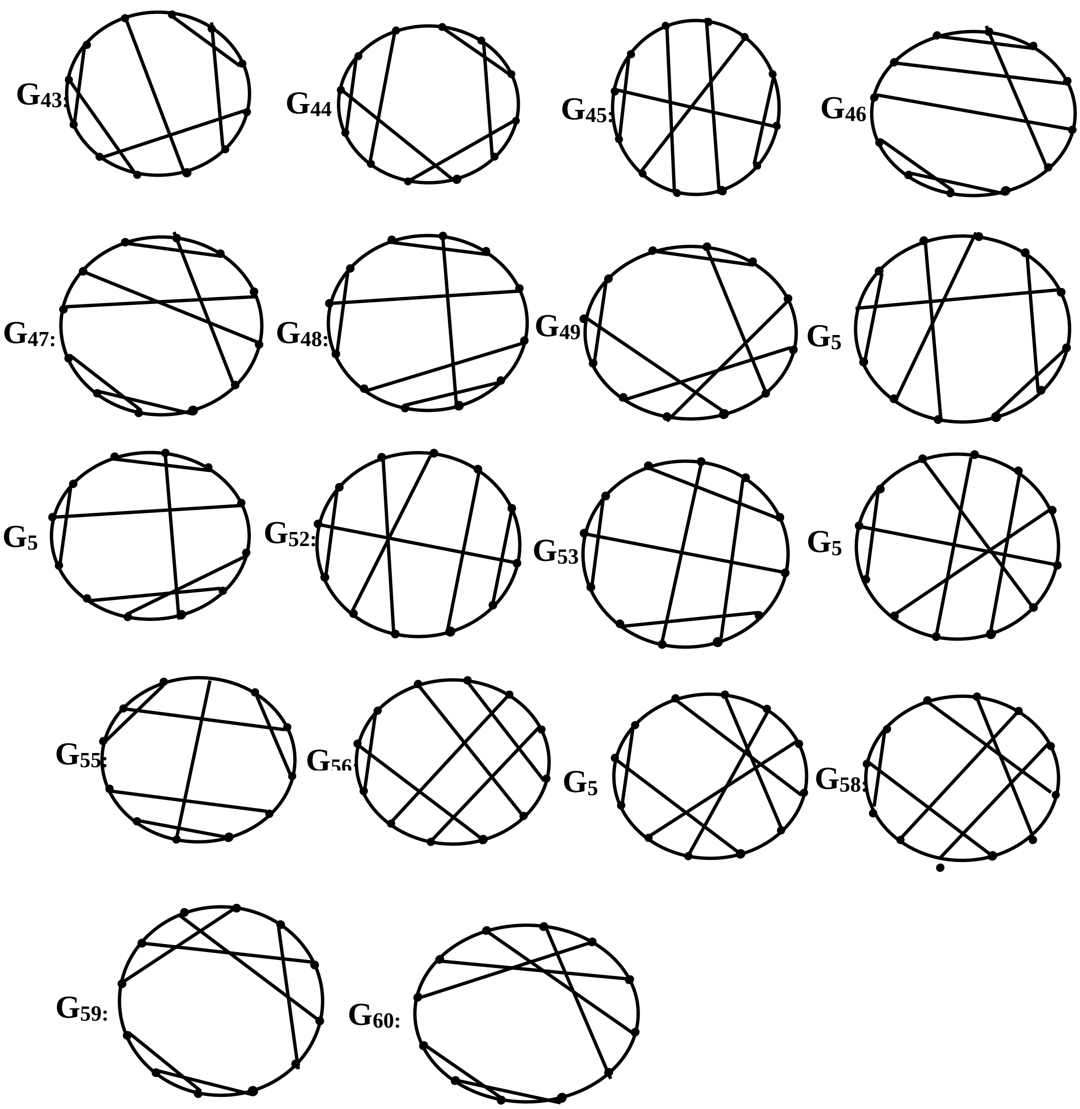


Proof.

Let D be a ctnd-set of G with |D| = 5.

Since D is a dominating set, <V – D> is a tree and since G is cubic, each vertex in <V – D> is of degree 1 or 2.

Therefore <V – D> is a path.

Since G has an even number of vertices, the number of vertices in <V – D>is odd.

Let<V – D> have seven vertices. Since G is cubic and |V(G)| = 14, V(G) has 21 edges and each vertex in <D> is of degree atmost 3.

Therefore, <D> has t edges, where $3 \leq t \leq 6$ and <V – D> has 9 edges and hence number of edges between D and V – D is atleast 11. There exists no cubic graph in this case.

Similar is the case, when <V – D> has more than m vertices where m is odd and $m \geq 13$. Therefore, <V – D> is nonempty and has atmost 7 vertices.

Case 1: $|V - D| = 1$.

Then the vertex in V – D is adjacent to three vertices in D.

Therefore, <D> is a graph on 5 vertices in which 2 vertices have degree 3 and 3 vertices have degree 2.Therefore $G \cong G_1$ or G_7.

But $\gamma_{ctnd}(G_1) = 4$. Therefore $G \cong G_7$.

Case 2: $|V - D| = 3$.

Then $\langle V - D \rangle \cong P_3$ and the number of edges between D and V – D is 5.

Since G has 8 vertices and 12 edges, number of edges in <D> is $|E(G)| - |E(P_3)| - 5 = 5$.

Since there exists a vertex $u \in D$ such that $N(u) \subseteq D$, <D> is one of the following graphs

(i) <D> is obtained from C_3 by attaching a pendant edge at two vertices of C_3.

(ii) <D> is obtained from C_4 by attaching a pendant edge at a vertex of C_4.

(iii) <D> is obtained from C_3 by attaching a path of length 2 at a vertex of C_3.

(iv) $\langle D \rangle \cong (K_4 - e) \cup K_1$.

Subcase 2.1: <D> is obtained from C_3 by attaching a pendant edge at two vertices of C_3.

Then G is one of the following cubic graphs: G_4, G_5 and G_6.

But in this case, $\gamma_{ctnd}(G) = 4$

Subcase 2.2: <D> is obtained from C_4 by attaching a pendant edge at a vertex of C_4.

Then G is one of the following cubic graphs: G_4, G_6 and G_8.

But if $G \cong G_4$ or G_6, then $\gamma_{ctnd}(G) = 4$.

Therefore $G \cong G_8$.

Subcase 2.3:<D> is obtained from C_3 by attaching a path of length 2 at a vertex of C_3.

Then G is one of the following cubic graphs: G_4, G_6, G_8 and G_9.

If $G \cong G_4$ or G_6, then $\gamma_{ctnd}(G) = 4$.

Therefore $G \cong G_8, G_9$.

Subcase 2.4: $\langle D \rangle \cong (K_4 - e) \cup K_1$.

Then $G \cong G_9$.

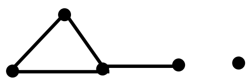
Case 3: $|V - D| = 5$.

Then $\langle V - D \rangle \cong P_5$ and $|E(\langle V - D \rangle)| = 4$.

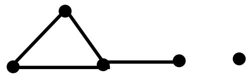
G has 10 vertices and 15 edges and number of edges between D and V – D is 7.

Therefore, $|E(\langle D \rangle)| = 15 - (7 + 4) = 4$. Also, there exists a vertex $u \in D$ such that $N(u) \subseteq D$, Therefore <D> has atleast one vertex of degree 3 in <D>.

Therefore, <D> is one of the following graphs:

(i) $\langle D \rangle \cong$ 

(ii) <D> is obtained from P_3 by attaching two pendant edges at a pendant vertex of P_3.

Subcase 3.1:<D> ≅ 

Then G is one of the cubic graphs on 10 vertices: G_i, (i=10,11, …,23)

Subcase 3.2: <D> is obtained from P_3 by attaching two pendant edges at a pendant vertex of P_3.

Then G is one of the cubic graphs: G_{10}, G_{16}, G_i, i=22, 23, …, 32.

Case 4: $|V - D| = 7$.

Then $\langle V - D\rangle \cong P_7$ and the number of edges between D and $V - D$ is 9.

Since G has 12 vertices and 18 edges, number of edges in <D> is $|E(G)| - |E(P_7)| - 9 = 3$.

Since there exists a vertex $u \in D$ such that $N(u) \subseteq D$, $\langle D\rangle \cong K_{1,3} \cup K_1$.

Then G is one of the cubic graphs on 12 vertices: G_i, i=33, 34, …, 60.

Conversely, if $G \cong G_i$, i=7, 8, …, 60, then $\gamma_{ctnd}(G) = 5$.

References:

[1] F. Harary, Graph Theory, Addison-Wesley, Reading Mass, 1972.

[2] S. Muthammai and G. Ananthavalli, Complementary Tree Nil Domination Number of Graphs, International Journal of Engineering Science, Advanced Computing and Bio-Technology, Volume 8, No.1(January-March, 2017) pp 1-16.

[3] S. Muthammai, M. Bhanumathi and P. Vidhya, Complementary tree domination in graphs,International Mathematical Forum, Vol. 6, 2011, 26, 1273 – 1283.

[4] O. Ore, Theory of graphs, Amer. Math. Soc. Colloq. Publ., 38, (1962).

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