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Further Results of Extended Medium Domination Number of Some Special Types of Graphs

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Abstract: *The Medium domination number of a graph was introduced by Duygu Vargor et,al., in [1]. Motivated by the above in [5], Mahadevan, Vijayalakshmi and Sivagnanam introduced the concept of extended medium domination number of a graph. This concept has lot of application in computer communication networks. edom(u,v) is sum of number of u-v paths of length one, two and three. The total number of vertices that dominate every pair of vertices ETDV(G) =* $\sum edom(u,v)$ *for u, v* $\in$ *V(G). In any simple graph G of p number of vertices, the extended medium domination number of G is defined as EMD(G)=* $\frac{\text{ETDV(G)}}{\binom{p}{2}}$ *. In this paper, we investigate the extended medium domination number for some special types of graphs.*

Key Words: *Extended medium domination Number of a graph.*
AMS Subject Classification: 05C69

1. Introduction

In graph theory a graph G is denoted by G = (V,E) where V is a vertex set and E is a edge set of G. Path graph is denoted by P_n where n is the total number of vertices in P_n. C_n is a cycle graph with n vertices. The Bistar is a graph joining the root vertex of $K_{1,n}$ to end vertices of K_2 and is denoted by B(n, n). In this paper we investigate the general result for the extended medium domination number of the graphs bistar, jellyfish graph and $C_m \Theta K_n{}^c$.

Definition 1.1:[5] Let G = (V,E) be a graph, V , E be the vertex set and edge set respectively. edom(u,v) is sum of number of u-v paths of length one, two and three.

Definition 1.2:[5] Let G be the graph. The total number of vertices that dominate every pair of vertices ETDV(G) = $\sum edom(u,v)$ for u,v$\in$V(G).

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Definition 1.3:[5] The extended medium domination number of G is defined as EMD(G)= $\frac{ETDV(G)}{\binom{p}{2}}$ where p is the total number of vertices in G.

Example 1.4:

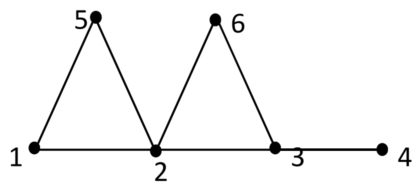

Figure 1.1

From the above figure, we have edom(1, 2) = 2; edom(1, 3) = 3; edom(1, 4) = 1; edom(1, 5) = 2; edom(1, 6) = 3; edom(2, 3) = 2; edom(2, 4) = 2; edom(2, 5) = 2; edom(2, 6) = 2; edom(3, 4) = 1; edom(3, 5) = 3; edom(3, 6) = 2; edom(4, 5) = 1; edom(4, 6) = 2; edom(5, 6) = 3; . ETDV(G) = 31; EMD(G) = $\frac{31}{15}$.

Observation1.5 [5] ETDV($K_{1,n}$) = $\frac{n(n+1)}{2}$

Observation 1.6 [5] ETDV(C_m) = 3m

Observation 1.7 [5] ETDV(P_m) = 3m-6

Definition 1.8: The peacock head graph is obtained by joining n pendent edges to any one vertex of the cycle C_m and it is denoted by PC(n, m).

Definition 1.9: Let G = (V, E) be a ladder graph L_k such that V={A_1 , A_2,…….A_k, B_1, B_2,……B_k}, $E = E_1 \cup E_2 \cup E_3$, where E_1 = {(A_i, A_{i+1}), 1≤ i≤ k-1}, E_2 = {(B_i, B_{i+1}), 1≤ i≤ k-1}, E_3 ={(A_i, B_i),1≤i≤k}.

Definition 1.10: A uniform t-ply graph is a graph obtained from t distinct $P_s, s \geq 3$ paths by merging all the initial vertices to a vertex u and all the terminal vertices to a vertex v. The uniform t-ply graph is denoted by P_t(u,v).

Notation 1.11: $P_m(K_{1,l})$ is a graph obtained by attaching the root vertex of $K_{1,l}$ to the end vertex of the path P_m.

Notation 1.12: umbrella graph U(n,m) is a collection of vertices V and edges E such that V[U(n,m)] = {x_1,x_2,…..x_n,y_1,y_2,……y_m};

$$E[U(n,m)] = \begin{cases} (x_i, x_{i+1}); i = 1,2,....n-1 \\ (y_i, y_{i+1}); i = 1,2,.....m-1 \\ (x_i, y_1); i = 1,2,...........n \end{cases}$$

Notation 1.13: S'($B_{k,k}$) is a graph whose vertices are a, b, c, d, x_i, y_i where 1≤ i ≤2k and the edges are {ab, bc, ad} ∪ {ax_i / 1≤ i ≤k} ∪ {ay_i / 1≤ i ≤k} ∪ {bx_i / k+1≤ i ≤2k} ∪{by_i / k+1≤ i ≤2k} ∪ {cy_i / 1≤ i ≤k} ∪ {dy_i / k+1≤ i ≤2k}. Here {ax_i / 1≤ i ≤k} and {bx_i / k+1≤ i ≤2k} are the pendent edges.

Notation 1.14: $P_m \Theta K_n^c$ is a graph obtained by attaching the root vertex of star $K_{1,n}$ to all the vertices of the path P_m.

Definition 1.15: Let P_n be a path on n vertices denoted by (1,1), (1,2),…(1,n) with n-1 edges denoted by e_1, e_2,….e_{n-1} where e_i is the edge joining the vertices (1,i) and (1,i+1) on each edge e_i = i, where 1 ≤ i ≤ n-1. We erect a ladder with n-(i-1) steps including the edge e_i. The graph obtained is called a step ladder graph and is denoted by S(T_n) where n denotes the number of vertices in base.

Definition 1.16: Let G be a graph. Let G' be a copy of G. The mirror graph M(G) of G is defined as the disjoint union of G and G' with additional edges joining each vertex of G to its corresponding vertex in G'.

2. Main Result :

In this section, we discuss the Medium domination number for step ladder graph, mirror graph of path and Extended Medium domination number of some special type of graphs like $P_m(K_{1,l})$, peacock head graph, Ladder graph, Umbrella graph, $P_m \Theta K_n^c$, S'($B_{k,k}$), uniform t-play graph and ladder graph.

2. Medium domination number

Theorem 2.1: If G = S(T_n) then MD(G) = $\dfrac{4n^2 - n - 6}{\binom{n+\sum_{i=2}^{n} i}{2}}$ where n ≥ 3.

Proof: Consider the step ladder graph. Let the vertices of P_n be (1,1), (1,2),…(1,n). The step ladder graph S(T_n) has vertices denoted by (1,1), (1,2),…(1,n), (2,1), (2,2),…(2,n), (3,1), (3,2),…(3,n-1),………(n,1), (n,2). In the ordered pair (i, j) i denotes the row (bottom to top) j denotes the column (left to right) respectively. TDV(G)= $\sum dom(u,v)$ for u,v∈ V(G).

TDV(P_n) = 2n -3.

In the step ladder graph there are n paths in row wise which has the vertices n, n, n-1, n-2, ….2 respectively. Therefore total dominating vertices of the above n paths is

$= (2n-3)+(2n-3)+(2(n-1)-3)+\ldots\ldots.+1 = (2n-3)+(2n-3)+(2(n-1)-3)+\ldots\ldots.+(4-3)$

$= (2n-3)+\{2[n+(n-1)+\ldots\ldots+2]\}-(n-1)3 = (2n-3)+ \{2[n+(n-1)+\ldots\ldots+2+1-1]\}-(n-1)3$

$= (2n-3)+2\left(\frac{n(n+1)}{2}\right)-2-3n-6 = n^2-2.$

Similarly column wise n^2-2.

Step ladder graph has $\left(\frac{n^2-n}{2}\right)$ squares which has the vertices (i, j), (i, j+1), (i+1, j), (i+1, j+1)

dom((i, j),(i+1, j+1))=2 and dom((i+1, j),(i, j+1))=2. Therefore $4\left(\frac{n^2-n}{2}\right) = 2n^2-2n$.

dom((i, j),(i+1, j-1)) = 1 for i = 2 to n; j = 3 to n; i+j=n+2;Therefore $\sum$ dom((i, j),(i+1, j-1)) = n-2 for i = 2 to n; j = 3 to n; i+j = n+2.

Therefore TDV(G) = $2(n^2-2)+2n^2-2n+n-2 = 4n^2-n-6$.

$$MD(G) = \frac{TDV(G)}{\binom{p}{2}} = \frac{4n^2-n-6}{\binom{n+\sum_{i=2}^{n} i}{2}}$$

Example 2.2 for the graph $S(T_4)$,

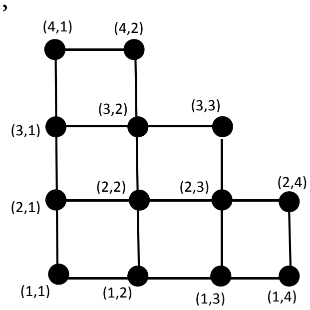

Figure 2.1

From the the above figure 2.1, we have dom((1,1),(1,2)) = 1; dom((1,1),(1,3)) = 1; dom((1,2),(1,3)) = 1; dom((1,2),(1,4)) = 1; dom((1,3),(1,4)) = 1; dom((2,1),(2,2)) = 1; dom((2,1),(2,3)) = 1; dom((2,2),(2,3)) = 1; dom((2,2),(2,4)) = 1; dom((2,3),(2,4)) = 1; dom((3,1),(3,2)) = 1; dom((3,1),(3,3)) = 1; dom((3,2),(3,3)) = 1; dom((4,1),(4,2)) = 1; dom((1,1),(2,1)) = 1; dom((1,1),(3,1)) = 1; dom((2,1),(3,1)) = 1; dom((2,1),(4,1)) = 1; dom((3,1),(4,1)) = 1; dom((1,2),(2,2)) = 1; dom((1,2),(3,2)) = 1;dom((2,2),(3,2)) = 1; dom((2,2),(4,2)) = 1; dom((3,2),(4,2)) = 1; dom((1,3),(2,3)) = 1;dom((1,3),(3,3)) = 1; dom((2,3),(3,3)) = 1; dom((1,4),(2,4)) = 1; dom((2,4),(3,3)) = 1; dom((3,3),(4,2)) = 1; dom((1,1),(2,2)) = 2; dom((1,2),(2,1)) = 2; dom((1,2),(2,3)) = 2; dom((1,3),(2,2)) = 2;

dom((1,3),(2,4)) = 2; dom((1,4),(2,3)) = 2; dom((2,1),(3,2)) = 2; dom((2,2),(3,1)) = 2; dom((2,2),(3,3)) = 2; dom((2,3),(3,2)) = 2; dom((3,1),(4,2)) = 2; dom((3,2),(4,1)) = 2.

TDV(G) = 54; MD(G) = $\frac{54}{78}$.

TDV(G) = $4n^2$-n-6 = 4(16) – 4 – 6 = 54.

$$MD(G) = \frac{4n^2 - n - 6}{\binom{n + \sum_{i=2}^{n} i}{2}} = \frac{54}{\binom{13}{2}} = \frac{54}{78}.$$

Theorem 2.3: If G = M (P_n) then MD(G) = $\frac{9n-10}{\binom{2n}{2}}$ where n ≥ 3.

Proof: Consider the mirror graph of a path P_n. Let the vertices of P_n are $A_1, A_2, \ldots\ldots A_n$ and the vertices of P_n' are $B_1, B_2, \ldots. B_n$. Now join A_i to B_i for i = 1 to n. For any path P_n, TDV(P_m) = 2n-3 for any n. we have two paths P_n.

dom(A_i,B_i) = 1 for i = 1 to n; therefore $\sum_{i=1}^{n}$ dom(A_i,B_i) = n;

dom(A_i,B_{i+1}) = 2 for i = 1 to n-1; therefore $\sum_{i=1}^{n-1}$ dom(A_i,B_{i+1}) = 2(n-1);

dom(A_i,B_{i-1}) = 2 for i = 2 to n; therefore $\sum_{i=2}^{n}$ dom(A_i,B_{i-1}) = 2(n-1).

TDV(G) = 2(2n-3) + n + 4(n-1) = 9n-10.

$$MD(G) = \frac{TDV(G)}{\binom{p}{2}} = \frac{9n-10}{\binom{2n}{2}}$$

Example 2.4 consider the graph M(P_3),

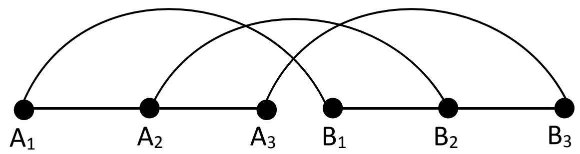

Figure 2.2

From the above figure 2.2, we have dom(A_1,B_1) = 1; dom(A_2,B_2) = 1;dom(A_3,B_3)= 1; dom(A_1,B_2) = 2; dom(A_2,B_1) = 2; dom(A_2,B_3) = 2;dom(A_3,B_2) = 2; dom(A_1,A_2) = 1; dom(A_1,A_3) = 1; dom(A_2,A_3) = 1; dom(B_1,B_2) = 1; dom(B_1,B_3) = 1; dom(B_2,B_3) = 1.

TDV(G) = 17; MD(G) = $\frac{17}{15}$

TDV(G) = 9n-10 = 9(3) – 10 = 17. MD(G) = $\dfrac{9n-10}{\binom{2n}{2}} = \dfrac{17}{15}$

3. Extended Medium domination number

Authors obtained the medium domination number of some different types of graphs. Each result plays an vital role in some real life situations. But still lot of some new types of graphs are available in the literature and it needs to investigate it Extended medium domination number. Therefore, in this section, we obtain the Extended medium domination number of some special types of graphs.

Theorem 3.1: Let G be the peacock head graph PC(n, m) where n > 2 and m > 3, then EMD(G) is $\dfrac{n(n+9)+6m}{2\binom{n+m}{2}}$

Proof: Let (a_1, a_2,…..a_i,….a_m) be the vertices of the cycle C_m. (b_1, b_2,….b_n) be the pendent vertices of the star $K_{1,n}$. Now attach the root vertex of the star $K_{1,n}$ to any vertex of the cycle C_m say (a_1). ETDV(G)= $\sum edom(u,v)$ for u, v$\in$ V(G). For any cycle C_m, ETDV(C_m) = 3m, for any m. For any strar $K_{1,n}$ ETDV($K_{1,n}$) = $\dfrac{n(n+1)}{2}$.

edom(a_2, b_i) = 1 for i = 1 to n; therefore $\sum_{i=1}^{n}$ edom(a_2, b_i) = n;

edom(a_3, b_i) = 1 for i = 1 to n; therefore $\sum_{i=1}^{n}$ edom(a_3, b_i) = n;

edom(a_m, b_i) = 1 for i = 1 to n; therefore $\sum_{i=1}^{n}$ edom(a_m, b_i) = n;

edom(a_{m-1}, b_i) = 1 for i = 1 to n; therefore $\sum_{i=1}^{n}$ edom(a_{m-1}, b_i) = n.

ETDV(G) = ETDV(C_m) + ETDV($K_{1,n}$) + 4n

$$= 3m + \frac{n(n+1)}{2} + 4n = [\,6m + n^2 + n + 8n]/2$$

$$= \frac{n^2+9n+6m}{2} = \frac{n(n+9)+6m}{2}$$

$$EMD(G) = \frac{ETDV(G)}{\binom{p}{2}} = \frac{n(n+9)+6m}{2\binom{n+m}{2}}$$

Example 3.2, for the graph PC(5,6)

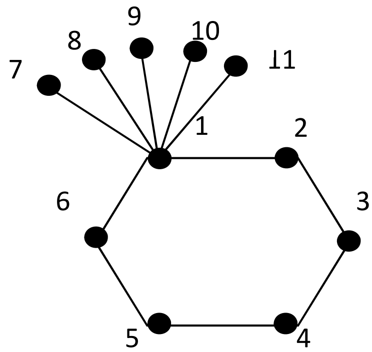

Figure 3.1

From the above figure 3.1, we have, edom(1, 2) = 1; edom(1, 3) = 1; edom(1, 4) = 2; edom(1, 5) = 1; edom(1, 6) = 1; edom(1, 7) = 1; edom(1, 8) = 1; edom(1, 9) = 1; edom(1, 10) = 1; edom(1, 11) = 1; edom(2, 3) = 1; edom(2, 4) = 1; edom(2, 5) = 2; edom(2, 6) = 1; edom(2, 7) = 1; edom(2, 8) = 1; edom(2, 9) = 1; edom(2, 10) = 1; edom(2, 11) = 1; edom(3, 4) = 1; edom(3, 5) = 1; edom(3, 6) = 2; edom(3, 7) = 1; edom(3, 8) = 1; edom(3, 9) = 1; edom(3, 10) = 1; edom(3, 11) = 1; edom(4, 5) = 1; edom(4, 6) = 1; edom(4, 7) = 1; edom(4, 8) = 1; edom(4, 9) = 1; edom(4, 10) = 1; edom(4, 11) = 1; edom(5, 6) = 1; edom(5, 7) = 1; edom(5, 8) = 1; edom(5, 9) = 1; edom(5, 10) = 1; edom(5, 11) = 1; edom(6, 7) = 1; edom(6, 8) = 1; edom(6, 9) = 1; edom(6, 10) = 1; edom(6, 11) = 1; edom(7, 8) = 1; edom(7, 9) = 1; edom(7, 10) = 1; edom(7, 11) = 1; edom(8, 9) = 1; edom(8, 10) = 1; edom(8, 11) = 1; edom(9, 10) = 1; edom(9, 11) = 1; edom(10, 11) = 1. ETDV(G) = 53; MD(G) = $\frac{53}{55}$.

$$ETDV(G) = \frac{n(n+9)+6m}{2} = \frac{5(14)+36}{2} = 53.$$

$$EMD(G)= \frac{ETDV(G)}{\binom{p}{2}} = \frac{53}{\binom{11}{2}} = \frac{53}{55}.$$

Theorem 3.3: Let G be a ladder graph L_k where k > 4 then EMD(G) is $\frac{k^2+16k-22}{\binom{2k}{2}}$

Proof: Let (A_1, A_2,……A_k) and (B_1, B_2,….B_k) be the vertices of two distinct paths with k vertices. Now join the vertices Ai to Bi for i = 1 to k. ETDV(G) = $\sum edom(u,v)$ for u, v $\in$ V(G). For any path P_k, ETDV(P_k) = 3k-6 for any k. we have two paths P_k.

edom(A_i, B_i) = 3 for i = 2 to k-1; therefore $\sum_{i=2}^{k-1}$ edom(A_i, B_i) = (k-2)3;

edom(A_i, B_i) = 2 for i = 1 and k; therefore $\sum_{i=1,k}$ edom(A_i, B_i) = 4;

edom(A_i, A_{i+1}) = 1 for i = 1 to k-1; therefore $\sum_{i=1}^{k-1}$ edom(A_i, B_{i+1}) = k-1;

edom(B_i, B_{i+1}) = 1 for i = 1 to k-1; therefore $\sum_{i=1}^{k-1}$ edom(A_i, B_{i+1}) = k-1;

edom(A_i, $B_{i\pm1}$) = 2 for i = 2 to k-1; therefore $\sum_{i=2}^{k-1}$ edom(A_i, $B_{i\pm1}$) = 4(k-2);

edom(A_i, $B_{i\pm2}$) = 2 for i = 3 to k-2; therefore $\sum_{i=3}^{k-2}$ edom(A_i, $B_{i\pm2}$) = 6(k-4);

edom(A_1, B_2) = 2; edom(A_n, B_{n-1}) = 2; edom(A_1, B_3) = 3; edom(A_2, B_4) = 3; edom(A_n, B_{n-2}) = 3; edom(A_{n-1}, B_{n-3}) = 3.

ETDV(G) = 2(3k-6)+3(k-2)+2(k-1)+4(k-2)+6(k-6) + 20

= 6k–12+3k–6+2k–2 4k 8+ 6k–24+20

= 21k – 32.

$$EMD(G) = \frac{ETDV(G)}{\binom{p}{2}} = \frac{21k-32}{\binom{2k}{2}}$$

Example 3.4, consider the graph L

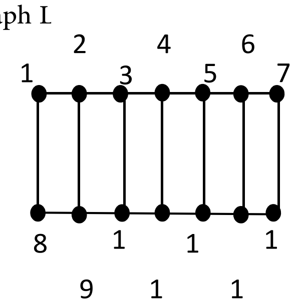

Figure 3.2

From the above figure 3.2, we have edom(1, 2) = 2; edom(1, 3) = 1; edom(1, 4) = 1; edom(1, 8) = 2; edom(1, 9) = 2; edom(1, 10) = 3; edom(2, 3) = 2; edom(2, 4) = 1; edom(2, 5) = 1; edom(2, 8) = 2; edom(2, 9) = 3; edom(2, 10) = 2; edom(2, 11) = 3; edom(3, 4) = 2; edom(3, 5) = 1; edom(3, 6) = 1; edom(3, 8) = 3; edom(3, 9) = 2; edom(3, 10) = 3; edom(3, 11) = 2; edom(3, 12) = 3; edom(4, 5) = 2; edom(4, 6) = 1; edom(4, 7) = 1; edom(4, 9) = 3; edom(4, 10) = 2; edom(4, 11) = 3; edom(4, 12) = 2; edom(4, 13) = 3; edom(5, 6) = 2; edom(5, 7) = 1; edom(5, 10) = 3; edom(5, 11) = 2; edom(5, 12) = 3; edom(5, 13) = 2; edom(5, 14) = 3; edom(6, 7) = 2; edom(6, 11) = 3; edom(6, 12) = 2; edom(6, 13) = 3; edom(6, 14) = 2; edom(7, 12) = 3; edom(7, 13) = 2; edom(7, 14) = 2; edom(8, 9) = 2; edom(8, 10) = 1; edom(8, 11) = 1; edom(9, 10) = 2; edom(9, 11) = 1; edom(9, 12) = 1; edom(10, 11) = 2; edom(10, 12)= 1; edom(10, 13)= 1;

edom(11, 12)= 2; edom(11, 13)= 1; edom(11, 14)= 1; edom(12, 13)= 2; edom(12, 14) = 1; edom(13, 14) = 2.

ETDV(G) = 115; EMD(G) = $\frac{115}{91}$.

ETDV(G) = 21k-32 = 21(7)-32 = 115; EMD(G)= $\frac{ETDV(G)}{\binom{p}{2}} = \frac{115}{\binom{14}{2}} = \frac{115}{91}$

Theorem 3.5: Let G be a graph $P_m(K_{1,l})$ where l,m > 2, then EMD(G) is $\frac{l(l+5)+6(m-2)}{2\binom{l+m}{2}}$

Proof: Let $P_m(K_{1,l})$ be a graph obtained by attaching the root vertex of $K_{1,l}$ to the end vertex of the path P_m. Let $K_{1,l}$ be a star with l pendent vertices and one root vertex, P_m be a path with m vertices. Let $(a_1, a_2, \ldots\ldots a_m)$ be the vertices of the path P_m and $(b_1, b_2, \ldots\ldots b_l)$ be the pendent vertices of the star $K_{1,l}$. Attach the pendent vertex a_1 of the path P_m to the root vertex of $K_{1,l}$.

ETDV(G)= $\sum edom(u,v)$ for u,v$\in$V(G).

For any path P_m, ETDV(P_m) = 3m-6 for any m. For any strar $K_{1,l}$, ETDV($K_{1,l}$) = $\frac{l(l+1)}{2}$

edom(a_2,b_i) = 1 for i = 1 to l; therefore, $\sum_{i=1}^{l}$ edom(a_2,b_i) = l;

edom(a_3,b_i) = 1 for i = 1 to l;therefore, $\sum_{i=1}^{l}$ edom(a_3,b_i) = l;

ETDV(G) = (3m-6)+ $\frac{l(l+1)}{2}$ +2l

= (6m - 12 + l^2 + l +4l) / 2

= [l(l+5) + 6(m-2)] / 2

EMD(G) = $\frac{ETDV(G)}{\binom{p}{2}} = \frac{l(l+5)+6(m-2)}{2\binom{l+m}{2}}$

Example 3.6 Consider the graph $P_6(K_{1,5})$,

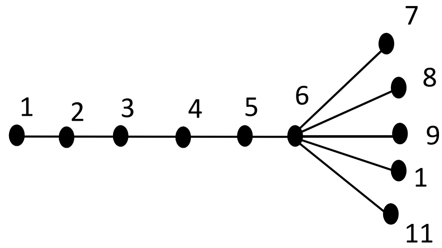

Figure 3.3

From the above figure 3.3, we have edom(1, 2) = 1; edom(1, 3) = 1; edom(1, 4) = 1; edom(2, 3) = 1; edom(2, 4) = 1; edom(2, 5) = 1; edom(3, 4) = 1; edom(3, 5) = 1; edom(3, 6) = 1; edom(4, 5) = 1; edom(4, 6) = 1; edom(4, 7) = 1; edom(4, 8) = 1; edom(4, 9) = 1; edom(4, 10) = 1; edom(4, 11) = 1; edom(5, 6) = 1; edom(5, 7) = 1; edom(5, 8) = 1; edom(5, 9) = 1; edom(5, 10) = 1; edom(5, 11) = 1; edom(6, 7) = 1; edom(6, 8) = 1; edom(6, 9) = 1; edom(6, 10) = 1; edom(6, 11) = 1; edom(7, 8) = 1; edom(7, 9) = 1; edom(7, 10) = 1; edom(7, 11) = 1; edom(8, 9) = 1; edom(8, 10) = 1; edom(8, 11) = 1; edom(9, 10) = 1; edom(9, 11) = 1; edom(10, 11) = 1;ETDV(G) = 37, EMD(G) = $\frac{37}{55}$.

ETDV(G) = [l(l+5) + 6(m-2)] /2 = [5(10)+6(4)] / 2 = 37.

$$\text{EMD(G)} = \frac{ETDV(G)}{\binom{p}{2}} = \frac{37}{\binom{11}{2}} = \frac{37}{55}$$

Theorem 3.7: Let G be a graph U(n,m) where n > 2 and m > 6, then EMD(G) is $\frac{3m+29n-67}{\binom{m+n}{2}}$

Proof: Let (a_1, a_2,…….a_m) be the vertices of the path P_m and (b_1, b_2,…….b_l) be the pendent vertices of the star $K_{1,l}$. Now attach the root vertex of $K_{1,l}$ to end vertex (say) a_1 of the path P_m. Now join b_i to b_{i+1} for i = 1 to n-1. This graph is known as umbrella graph m+n vertices.

ETDV(G)= $\sum edom(u,v)$ for all u,v $\in$ V(G). edom(a_1, b_i) = 5 for i = 3 to n-2; Therefore, $\sum_{i=3}^{n-2}$ edom(a_1, b_i) = 5(n-4); edom(a_2, b_i) = 3 for i = 2 to n-1; Therefore, $\sum_{i=2}^{n-1}$ edom(a_2, b_i) = 3(n-2); edom(a_3, b_i) = 1 for i = 1 to n; Therefore, $\sum_{i=1}^{n}$ edom(a_3, b_i) = n ;

edom(a_1, b_i) = 4 for i = 2 and n-1; Therefore, $\sum_{i=2,n-1}$ edom(a_1, b_i) = 8;

edom(a_1, b_i) = 3 for i = 1 and n; Therefore, $\sum_{i=1,n}$ edom(a_1, b_i) = 6;

edom(a_2, b_i) = 2 for i = 1 and n; Therefore, $\sum_{i=1,n}$ edom(a_2, b_i) = 4;

edom(b_i, b_{i+1}) = 4 for i = 2 to n-2; Therefore, $\sum_{i=2}^{n-2}$ edom(b_i, b_{i+1}) = 4(n-3);

edom(b_i, b_{i+1}) = 3 for i = 1 and n-1 ; Therefore, $\sum_{i=1,n-1}$ edom(b_i, b_{i+1}) = 6;

edom(b_i, b_{i+2}) = 6 for i = 2 to m-3 ; Therefore, $\sum_{i=2}^{m-3}$ edom(b_i, b_{i+2}) = 6(n-4);

edom(b_i, b_{i+3}) = 5 for i = 2 to n-4; Therefore, $\sum_{i=2}^{m-4}$ edom(b_i, b_{i+3}) = 5(n-5);

edom(b_i, b_j) = 5 for i = 2 to n-5, j = 6 to n-1, j-i>3 ; Therefore, $\sum$ edom(b_i, b_j) = 5(n-6)
for i = 2 to n-5, j = 6 to n-1, j-i>3;
edom(b_1, b_3) = edom(b_{n-2}, b_n) = 5; edom(b_1, b_4) = edom(b_{n-3}, b_n) = 4; edom(b_1, b_5) = edom(b_{n-4}, b_n) = 3; edom(b_1, b_{n-1}) = edom(b_2, b_n) = 3; edom(b_1, b_n) = 2.
ETDV(G) = 3m-6 + 5(n-4) + 3(n-2) + n + 4(n-3) + 6(n-4) + 5(n-5) + 5(n-6) + 56.
= 3m+5n+3n+n+4n+6n+5n+5n+56-123 = 3m + 29n - 67

$$EMD(G) = \frac{ETDV(G)}{\binom{p}{2}} = \frac{3m+29n-67}{\binom{m+n}{2}}$$

Example 3.8 for the graph U(7,4)

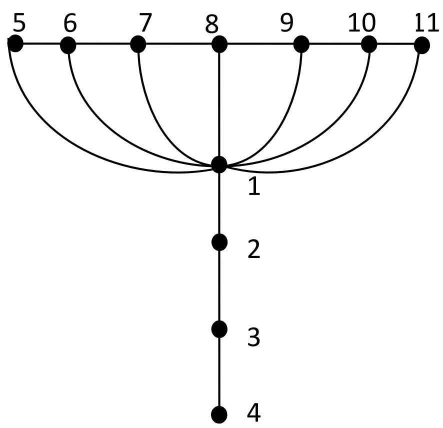

Figure3.4

from the above figure 3.4, we have edom(1, 2) = 1; edom(1, 3) = 1; edom(1, 4) = 4; edom(1, 5) = 3; edom(1, 6) = 4; edom(1, 7) = 5; edom(1, 8) = 5; edom(1, 9) = 5; edom(1, 10) = 4; edom(1, 11) = 3; edom(2, 3) = 1; edom(2, 4) = 4; edom(2, 5) = 2; edom(2, 6) = 3; edom(2, 7) = 3; edom(2, 8) = 3; edom(2, 9) = 3; edom(2, 10) = 3; edom(2, 11) = 2; edom(3, 4) = 1; edom(3, 5) = 1; edom(3, 6) = 1; edom(3, 7) = 1; edom(3, 8) = 1; edom(3, 9) = 1; edom(3, 10) = 1; edom(3, 11) = 1; edom(5, 6) = 3; edom(5, 7) = 5; edom(5, 8) = 4; edom(5, 9) = 3; edom(5, 10) = 3; edom(5, 11) = 2; edom(6, 7) = 4; edom(6, 8) = 6; edom(6, 9) = 5; edom(6, 10) = 5; edom(6, 11) = 3; edom(7, 8) = 4; edom(7, 9) = 6; edom(7, 10) = 5; edom(6, 11) = 3; edom(8, 9) = 4; edom(8, 10) = 6; edom(8, 11) = 4; edom(9, 10) = 4; edom(9, 11) = 5; edom(10, 11) = 3.

ETDV(G) = 148, EMD(G) = $\frac{148}{55}$. ETDV(G) = 3m + 29n - 67 = 3(4) + 29 (7) - 67 = 148

$$\text{EMD(G)} = \frac{ETDV(G)}{\binom{p}{2}} = \frac{148}{\binom{11}{2}} = \frac{148}{55}$$

Theorem 3.9: Let G be the graph S'($B_{k,k}$), then EMD(G) = $\frac{17k^2+25k+6}{\binom{4(k+1)}{2}}$ where k ≥ 2 .

Proof: Let G be the graph S'($B_{k,k}$) construct by the vertices a, b, c, d, x_i, y_i where 1≤ i ≤2k and the edges {ab, bc, ad} ∪ {ax_i / 1≤ i ≤k} ∪ {ay_i / 1≤ i ≤k} ∪ {bx_i / k+1≤ i ≤2k} ∪ {by_i / k+1≤ i ≤2k} ∪ {cy_i / 1≤ i ≤k} ∪ {dy_i / k+1≤ i ≤2k}. Here {ax_i / 1≤ i ≤k} and {bx_i / k+1≤ i ≤2k} are the pendent edges.

edom(a,b) = 2k+1; edom(a,c) = k+1; edom(a,d) = k+1;

edom(a,x_i) = 1 for i = 1 to 2k; therefore $\sum_{i=1}^{2k}$ edom(a, x_i) = 2k;

edom(a,y_i) = k+1 for i = 1 to k; therefore $\sum_{i=1}^{k}$ edom(a, y_i) = k(k+1);

edom(a,y_i) = 2 for i = k+1 to 2k; therefore $\sum_{i=k+1}^{2k}$ edom(a, y_i) = 2k;

edom(b,c) = k+1; edom(b,d) = k+1;

edom(b,x_i) = 1 for i = 1 to 2k; therefore $\sum_{i=1}^{2k}$ edom(b, x_i) = 2k;

edom(b,y_i) = 2 for i = 1 to k; therefore $\sum_{i=1}^{k}$ edom(b, y_i) = 2k;

edom(b,y_i) = k+1 for i = k+1 to 2k; therefore $\sum_{i=k+1}^{2k}$ edom(b, y_i) = k(k+1); edom(c, d) = 2k+1;

edom(c,x_i) = 2 for i = 1 to k; therefore $\sum_{i=1}^{k}$ edom(c, x_i) = 2k;

edom(c,x_i) = 1 for i = k+1 to 2k; therefore $\sum_{i=k+1}^{2k}$ edom(c, x_i) = k;

edom(c,y_i) = k+1 for i = 1 to k; therefore $\sum_{i=1}^{k}$ edom(c, y_i) = k(k+1);

edom(c,y_i) = 1 for i = k+1 to 2k; therefore $\sum_{i=k+1}^{2k}$ edom(c, y_i) = k;

edom(d,x_i) = 1 for i = 1 to k; therefore $\sum_{i=1}^{k}$ edom(d, x_i) = k;

edom(d,x_i) = 2 for i = k+1 to 2k; therefore $\sum_{i=k+1}^{2k}$ edom(d, x_i) = 2k;

edom(d,y_i) = 1 for i = 1 to k; therefore $\sum_{i=1}^{k}$ edom(d, y_i) = k;

edom(d,y_i) = k+1 for i = k+1 to 2k; therefore $\sum_{i=k+1}^{2k}$ edom(d, y_i) = k(k+1);

edom(x_i,x_j) = 1 for i = 1 to k; j = k+1 to 2k; therefore $\sum$ edom(x_i, x_j) = k^2 for i = 1 to k; j = k+1 to 2k;

edom(x_i,y_j) = 1 for i,j = 1 to k; therefore $\sum$ edom(x_i, y_j) = k^2 for i,j = 1 to k;

edom(x_i,y_j) = 2 for i = 1 to k; j = k+1 to 2k; therefore $\sum$ edom(x_i, y_j) = $2k^2$ for i = 1 to k; j = k+1 to 2k;

edom(x_i,y_j) = 1 for i,j = k+1 to 2k; therefore $\sum$ edom(x_i, y_j) = k^2 for i,j = k+1 to 2k;

edom(x_i,y_j) = 2 for j = 1 to k; i = k+1 to 2k; therefore $\sum$ edom(x_i, y_j) = $2k^2$ for j = 1 to k; i = k+1 to 2k;

edom(y_i,y_j) = 3 for i = 1 to k; j = k+1 to 2k; therefore $\sum$ edom(y_i, y_j) = $3k^2$ for i = 1 to k; j = k+1 to 2k;

edom(x_i,x_j) = 1 for i,j = 1 to k; i ≠ j; therefore $\sum$ edom(x_i, x_j) = $\frac{k(k-1)}{2}$ for i,j = 1 to k; i ≠ j;

edom(x_i,x_j) = 1 for i,j = k+1 to 2k; i ≠ j; therefore $\sum$ edom(x_i, x_j) = $\frac{k(k-1)}{2}$ for i,j = k+1 to 2k; i ≠ j;

edom(y_i,y_j) = 2 for i,j = 1 to k; i ≠ j; therefore $\sum$ edom(y_i, y_j) = $2\left(\frac{k(k-1)}{2}\right)$ for i,j = 1 to k; i ≠ j;

edom(y_i,y_j) = 2 for i,j = k+1 to 2k; i ≠ j; therefore $\sum$ edom(y_i, y_j) = $2\left(\frac{k(k-1)}{2}\right)$ for i,j = k+1 to 2k; i ≠ j;

ETDV(G) = 6 + 28k + 14k^2 + 3k(k-1) = 17k^2 + 25k +6

EMD(G) = $\frac{ETDV(G)}{\binom{p}{2}} = \frac{17k^2+25k+6}{\binom{4(k+1)}{2}}$

Example 3.10, For the graph S'($B_{3,3}$),

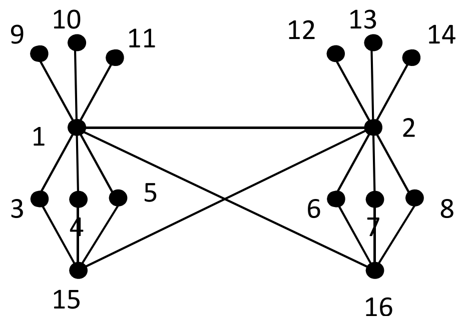

Figure 3.5

From the above figure 3.5, We have edom(1, 2) = 7; edom(1, 3) = 2; edom(1, 4) = 2; edom(1, 5) = 2; edom(1, 6) = 2; edom(1, 7) = 2; edom(1, 8) = 2; edom(1, 9) = 2; edom(1, 10) = 2; dom(1, 11) = 2; edom(1, 12) = 2; edom(1, 13) = 2; edom(1, 14) = 2; edom(1, 15) = 4; edom(1, 16) = 4; edom(2, 3) = 2; dom(2, 4) = 2; edom(2, 5) = 2; edom(2, 6) = 2; edom(2, 7) = 2; edom(2, 8) =2; edom(2, 9) = 1; edom(2, 10) = 1; edom(2, 11) = 1; edom(2, 12) = 1; edom(2, 13) = 1; edom(2, 14) = 1; edom(2, 15) = 4; edom(2, 16) = 4; edom(3, 4) = 2; edom(3, 5) = 2; edom(3, 6) = 3; edom(3, 7) = 3; edom(3, 8) =3; edom(3, 9) = 1; edom(3, 10) = 1; edom(3, 11) = 1; edom(3, 12) = 2; edom(3, 13) = 2; edom(3, 14) = 2; edom(3, 15) = 1; edom(3, 16) = 1; edom(4, 5) = 2; edom(4, 9) = 1; edom(4, 10) = 1; edom(4, 11) = 1; edom(4, 15) = 1; edom(4, 16) = 1; edom(5, 9) = 1; edom(5, 10) = 1; edom(5, 11) = 1; edom(5, 15) = 1; edom(5, 16) = 1; ; edom(6, 7) = 2; edom(6, 8) = 2; edom(6, 12) = 1; edom(6, 13) = 1; edom(6, 14) = 1; edom(6, 15) = 1; edom(6, 16) = 1; edom(7, 8) = 2; edom(7, 12) = 1; edom(7, 13) = 1; edom(7, 14) = 1; edom(7, 15) = 1; edom(7, 16) = 1; edom(8, 12) = 1; edom(8, 13) = 1; edom(8, 14) = 1; edom(8, 15) = 2; edom(8, 16) = 1; edom(9, 10) = 1; edom(9, 11) = 1;

edom(9, 16) = 1; edom(10, 11) = 1; edom(10, 16) = 1; edom(12, 13) = 1; edom(12, 14) = 1; edom(12, 15) = 1; edom(13, 14) = 1; edom(13, 15) = 1; edom(14, 15) = 1.

ETDV(G) = 234; EMD(G) = $\frac{234}{120}$.

ETDV(G) = $17k^2 + 25k + 6 = 153 + 75 + 6 = 234$.EMD(G) = $\frac{ETDV(G)}{\binom{p}{2}} = \frac{234}{\binom{16}{2}} = \frac{234}{120}$

.

Theorem 3.11: Let G be the uniform t-ply graph $P_t(u,v)$ then EMD(G) = $\frac{3t(s+t-1)}{\binom{st+2}{2}}$ where s > 4, t > 2

Proof: Let $(a_{i1},a_{i2},\ldots..a_{is})$ be the vertices of the i^{th} path P_s for i = 1 to t. Join the initial vertices $(a_{11},a_{21},\ldots..a_{t1})$ to the vertex u and the terminal vertices $(a_{1s},a_{2s},\ldots\ldots..a_{ts})$ to the vertex v. ETDV(G) = $\sum edom(u,v)$ for u, v $\in$ V(G).

For any path P_s, ETDV(P_s) = 3s-6 for any s. we have t copies of P_s;

For any star $K_{1,t}$, ETDV($K_{1,t}$) = $\frac{t(t+1)}{2}$; we have two stars $K_{1,t}$.

edom(u, x) = 1 for x = a_{i1}, a_{i3}; i = 1 to t; therefore $\sum$ edom(u, a_{i1}) = 2t for x = a_{i1}, a_{i3}; i = 1 to t;

edom(v, x) = 1 for x = a_{s1}, $a_{i(s-1)}$; i = 1 to t; therefore $\sum$ edom(v, a_{i1}) = 2t for x = a_{s1}, $a_{i(s-1)}$; i = 1 to t;

edom(a_{11}, a_{i2}) = 1 for i = 2 to t; therefore $\sum_{i=2}^{t}$ edom(a_{11}, a_{i2}) = t-1;

edom(a_{21}, a_{i2}) = 1 for i = 3 to t; therefore $\sum_{i=3}^{t}$ edom(a_{21}, a_{i2}) = t-2;……. edom($a_{(t-1)1}$, a_{t2}) = 1;

edom(a_{12}, a_{i1}) = 1 for i = 2 to t; therefore $\sum_{i=2}^{t}$ edom(a_{12}, a_{i1}) = t-1;

edom(a_{22}, a_{i1}) = 1 for i = 3 to t; therefore $\sum_{i=3}^{t}$ edom(a_{22}, a_{i1}) = t-2;……. edom($a_{(t-1)2}$, a_{t1}) = 1;

edom(a_{1s}, $a_{i(s-1)}$) = 1 for i = 2 to t; therefore $\sum_{i=2}^{t}$ edom(a_{1s}, $a_{i(s-1)}$) = t-1;

edom(a_{2s}, $a_{i(s-1)}$) = 1 for i = 3 to t; therefore $\sum_{i=3}^{t}$ edom(a_{s1}, $a_{i(s-1)}$) = t-2;……. edom($a_{(t-1)s}$, $a_{t(s-1)}$) = 1;

edom($a_{1(s-1)}$, a_{is}) = 1 for i = 2 to t; therefore $\sum_{i=2}^{t}$ edom($a_{1(s-1)}$, a_{is}) = t-1;

edom($a_{2(s-1)}$, a_{is}) = 1 for i = 3 to t; therefore $\sum_{i=3}^{t}$ edom($a_{2(s-1)}$, a_{is}) = t-2;……. edom($a_{(t-1)(s-1)}$, a_{ts}) = 1.

$$ETDV(G) = t(3s - 6) + 2\left(\frac{t(t+1)}{2}\right) + 4t + 4\,[1+2+\ldots\ldots.+(t-1)]$$

$$= 3st - 6t + t^2 + t + 4t + 4\left(\frac{t(t-1)}{2}\right)$$

$$= 3st - t + t^2 + 2t^2 - 2t = 3t^2 + 3st - 3t = 3t(t + s - 1)$$

$$EMD(G) = \frac{ETDV(G)}{\binom{p}{2}} = \frac{3t(s+t-1)}{\binom{st+2}{2}}$$

Example 3.12, consider the graph $P_3(u, v)$,

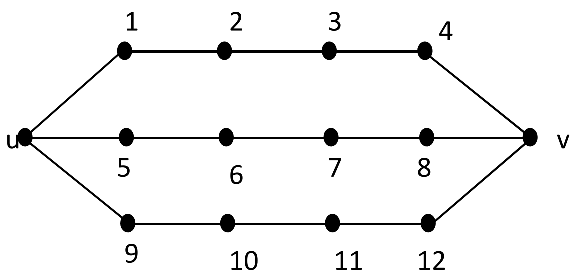

Figure 3.6

From the above figure 2.6, we have edom(u, 1) = 1; edom(u, 2) = 1; edom(u, 5) = 1; edom(u, 6) = 1; edom(u, 9) = 1; edom(u, 10) = 1; edom(u, 3) = 1; edom(u, 7) = 1; edom(u, 11) = 1; edom(v, 4) = 1; edom(v, 8) = 1; edom(v, 12) = 1; edom(v, 3) = 1; edom(v, 7) = 1; edom(v, 11) = 1; edom(v, 2) = 1; edom(v, 6) = 1; edom(v, 10) = 1; edom(1, 2) = 1; edom(1, 3) = 1; edom(1, 4) = 1; edom(1, 5) = 1; edom(1, 9) = 1; edom(1, 6) = 1; edom(1, 10) = 1; edom(2, 3) = 1; edom(2, 4) = 1; edom(2, 5) = 1; edom(2, 9) = 1; edom(3, 4) = 1; edom(3, 8) = 1; edom(3, 12) = 1; edom(4, 8) = 1; edom(4, 12) = 1; edom(4, 7) = 1; edom(4, 11) = 1; edom(5, 6) = 1; edom(5, 7) = 1; edom(5, 8) = 1; edom(5, 9) = 1; edom(5, 10) = 1; edom(6, 7) = 1; edom(6, 8) = 1; edom(6, 9) = 1; edom(7, 8) = 1; edom(7, 12) = 1; edom(8, 11) = 1; edom(8, 12) = 1; edom(9, 10) = 1; edom(9, 11) = 1; edom(9, 12) = 1; edom(10, 11) = 1; edom(10, 12)=1; edom(11, 12) = 1; ETDV(G) = 54; EMD(G) = $\frac{54}{91}$.

ETDV(G) = 3t(s + t -1) = 9(6) = 54. EMD(G) = $\frac{ETDV(G)}{\binom{p}{2}} = \frac{54}{\binom{14}{2}} = \frac{54}{91}$.

Theorem 3.13: If G be a graph $P_m \Theta K_n^c$, then EMD(G) = $\frac{m(3n^2+9n+6)-(2n^2+12n+12)}{2\binom{m(n+1)}{2}}$ where m ≥ 3, n ≥ 2.

Proof: Let $P_m \Theta K_n^c$ be a graph obtained by attaching the root vertex of the star $K_{1,n}$ to all the vertices of the path P_m. Let (a_1, a_2,.......a_m) be the vertices of the path P_m. Let (b_1, b_2, ...b_n) be the pendent vertices of the star S_1, (b_{n+1}, b_{n+2},.......b_{2n}) be the pendent vertices of the star S_2,..........($b_{(m-1)n+1}$,.......b_{mn}) be the pendent vertices of the star S_m. Now attach the root vertex of S_1 to a_1, S_2 to a_2,........ S_m to a_m respectively.

ETDV(G)= $\sum edom(u,v)$ for u,v∈ V(G).

For any path P_m, ETDV(P_m) = 3m - 6; For any star $K_{1,n}$, ETDV($K_{1,n}$) = $\frac{n(n+1)}{2}$;

edom(b_i,b_j) = 1 for i = 1 to n; j = n+1 to 2n; therefore, $\sum$ edom(b_i,b_j) = n^2 for i = 1 to n; j = n+1 to 2n;

edom(b_i,b_j) = 1 for i = n+1 to 2n; j = 2n+1 to 3n; therefore, $\sum$ edom(b_i,b_j) = n^2 for i = n+1 to 2n; j = 2n+1 to 3n; .... edom(b_i,b_j) = 1 for i = mn-2n+1 to mn-n; j = mn-n+1 to mn; therefore, $\sum$ edom(b_i,b_j) = n^2 for i = mn-2n+1 to mn-n; j = mn-n+1 to mn;

edom(a_1,b_i) = 1 for i = n+1 to 3n; therefore, $\sum_{i=n+1}^{3n}$ edom(a_1,b_i) = 2n;

edom(a_2,b_i) = 1 for i = 1 to n and 2n+1 to 4n; therefore, $\sum$ edom(a_2,b_i) = 3n for i = 1 to n and 2n+1 to 4n;

edom(a_3,b_i) = 1 for i = 1 to 2n and 3n+1 to 5n; therefore, $\sum$ edom(a_3,b_i) = 4n for i = 1 to 2n and 3n+1 to 5n;

edom(a_4,b_i) = 1 for i = n+1 to 3n and 4n+1 to 6n; therefore, $\sum$ edom(a_4,b_i) = 4n;..............

edom(a_m,b_i) = 1 for i = (m-3)n+1 to mn-n; therefore, $\sum$ edom(a_m,b_i) = 2n for i = (m-3)n+1 to mn-n.

$$ETDV(G) = 3m - 6 + m\left(\frac{n(n-1)}{2}\right) + (m-1)n^2 + 10n + (m-4)4n$$

$$= [6m - 12 + mn^2 + mn + 2mn^2 - 2n^2 + 20n + 8mn - 32n]/2$$

$$= [2mn^2 + mn^2 - 2n^2 + 9mn + 6m - 12n - 12]/2$$

$$= [m(3n^2 + 9n + 6) - (2n^2 + 12n + 12)]/2$$

$$EMD(G) = \frac{ETDV(G)}{\binom{p}{2}} = \frac{m(3n^2+9n+6)-(2n^2+12n+12)}{2\binom{m(n+1)}{2}}$$

Example 3.14 Consider the graph $P_4 \Theta K_3^c$,

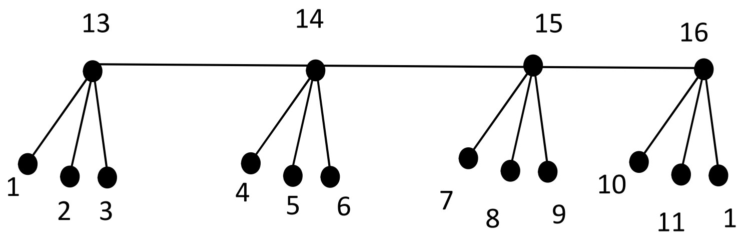

Figure 3.7

From the above figure 3.7, We have, edom(1, 2) = 1; edom(1, 3) = 1; edom(1, 4) = 1; edom(1, 5) = 1; edom(1, 6) = 1; edom(1, 13) = 1; edom(1, 14) = 1; edom(1, 15) = 1; edom(2, 3) = 1; edom(2, 4) = 1; edom(2, 5) = 1; edom(2, 6) = 1; edom(2, 13) = 1; edom(2, 14) = 1; edom(2, 15) = 1; ; edom(3, 4) = 1; edom(3, 5) = 1; edom(3, 6) = 1; edom(3, 13) = 1; edom(3, 14) = 1; edom(3, 15) = 1; edom(4, 5) = 1; edom(4, 6) = 1; edom(4, 7) = 1; edom(4, 8) = 1; edom(4, 9) = 1; edom(4, 13) = 1; edom(4, 14) = 1; edom(4, 15) = 1; edom(5, 6) = 1; edom(5, 7) = 1; edom(5, 8) = 1; edom(5, 9) = 1; edom(5, 13) = 1; edom(5, 14) = 1; edom(5, 15) = 1; edom(6, 7) = 1; edom(6, 8) = 1; edom(6, 9) = 1; edom(6, 13) = 1; edom(6, 14) = 1; edom(6, 15) = 1; edom(7, 8) = 1; edom(7, 9) = 1; edom(7, 10) = 1; edom(7, 11) = 1; edom(7, 12) = 1; edom(7, 13) = 1; edom(7, 14) = 1; edom(7, 15) = 1; edom(7, 16) = 1; edom(8, 9) = 1; edom(8, 10) = 1; edom(8, 11) = 1; edom(8, 12) = 1; edom(8, 13) = 1; edom(8, 14) = 1; edom(8, 15) = 1; edom(8, 16) = 1; edom(9, 10) = 1;edom(9, 11) = 1; edom(9, 12) = 1; edom(9, 13) = 1; edom(9, 14) = 1; edom(9, 15) = 1; edom(9, 16) = 1; edom(10, 11) = 1; edom(10, 12) = 1; edom(10, 14) = 1; edom(10, 15) = 1; edom(10, 16) = 1; edom(11, 12) = 1; edom(11, 14) = 1; edom(11, 15) = 1; edom(11, 16) = 1; edom(12, 14) = 1; edom(12, 15) = 1; edom(12, 16) = 1.

ETDV(G) = 87; MD(G) = $\frac{87}{120}$

ETDV(G) = [m(3n ^{2} + 9n + 6) - (2n^2 + 12n + 12)] /2

= [3(27 + 27 + 6) - (18 + 36 + 12)] / 2 = 87

$$EMD(G) = \frac{ETDV(G)}{\binom{p}{2}} = \frac{ETDV(G)}{\binom{m(n+1)}{2}} = \frac{87}{120}$$

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References:

[1] D.Vargor, P.Dundar,(2011), The medium domination number of a graph, International Journal of Pure and Applied Mathematics, Vol :70, pp. (297-306).

[2] M.Ramachandran, N. Parvathi,(2015) The medium domination number of Jahangir graph J $_{m,n}$, Indian Journal of Science Technology, Vol: 8(5), pp.(400-406).

[3] F. Buckley, F. Harary,(1990), Distance in graphs, Addison Wesley Pub., California.

[4] DA. Mojdeh, AN. Ghamesholu,(2007), Domination in Jahangir graph J $_{2,m}$, International Journal of Math Sciences, Vol: 2(24), Page number (1193-9).

[5] G. Mahadevan, V. Vijayalakshmi, C. Sivagnanam, (2015) Extended Medium domination number of a graph, International Journal of Applied Engineering Research, Vol.10, No.92.

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