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Super Root Square Mean Labeling of Some Graphs

R. Gopi

Department of Mathematics, Srimad Andavan Arts and Science College (Autonomous)

Tiruchirappalli – 62005, Tamil Nadu, India

E-Mail: *drrgmaths@gmail.com*

Abstract: *Let G be a graph with p vertices and q edges. Let* $f : V(G) \rightarrow \{1,2,3,..., p+q\}$ *be an injective function. For a vertex labeling f, the induced edge labeling* $f^*(e = uv)$ *is defined by* $f^*(uv) = \left\lceil \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rceil$ *or* $\left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor$ *then f is called a super root square mean if* $f(V(G) \cup \{f(e)/e \in E(G)\} = \{1,2,3,..., p+q\}$. *A graph which admits super root square mean labeling is called super root square mean graph. In this paper, we investigate super root square mean labeling of some graphs.*

Key words: *Super root square mean labeling, super root square mean graph.*
AMS subject classification (2010):05C78

1. Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here used in the sense of Harary[5]. The symbols $V(G)$ and $E(G)$ will denote the vertex set and edge set of a graph.

Graph labeling were first introduced in the late 1960's. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling). For a detail survey of graph labeling we refer to Gallian[1].

Mean labeling of graphs was discussed in [3, 4, 6], the concept of *k*-even mean and (*k*, *d*)-even mean are introduced and discussed in [2], the concept of root square mean labeling was introduced and discussed in [7] and the concept of super root square mean labeling was introduced and discussed in [8]. In this paper, we investigate super root square mean labeling of P_n^2, slanting ladder, $T_n \Theta K_1$, $VD(P_n)$.

2. Main Results

Theorem 2.1: The graph $P_n^2\ (n \geq 4)$ is a super root square mean graph.

Proof: Let $\{v_i, 1 \leq i \leq n\}$ be the vertices and $\{e_i, 1 \leq i \leq n-1, a_i, 1 \leq i \leq n-2\}$ be the edges are denoted as in Figure 1.1

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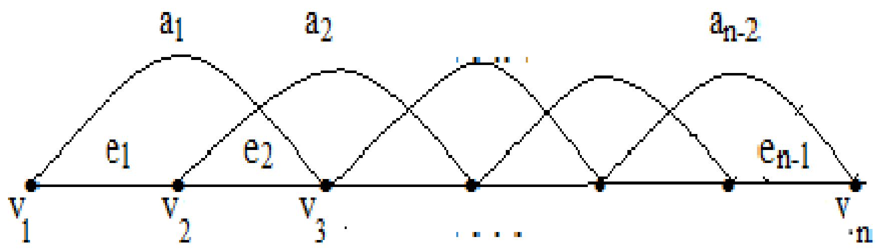

Figure 1.1: Ordinary labeling of P_n^2

First we label the vertices as follows

Define $f: V \rightarrow \{1,2,3,\dots,p+q\}$ by

Case i): *n* is even

$f(v_1)=1, \qquad f(v_3)=6$

For $2 \le i \le n \quad f(v_i)=3i-3 \quad i$ is even

For $5 \le i \le n \quad f(v_i)=3i-2 \quad i$ is odd

Then the induced edge labels are:

For $1 \le i \le n-1 \quad f^*(e_i)=3i-1 \quad f(a_2)=7$

For $4 \le i \le n-2 \quad f(a_i)=3i \quad i$ is even

For $1 \le i \le n-2 \quad f^*(a_i)=3i+1 \quad i$ is odd

Case ii): *n* is odd

$f(v_1)=1, \quad f(v_2)=3$

For $3 \le i \le n$

$$f(v_i)=\begin{cases} 3i-3 & i \text{ is odd} \\ 3i-2 & i \text{ is even} \end{cases}$$

Then the induced edge labels are:

$f^*(a_1)=4 \quad f^*(a_2)=7$

For $3 \le i \le n-2$

$$f^*(a_i)=\begin{cases} 3i & i \text{ is odd} \\ 3i+1 & i \text{ is even} \end{cases}$$

For $1 \le i \le n-1 \quad f^*(e_i)=3i-1$

Thus the vertices and edges together get distinct labels. Hence the graph $P_n^2\,(n \ge 4)$ is a super root square mean graph.

Super root square mean labeling of P_6^2 is shown in Figure 1.2

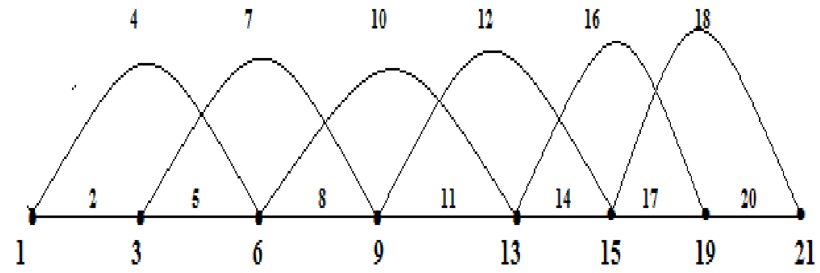

Figure 1.2: Super root square mean labeling of P_6^2

Theorem 2.2: The slanting ladder $SL_n\,(n \geq 3)$ is a super root square mean graph.

Proof: Let $\{u_i, v_i, 1 \leq i \leq n\}$ be the vertices and $\{a_i, b_i, e_i, 1 \leq i \leq n-1\}$ be the edges which are denoted as in figure 1.3

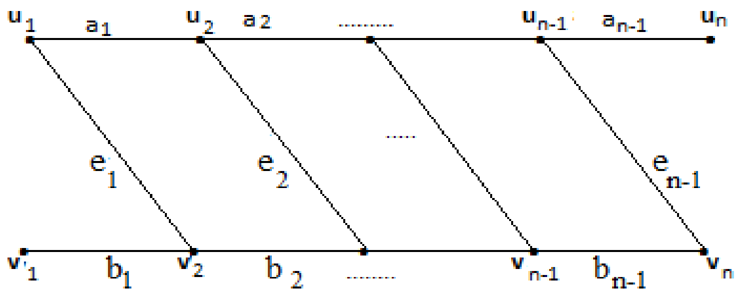

Figure 1.3: Ordinary labeling of SL_n

First we label the vertices as follows

Define $f : V \rightarrow \{1,2,3,\ldots, p+q\}$ by $f(u_1) = 1$

For $2 \leq i \leq n$ $\quad f(u_i) = 5i - 3$

$f(v_1) = 6 \quad f(v_2) = 3$

For $3 \leq i \leq n$ $\quad f(v_i) = 5i - 4$

Then the induced edge labels are:

For $1 \leq i \leq n-1$ $\quad f^*(a_i) = 5i \quad f^*(b_1) = 4$

For $2 \leq i \leq n-1$ $\quad f^*(b_i) = 5i - 2 \quad f^*(e_1) = 2$

For $2 \leq i \leq n-1$ $\quad f^*(e_i) = 5i - 1$

Thus the vertices and edges together get distinct labels. Hence the graph $SL_n\,(n \geq 3)$ is a super root square mean graph.

Super root square mean labeling of SL_6 is shown in Figure 1.4

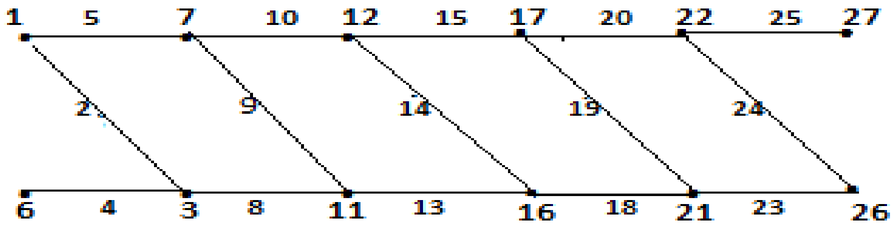

Figure 1.4: Super root square mean labeling of SL_6

Theorem 2.3: The graph $T_n \Theta K_1 \,(n \geq 3)$ is a super root square mean graph.

Proof: Let $\{u_i, u_i^{'}, 1 \leq i \leq n, v_i, v_i^{'}, 1 \leq i \leq n+1\}$ be the vertices and $\{a_i, b_i, c_i, e_i, 1 \leq i \leq n,\ d_i, 1 \leq i \leq n+1\}$ be the edges which are denoted as in Figure 1.5

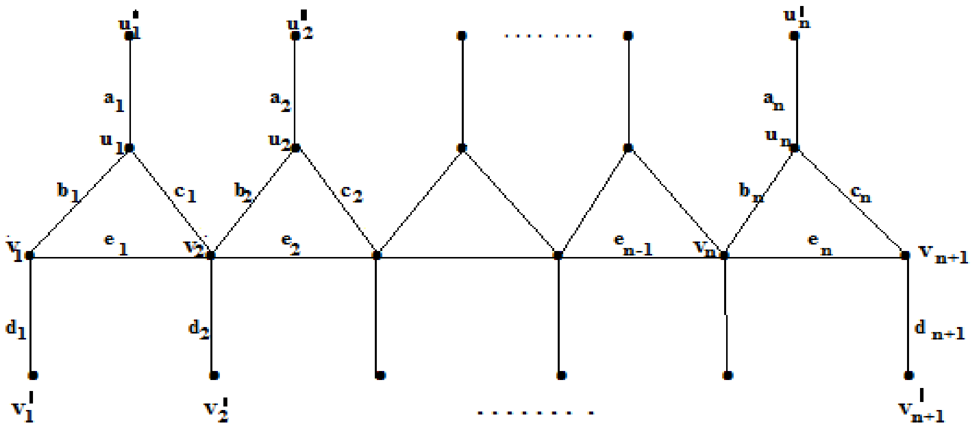

Figure 1.5: Ordinary labeling of $T_n \Theta K_1$

First we label the vertices as follows

Define $f : V \rightarrow \{1,2,3,...,p+q\}$ by

$f(v_1) = 3 \quad f(v_2) = 6$, For $3 \leq i \leq n+1 \quad f(v_i) = 9i - 8$, $f(v_1^{'}) = 10 \quad f(v_2^{'}) = 11$,

For $3 \leq i \leq n+1 \quad f(v_i^{'}) = 9i - 6$, $f(u_1) = 1 \quad f(u_2) = 17$

For $3 \leq i \leq n \quad f(u_i) = 9i - 3$, $f(u_1^{'}) = 12 \quad f(u_2^{'}) = 15$, For $3 \leq i \leq n \quad f(u_i^{'}) = 9i$

Then the induced edge labels are:

$f^*(a_1) = 8$, For $2 \leq i \leq n \quad f^*(a_i) = 9i - 2$, $f^*(b_1) = 2$

For $2 \leq i \leq n \quad f^*(b_i) = 9i - 5$, $f^*(c_1) = 4 \quad f^*(c_2) = 18$

For $3 \leq i \leq n \quad f^*(c_i) = 9i - 1$, For $1 \leq i \leq n \quad f^*(e_i) = 9i - 4$

$f^*(d_1) = 7 \quad f^*(d_2) = 9$, For $3 \leq i \leq n+1 \quad f^*(d_i) = 9i + 7$

Thus the vertices and edges together get distinct labels. Hence the graph $T_n \Theta K_1 \,(n \geq 3)$ is a super root square mean graph.

Super root square mean labeling of $T_5 \Theta K_1$ is shown in Figure 1.6

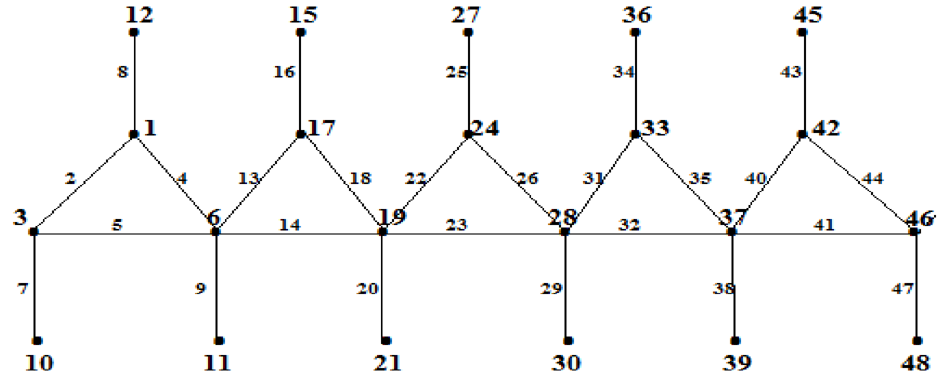

Figure1. 6: Super root square mean labeling of $T_5 \Theta K_1$

Theorem 2.4: The graph $Q_n \Theta K_1 (n \geq 3)$ is a super root square mean graph.

Proof: Let $\{u_i, u_i', 1 \leq i \leq n+1, v_i, w_i, v_i', w_i', 1 \leq i \leq n\}$ be the vertices and $\{a_i, b_i, c_i, a_i', b_i', c_i', 1 \leq i \leq n,\ d_i, 1 \leq i \leq n+1\}$ be the edges which are denoted as in Figure 1.7

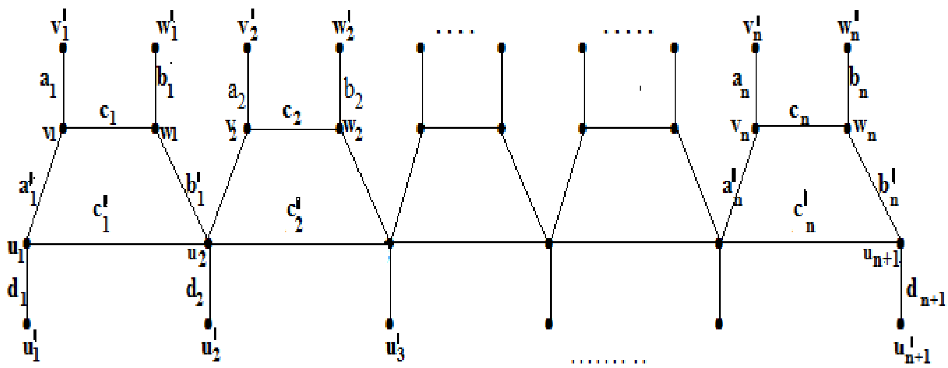

Figure 1.7: Ordinary labeling of $Q_n \Theta K_1$

First we label the vertices as follows

Define $f : V \rightarrow \{1,2,3,\ldots, p+q\}$ by $f(u_1) = 5 \qquad f(u_2) = 13$

For $3 \leq i \leq n+1 \quad f(u_i) = 13i - 12$, $f(u_1') = 9 \quad f(u_2') = 15$

For $3 \leq i \leq n+1 \quad f(u_i') = 13i - 10$, $\quad f(v_1) = 1$

For $2 \leq i \leq n \quad f(v_i) = 13i - 6$, $f(v_1') = 8$

For $2 \leq i \leq n \quad f(v_i) = 13i - 3$, $f(v_1) = 3$

For $2 \leq i \leq n \quad f(v_i') = 13i - 8$, $f(w_1') = 16$

For $2 \le i \le n \quad f(v_i') = 13i$

Then the induced edge labels are:

$f^*(a_1) = 2$, For $2 \le i \le n$, $f^*(a_i) = 13i - 7$

$f^*(b_1) = 12$, For $2 \le i \le n$, $f^*(b_i) = 13i - 2$

$f^*(c_1) = 6$, For $2 \le i \le n$, $f^*(c_i) = 13i - 4$

$f^*(a_1') = 4$, For $2 \le i \le n$, $f^*(a_i') = 13i - 9$

$f^*(b_1') = 11$, For $2 \le i \le n$, $f^*(b_i') = 13i - 1$

$f^*(c_1') = 10$, For $2 \le i \le n$, $f^*(c_i') = 13i - 5$

$f^*(d_1) = 7 \qquad f^*(d_2) = 14$, For $3 \le i \le n+1$, $f^*(d_i) = 13i - 11$

Thus the vertices and edges together get distinct labels. Hence the graph $Q_n \Theta K_1 (n \ge 3)$ is a super root square mean graph.

Super root square mean labeling of $Q_5 \Theta K_1$ is shown in Figure 1. 8

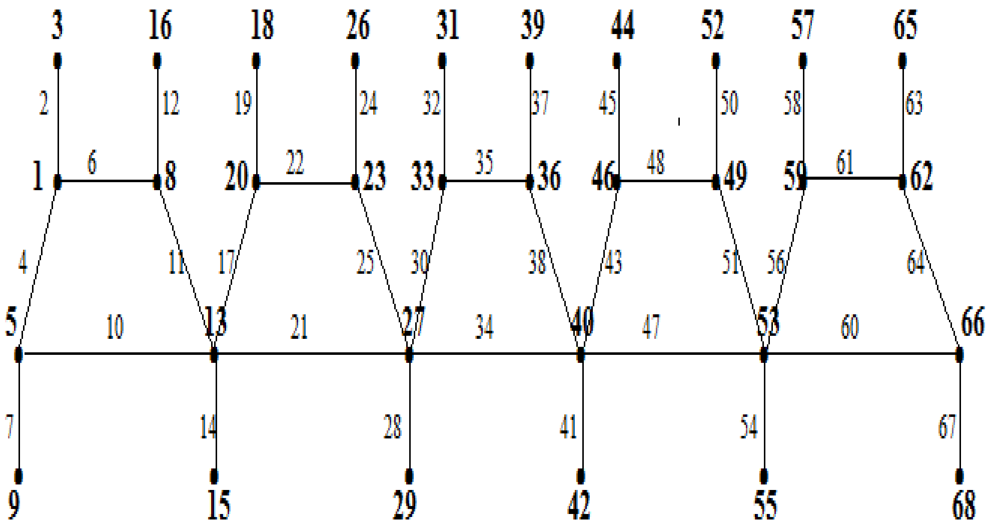

Figure 1.8: Super root square mean labeling of $Q_5 \Theta K_1$

References:

[1] J.A. Gallian, A dynamic survey of graph labeling, The Electronic Journal of Combinatorics, 18(2015), #DS6.

[2] B. Gayathri and R. Gopi, (*k*, *d*) – even mean labeling of $P_m \Theta n K_1$, International Journal of Mathematics and Soft Computing, Vol.1(1), August (2011), 17 – 23.

[3] B. Gayathri and R. Gopi, Necessary Condition for mean labeling, International Journal of Engineering Sciences, Advanced Computing and Bio – Technology, Vol.4(3), July – Sep (2013), 43 – 52.

[4] B. Gayathri and R. Gopi, Cycle related mean graphs, Elixir International Journal of Applied Sciences, No. 71(2014), 25116 – 251124.

[5] F. Harary, Graph Theory, Narosa Publication House Reading, New Delhi 1998.

[6] S. Somaasundaram and R. Ponraj, Mean Labeling of Graphs National Academy Science Letters, 26(2003), 210 – 213.

[7] S.S. Sandhya, S. Somasundaram and S. Anusa, Root Square Mean Labeling of Graphs, International Journal Contemporary Math. Science, Vol. 9, 14(2014), 667 – 676.

[8] K. Thirugnansambandam and K. Venkatesan, Super Root Square Mean labeling of graphs, International Journal of Mathematics and Soft Computing, Vol. 5(2)(2015), 189 – 195.

Authors' Profile:

R. Gopi was born in Kalluppatti, Puddukkottai, India, in 1985. He received the B.Sc., M.Sc., M.Phil., Ph.D., degrees in Mathematics from Periyar E.V.R. College, Tiruchirappalli, India, in 2005, 2008, 2009 and 2013 respectively, and the B.Ed., degree in Institute of Advanced study in Education, Saidapet, Chennai, India. In 2015, he joined the Department of Mathematics, Srimad Andavan Arts and Science College(Autonomous), Tiruchirappalli, India, as an Assistant Professor to till date. His current research interests in Graph Labeling.