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On Ideal Closure Spaces

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Abstract: *In this paper, we initiate the concepts of Ideal closure spaces. In particular, we deliberate the properties of open and closed sets, union and intersection of subsets, characterization of subspaces in Ideal closure spaces. Along with we make some views on separation axioms in ideal closure spaces.*

Keywords: *Closure space, Ideal Closure space, T_o–space, T_1–space, T_2–space, Pseudo Hausdorff, Uryshon space.*
AMS Subject Classification: *54A05, 54B05, 54C08, 54C10.*

1. Introduction

In topology, the Kurotowski Closure axioms are a set of axioms that can be used to define a topological structure on a set, which defines the closed sets as the fixed points of an operator on the power sets of X[2]. Ideals in topological space has been considered since 1930 by the author vaidyanathaswamy[12]. Jankovic and Hamlet[7] introduced new topologies from old via ideals. In this paper, we introduce and analysis the concepts of Ideal closure space. This is an Ideal space which satisfied kuratowski closure axioms. Separation axioms in closure space has different implications in comparison with the corresponding topological spaces. In cech closure space, a closure space is to be separated by distinct neighbourhood. K. Chandrasekhara Rao and R. Gowri[3] studied separation axioms in bicech closure space. In addition we confer the relation between separation properties in Ideal closure space (X, I, k*) and those in associated topological space (X, I, $\mathcal{J}$*).

2. Prior Results

Definition 2.1:[1]

(X, $\mathcal{J}$) be a topological space. An ideal I on a topological space is a non empty collection of subsets of X which satisfies :

(i) $\varphi \in I$

(ii) $A \in I$, $B \subseteq A$ implies $B \in I$,

(iii) $A \in I$ and $B \in I$ implies $A \cup B \in I$.

If (X, $\mathcal{J}$) is a topological space and I is an ideal on X, then (X, $\mathcal{J}$, I) is called an Ideal topological space or an Ideal space.

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Definition 2.2: [8]

Let P(X) be the power set of X. Then the operator (.)* : P(X) $\rightarrow$ P(X) is called a **local function** of A with respect to $\mathcal{J}$ and I, is defines as follows : For A $\subseteq$ X,
A*(I, $\mathcal{J}$*) = { x $\in$ X : U $\cap$ A $\notin$ I for every open set U containing x}. We simply write A*.

Additionally, cl* (A) = A $\cup$ A* defines Kuratowski closure operator for a topology $\mathcal{J}$* finer than $\mathcal{J}$ [11].

Definition 2.3: [4]

Let X be a non empty set. Let P(X) denote the collection of all subsets of X. the function k: P(X) $\rightarrow$ P(X) satisfying

(i) $k(\varphi) = \varphi$

(ii) $A \subset k(A) \quad \forall A \subset X$

(iii) $k(A \cup B) = k(A) \cup k(B) \quad \forall A \subset X, \; B \subset X$

(iv) $k(A) = k(k(A)) \quad \forall A \subset X$

is called the closure operator on X. The structure (X, k) is called closure spaces.

Definition 2.4:[4]

A subset A of a closure space (X, k) is said to be closed if k(A) = A.

Definition 2.5:[4]

A subset A of a closure space (X, k) is said to be open if k(X-A) = X - A.

Definition 2.6: [4]

The set Int A with respect to the closure operator k is defined as
Int A = X – k(X-A) (i.e.) $[k(A^C)]^C$, where A^C = X - A.

Definition 2.7: [3]

If (X, k) is a closure space than the associate topology on X is $\mathcal{J} = \{A^C; k(A)=A\}$

Definition 2.8: [4]

A subset A in a Closure space (X, k) is called neighbourhood of x if x $\in$ Int (A)

Definition 2.9: [4]

Let (X, k) be a Closure space. A Closure space (Y, k_Y) is called a subspace of (X, k) if Y $\subseteq$ X and $k_Y(A) = k(A) \cap Y$, for each subset A $\subseteq$ Y.

3. Ideal Closure Spaces

Definition 3.1:

Let X be a non-empty set. I be an Ideal on X. Let $A^* : P(X) \to P(X)$ be a local function of A with respect to $\mathcal{J}$ & I.

Let $k^*(A) = A \cup A^*$ defines Kuratowski closure operator for a topology

Then the function $k^* : P(X) \to P(X)$ satisfying,

(i) $k^*(\varphi) = \varphi$

(ii) $A \subset k^*(A)$ $\forall A \subset X$

(iii) $k^*(A \cup B) = k^*(A) \cup k^*(B)$ $\forall A \subset X,\ B \subset X$

(iv) $k^*(A) = k^*(k^*(A))$ $\forall A \subset X$

is called a closure operator on X. The structure (X, I, k*) is called an **Ideal Closure Space.**

Example 3.2:

$X = \{a,b,c\}$

$\mathcal{J} = \{X, \varphi, \{a\}, \{c\}, \{a,c\}\}$

$I = \{\varphi, \{c\}\}$

(i) $A=\{a,c\}$ $A^* = \{a,b\}$ $k^*(A) = A \cup A^* \Rightarrow k^*\{a,c\} = X$

(ii) $A = \{b,c\}$ $A^* = \{b\}$ $k^*(A) = A \cup A^* \Rightarrow k^*\{b,c\} = \{b,c\}$

(iii) $A = \{a,b\}$ $A^* = \{a,b\}$ $k^*(A) = A \cup A^* \Rightarrow k^*\{a,b\} = \{a,b\}$

(iv) $A = X$ $A^* = \{a,b\}$ $k^*(A) = A \cup A^* \Rightarrow k^*(X) = X$

(v) $A = \varphi$ $A^* = \varphi$ $k^*(A) = A \cup A^* \Rightarrow k^*(\varphi) = (\varphi)$

(vi) $A = \{a\}$ $A^* = \{a,b\}$ $k^*(A) = A \cup A^* \Rightarrow k^*(a) = \{a,b\}$

(vii) $A = \{b\}$ $A^* = \{b\}$ $k^*(A) = A \cup A^* \Rightarrow k^*(b) = \{b\}$

(viii) $A = \{c\}$ $A^* = \varphi$ $k^*(A) = A \cup A^* \Rightarrow k^*(c) = \{c\}$

Then(X, I, k*) is an **Ideal Closure Space.**

Definition 3.3:

A subset A of an Ideal closure space (X, I, k*) is said to be closed if $k^*(A) = A$.

Definition 3.4:

A subset A of an Ideal closure space (X, I, k*) is said to be open if $k^*(X - A) = X - A$ (i.e) $Int(A) = A$.

Definition 3.5:

The set Int A with respect to the closure operator k* is defined as $Int\ A = X - k^*(X-A)$ (i.e.) $[k^*(A^C)]^C$, where $A^C = X - A$.

Definition 3.6:

If (X, I, k*) is anIdeal closure space than the associate topology on X is $\mathcal{J}^* = \{A^C;\ k^*(A)=A\}$. Here $\mathcal{J}$ is not equal to $\mathcal{J}^*$.

Definition 3.7:

A subset A in an Ideal Closure space (X, I, k^*) is called neighbourhood of x if $x \in Int (A)$

Definition 3.8:

Let (X, I, k^*) be an Ideal Closure space. An Closure space (Y, I, k^*_Y) is called a subspace of (X, I, k^*) if $Y \subseteq X$ and $k^*_Y (A) = k^*(A) \cap Y$, for each subset $A \subseteq Y$.

Proposition 3.9:

Let (X, I, k^*) be an Ideal closure space and let $A \subseteq X$, then

(i) A is open if and only if $A = X - k^*(X - A)$.

(ii) If B is open and $B \subset A$, then $B \subseteq X - k^* (X - A)$

Proof:(i)Assume that A is open. Then $X - A$ is closed. This implies $k^*(X - A) = X - A$. So, $X - k^*(X - A) = X - (X - A)$. Therefore $A = X - k^*(X - A)$.

Conversely, let B be open subset of (X, I, k^*) such that $X - A \subseteq B$. Then $X - B \subseteq A$. Since $X - B$ is closed subset of (X, I, k^*).

We have, $X - B \subseteq X - k^*(X - A)$. Consequently, $k^*(X - A) \subseteq B$. Hence $X - A$ is closed and so A is open.

(ii)Let B is open and $B \subseteq A$, then by (i), we get $B \subseteq X - k^*(X - A)$.

Proposition 3.10:

Let (X, I, k^*) be an Ideal closure space and let (Y, I, k^*_Y) be a closed subspace of (X, I, k^*). If A is closed subset of (Y, I, k^*_Y), then A is closed subset of (X, I , k^*).

Proof: Let A is closed set of (Y, I, k^*_Y). Then $k^*_Y(A) = A$. Since Y is closed subset of (X, I, k^*). This implies $k^*(A) = A$. Therefore, A is a closed subset of (X, I, k^*).

Proposition 3.11:

Let (X, I, k^*) be an Ideal closure space, if A and B are closed sets then $A \cup B$ also closed.

Proof: Let (X, I, k^*) be an Ideal closure space. Let A and B be two closed sets. $k^*(A) = A$ and $k^*(B) = B$. Since, by additivity, $k^*(A \cup B) = k^*(A) \cup k^*(B) = A \cup B$. Hence, $A \cup B$ is also closed.

Proposition 3.12:

Let (X, I, k^*) be an Ideal closure space and let $A \subseteq X$. If A is closed set then $k^*(A) - A$ contains no non empty sets.

Proof: Let (X, I, k^*) be an Ideal closure space. Let B be a closed subset of (X, I, k^*) such that $B \subseteq k^*(A) - A$. Then $A \subseteq X - B$. Since, A is closed and $X - B$ is open subset of (X, I, k^*). Then, $k^*(A) \subseteq X - B$. This implies, $B \subseteq X - k^*(A)$ and we get

$B \subseteq (X - k^*(A)) \cap k^*(A) = \varphi$. Therefore $B = \varphi$. Hence, $k^*(A) - A$ contains no non empty sets.

Proposition 3.13:

Let (X, I, k^*) be an Ideal closure space. If $A \subseteq X$ is closed then $k^*(A) - A$ is open.

Proof: Let (X, I, k^*) be an Ideal closure space. Suppose that $A \subseteq X$ is closed and let B be a closed subset of (X, I, k^*) such that $B \subseteq k^*(A) - A$. By proposition 3.12 $B = \varphi$ and hence $B \subseteq X - k^*(X - (k^*(A) - A))$. By proposition 3.9(ii), $k^*(A) - A$ is open.

Proposition 3.14:

Let (X, I, k^*) be an Ideal closure space. If A and B be two open sets, then $A \cap B$ also open.

Proof: Let (X, I, k^*) be an ideal Closure space. Let A and B be two open sets, then A^C and B^C are closed set. This implies $k^*(A^C) = A^C$ and $k^*(B^C) = B^C$. Since, by additivity, $k^*(A^C \cup B^C) = k^*(A^C) \cup k^*(B^C) = A^C \cup B^C$. Therefore, $A^C \cup B^C$ is closed. That is $A \cap B$ is open.

4. Separation Axioms on Ideal Closure Spaces

Definition 4.1:

An Ideal Closure space (X, I, k^*) is said to be T_0 - space iff for every distinct points $x \neq y$ and $x \notin k^*(\{y\})$ **or** $y \notin k^*(\{x\})$.

Example 4.2:

X $=\{a,b,c\}$ $\mathcal{J} = \{X, \varphi, \{b\}, \{c\}, \{b,c\}\}$ $I = \{\varphi, \{a\}\}$

$k^*(a) = \{a\}$; $k^*(b) = \{a,b\}$; $k^*(c) = \{a,c\}$; $k^*\{a,b\} = \{a,b\}$;

$k^*\{b,c\} = X$; $k^*\{c,a\} = \{a,c\}$; $k^*(X) = X$; $k^*(\varphi) = \varphi$

Let $a,b \in X$. Then there is a $k^*(a) = \{a\}$ and $k^*(b) = \{a,b\}$ such that $a \in k^*(a)$, $b \notin k^*(a)$.

Let $b,c \in X$. Then there is a $k^*(b) = \{a,b\}$ and $k^*(c) = \{a,c\}$ such that $b \in k^*(b)$, $c \notin k^*(b)$.

Let $c,a \in X$. Then there is a $k^*(c) = \{a,c\}$ and $k^*(a) = \{a\}$ such that $a \in k^*(a)$, $c \notin k^*(a)$.

Therefore (X, I, k^*) is T_0 – space.

Definition 4.3:

An Ideal closure space (X, I, k^*) is said to be T_1-space iff for every distinct points $x \neq y$ andx $\notin k^*(\{y\})$ **and** $y \notin k^*(\{x\})$.

Example 4.4:

X $=\{a,b\}$ $\mathcal{J} = \{X, \varphi, \{a\}, \{b\}\}$ $I = \{\varphi, \{b\}\}$

$k^*(a) = \{a\}$; $k^*(b) = \{b\}$; $k^*(X) = X$; $k^*(\varphi) = \varphi$

Let $a,b \in X$. Then there is a $k^*(a) = \{a\}$ and $k^*(b) = \{b\}$ such that $a \in k^*(a)$, $b \notin k^*(a)$ and $b \in k^*(b)$, $a \notin k^*(b)$.

Therefore (X, I, k^*) is T_1-space.

Theorem 4.5:

An Ideal Closure subspace of a T_0- space is T_0.

Proof: Let (X, I, k^*) be an Ideal Closure T_0-space and (Y, I, k^*_Y) be the subspace of (X, I, k^*).Let x and y are two distinct points in Y. since $(Y, I, k^*_Y) \subseteq (X, I, k^*)$, Then either $x \notin k^*(\{y\})$ or $y \notin k^*(\{x\})$ implies that either $x \notin k^*(\{y\}) \cap Y$ or $y \notin k^*(\{x\}) \cap Y$. Hence (Y, I, k^*_Y) is a T_o – space.

Result 4.6:

Let (X, I, k^*) be an Ideal Closure space then $k^*(A) \subset \mathcal{J}^*$- cl(A), where $\mathcal{J}^*$ - cl(A) is a topological closure with respect to k^*, $\forall A \subset X$.

Proof: Let (X, I, k^*)be an Ideal closure space. We have $A \subset \mathcal{J}^*$- cl (A),

$k^*(A) \subset k^*(\mathcal{J}^*$– cl(A))….. (1) Since $\mathcal{J}^*$ – cl(A) is closed.

$\because k^*(\mathcal{J}^*$ – cl(A)) $= \mathcal{J}^*$ – cl(A)…… (2) From (1) & (2), We have,

$k^*(A) \subset \mathcal{J}^*$- cl (A), $\forall A \subset X$

Theorem 4.7:

If $(X, I, \mathcal{J}^*)$ is T_0 - space then (X, I, k^*) is also T_0 - space.

Proof: Let (X, I, k^*) be an Ideal Closure space. Assume $(X, I, \mathcal{J}^*)$ be T_0– space. Let $x \neq y$ and either $x \notin \mathcal{J}^*$ - cl ({y}) or $y \notin \mathcal{J}^*$- cl ({x}) We have, $k^*(A) \subset \mathcal{J}^*$ - cl (A), $\forall A \subset X$ So, $x \notin \mathcal{J}^*$ - cl ({y}) implies that $x \notin k^*(\{y\})$ or $y \notin \mathcal{J}^*$ - cl ({x}) implies that $y \notin k^*(\{x\})$. Therefore $x \notin k^*(\{y\})$ or $y \notin k^*(\{x\})$.

Hence (X, I, k^*) is T_0 - space.

Theorem 4.8:

An Ideal Closure subspace of a T_1-space is T_1.

Proof: Let (X, I, k^*) be an Ideal Closure T_1 space and (Y, I, k^*_Y) be the subspace of (X, I, k^*). Let x and y are two distinct points in Y. Since $(Y, I, k^*_Y) \subseteq (X, I, k^*)$ then there exist $x \notin k^*(\{y\})$ and $y \notin k^*(\{x\})$. This implies $x \notin k^*(\{y\}) \cap Y$ and $y \notin k^*(\{x\}) \cap Y$. Hence (Y, I, k^*_Y) is a T_1 space.

Remark 4.9:

Let (X, I, k^*) be an Ideal closure space then for every T_1 space is also T_0 space. But the converse is not true.

Example 4.10:

X $=\{a,b,c\}$ $\mathcal{J} = \{X, \varphi, \{a\}, \{b\}, \{a,b\}\}$ $I = \{\varphi, \{c\}\}$

$k^*(a) = \{a,c\}$; $k^*(b) = \{b,c\}$; $k^*(c) = \{c\}$;

$k^*\{a,b\} = X$; $k^*\{b,c\} = \{b,c\}$; $k^*\{c,a\} = \{a,c\}$; $k^*(X) = X$; $k^*(\varphi) = \varphi$.

Here (X, I, k*) is an Ideal closure space, which is T_0 space but every singleton set is not closed therefore it is not T_1.

Theorem 4.11:

For an Ideal closure space (X, I, k*) the following are equivalent.

(i) The space (X, I, k*) is T_1.

(ii) For any $x \in X$, the singleton set {x} is closed with respect to k*.

(iii) Every finite subset of X is closed with respect to k*.

Proof:

(i) $\Longrightarrow$ (ii)

Let (X, I, k*) is T_1. Let $x \neq y$ in X. Suppose {x} is not closed with respect to k*. $k^*(x) \neq \{x\}$ there exists $y \neq x$, $y \in k^*(x)$. This contradicts (i) therefore {x} is closed.

(ii) $\Longrightarrow$ (iii)

For any $x \in X$ the singleton set {x} is closed. Since finite union of closed set is closed, therefore every finite subset of X is closed.

(iii) $\Longrightarrow$ (ii)

Since {x} is finite. by (iii) {x} is closed.

(ii) $\Longrightarrow$ (i)

Let $x \neq y$ in X. Since Singleton sets are closed. $k^*(x) = \{x\}$, $k^*(y) = \{y\}$ therefore $x \notin k^*(\{y\})$ and $y \notin k^*(\{x\})$. Then (X, I, k*) is T_1.

Definition 4.12:

An Ideal Closure space (X, I, k*) is said to be Hausdorff or T_2-space if every distinct points $x \neq y$ and there exists disjoint open sets G and H such that $x \in G$ and $y \in H$.

Example 4.13:

X ={a,b,c} $\mathcal{J}$ = {X, φ ,{a}, {b}, {a,b}} I = {φ , {a,b}}. Ideal closure space is defined by $k^*(a) = \{a,c\}$; $k^*(b) = \{b,c\}$; $k^*(c) = \{c\}$;

$k^*\{a,b\} = \{a,b\}$; $k^*\{b,c\} = \{b,c\}$; $k^*\{c,a\} = \{a,c\}$;

$k^*(X) = X$; $k^*(\varphi) = \varphi$.

Closed sets are X, φ, {a,b}, {b,c}, {c,a}, {c}.

Open sets are X, φ, {a,b}, {a}, {b}, {c}.

Let $a,b \in X$. Then there is a open set U = {a} and V = {b} such that $a \in U$, $b \in V$ and $U \cap V = \varphi$.

Let $b,c \in X$. Then there is a open set U = {b} and V = {c} such that $b \in U$, $c \in V$ and $U \cap V = \varphi$.

Let c,a $\in$ X. Then there is a open set U = {c} and V = {a} such that c $\in$ U, a $\in$ V and U $\cap$ V = φ.
Then (X, I, k*) is T_2-space.

Theorem 4.14:

If the space (X, I, $\mathcal{J}$*) is Hausdorff then the Ideal closure space (X, I, k*) is also Hausdorff.

Proof: Let (X, I, k*) be a Ideal topological space. Then for any two points x $\neq$ y, there exists $\mathcal{J}$* –open U and V of x and y such that U $\cap$ V = φ. Since each $\mathcal{J}$*- neighbourhood in(X, I, $\mathcal{J}$*) is also k*- neighbourhood in (X, I, k*). Therefore, there exists U and V are k*-neighbourhood of x and y in (X, I, k*) such that U $\cap$ V = φ.

Definition 4.15:

An ideal closure space (X, I, k*) is said to be Semi-Hausdorff if for every x $\neq$ y either there exists open sets x $\in$ U and y $\notin$ k*(U) **or** there exists open set V such that y $\in$ V and x $\notin$ k*(V).

Example 4.16:

X = {a,b,c} $\mathcal{J}$ = {X, φ,{a}, {b}, {a,b}} I = {φ, {a,b}}. Ideal closure space is defined by k*(a) = {a,c}; k* (b) = {b,c};k*(c) = {c};
k*{a,b} = {a,b}; k*{b,c} = {b,c}; k* {c,a} = {a,c}; k*(X) = X ; k* (φ) = φ.
Closed sets are X, φ, {a,b}, {b,c}, {c,a}, {c}.
Open sets are X, φ, {a,b}, {a}, {b}, {c}.
Let a,b $\in$ X. Then there exists open set U = {a} and V= {b} then
k*(U) = {a}, k*(V) = {b,c} such that a $\in$ U, b $\notin$ k*(U) **or** b $\in$ V, a $\notin$ k*(V)
Let b,c $\in$ X. Then there exists open set U = {b} and V= {c} then
k*(U) = {b,c}, k*(V) = {c} such that b $\in$ U, c $\notin$ k*(U) **or**c $\in$ V, b $\notin$ k*(V)
Let c,a $\in$ X. Then there exists open set U = {c} and V= {a,b} then
k*(U) = {c}, k*(V) = {a,b} such that c $\in$ U, a $\notin$ k*(U) **or** a $\in$ V, c $\notin$ k*(V)
Therefore (X, I, k*) is Semi-Hausdorff space.

Definition 4.17:

An ideal closure space (X, I, k*) is said to be Pseudo-Hausdorff if for every x $\neq$ y either there exists open sets x $\in$ U and y $\notin$ k*(U) **and** there exists open set V such that y $\in$ V and x $\notin$ k*(V).

Example 4.18:

X = {a,b,c} $\mathcal{J}$ = {X, φ,{a}, {b}, {c}, {a,b}, {b,c}, {c,a}}
I = {φ, {a,b}}. Ideal closure space is defined by k*(a) = {a}; k* (b) = {b}; k*(c) = {c};
k*{a,b} = {a,b}; k*{b,c} = {b,c}; k* {c,a} = {a,c};

$k^*(X) = X$; $k^*(\varphi) = \varphi$.

Closed sets are X, φ, {a,b}, {b,c}, {c,a}, {a}, {b}, {c}.

Open sets are X, φ, {a,b}, {b,c}, {c,a}, {a}, {b}, {c}.

Let a,b $\in$ X. Then there exists open set U = {a} and V= {b} then

$k^*(U) = \{a\}$, $k^*(V) = \{b\}$ such that $a \in U$, $b \notin k^*(U)$ **and** $b \in V$, $a \notin k^*(V)$

Let b,c $\in$ X. Then there exists open set U = {b} and V= {c} then

$k^*(U) = \{b\}$, $k^*(V) = \{c\}$ such that $b \in U$, $c \notin k^*(U)$ **and** $c \in V$, $b \notin k^*(V)$

Let c,a $\in$ X. Then there exists open set U = {c} and V= {a} then

$k^*(U) = \{c\}$, $k^*(V) = \{a\}$ such that $c \in U$, $a \notin k^*(U)$ **and** $a \in V$, $c \notin k^*(V)$

Therefore (X, I, k*) is Pseudo-Hausdorff space.

Theorem 4.19:

If the space (X, I, $\mathcal{J}^*$) is Pseudo Hausdorff then (X, I, k*) is also Pseudo Hausdorff.

Proof: If the space (X, I, $\mathcal{J}^*$) is Pseudo Hausdorff. Let $x \neq y$, there exists $\mathcal{J}^*$ –open U such that $x \in U$ and $y \notin \mathcal{J}^*$-cl ({U}) and there exists a open set V such that $y \in V$ and$x \notin \mathcal{J}^*$-cl (V). Since $k^*(A) \subset \mathcal{J}^*$ - cl(A), $\forall A \subset X$.
Therefore $k^*(U) \subset \mathcal{J}^*$ - cl(U) and $k^*(V) \subset \mathcal{J}^*$ - cl(V). This implies $y \notin \mathcal{J}^*$-cl (U)then $y \notin k^*(U)$ and $x \notin \mathcal{J}^*$-cl (V) then $x \notin k^*(V)$. Hence the Ideal closure space (X, I, k*) is Pseudo Hausdorff.

Theorem 4.20:

If (X, I, k*) is Pseudo Hausdorff then every subspace (Y, I, k^*_Y) of (X, I, k*) is also Pseudo Hausdorff.

Proof: If (X, I, k*) is Pseudo Hausdorff. Let (Y, I, k^*_Y) be the subspace of (X, I, k*). Since (X, I, k*) is Pseudo Hausdorff, then $x \neq y$, there exists open sets U and V such that $x \in U$, $y \notin k^*(U)$ and $y \in V$, $x \notin k^*(V)$. Then $U \cap Y$ and $V \cap Y$ are open sets in Y such that $x \in U \cap Y$ and $y \notin k^*(U \cap Y)$ also $y \in V \cap Y$ and $x \notin k^*(V \cap Y)$. Therefore (Y, I, k^*_Y) is Pseudo Hausdorff.

Definition 4.21:

An Ideal closure space (X, I, k*) is said to be Uryshon space if given $x \neq y$, there exists open sets U and V such that $x \in U$, $y \in V$ and $k^*(U) \cap k^*(V) = \varphi$.

Theorem 4.22:

If (X, I, $\mathcal{J}^*$) is Uryshon space, then the Ideal closure space (X, I, k*) is also Uryshon space.

Proof:

Let (X, I, $\mathcal{J}^*$) be a Uryshon space. Then any two points $x \neq y$, there exists

$\mathcal{J}^*$– open sets U and V such that $x \in U$, $y \in V$ and $\{\mathcal{J}^*\text{-}cl(U)\} \cap \{\mathcal{J}^*\text{-}cl(V)\} = \varphi$. Since each $\mathcal{J}^*$- neighbourhood in (X, I, $\mathcal{J}^*$) is also k*-neighbourhood in (X, I, k*) of x and y in (X, I, k*) such that $k^*(U) \cap k^*(V) = \varphi$.

Theorem 4.23:

If (X, I, k*) is Uryshon, then every subspace (Y, I, k^*_Y) of (X, I, k*) is also Uryshon.

Proof:

Let (X, I, k*) be an Ideal closure space. Let (Y, I, k^*_Y) be a subspace of (X, I, k*). Since (X, I, k*) is Uryshon, gives $x \neq y$, there exists open sets U and V such that $x \in U$, $y \in V$ and $k^*(U) \cap k^*(V) = \varphi$. Now $U \cap Y$ and $V \cap Y$ are open sets in (Y, I, k^*_Y) such that $x \in U \cap Y$, $x \in V \cap Y$. Consider,

$$\begin{aligned} k^*_Y(U \cap Y) \cap k^*_Y(V \cap Y) &= [k^*_Y(U) \cap k^*_Y(V)] \cap Y \\ &= [k^*(U) \cap k^*(V)] \cap Y \\ &= \varphi \cap Y \\ &= \varphi. \end{aligned}$$

Therefore (Y, I, k^*_Y) is Uryshon space.

5. Conclusion

In this paper, basic concepts of Ideal Closure space is introduced. Also the relation between separation properties of Ideal Closure space (X, I, k*) and the associated topological space (X, I, $\mathcal{J}^*$) are discussed.

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