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On Multiplicative K-Eccentric Indices and Multiplicative K Hyper- Eccentric Indices of Graphs

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Abstract: *In this paper, we define Multiplicative first and second K Eccentric indices as,* $B\pi_1E(G) = \prod_{ue}[e_G(u) + e_{L(G)}(e)]$ *and* $B\pi_2E(G) = \prod_{ue}[e_G(u)e_{L(G)}(e)]$*and we define Multiplicative first and second K Hyper Eccentric indices as,* $HB\pi_1E(G) = \prod_{ue}[e_G(u) + e_{L(G)}(e)]^2$ *and* $HB\pi_2E(G) = \prod_{ue}[e_G(u)e_{L(G)}(e)]^2$ *and we evaluate these indices for some classes of graphs.*

Keywords: *Eccentricity, First and second K Eccentric indices , First and second K Hyper Eccentric indices, Multiplicative first and second K Eccentric indices and Multiplicative first and second K Hyper Eccentric indices.*

1. Introduction

Topological indices are the numerical parameters of a graph which characterize its topology and are usually graph invariant. Topological indices are used for example in the development of quantitative structure activity relationship (QSAR'S) in which the biological activity or other properties of molecules are correlated with their chemical structure.

Let $G = (V, E)$ be a graph with $|V| = n$ and $|E| = m$. The eccentricity$e_G(v)$ of a vertex v is the distance of any vertex farthest from v. Let $e = uv \in E(G)$. Let $e_{L(G)}(e)$denote the eccentricity of an edge e in L(G), where L(G) is the line graph of G. The vertices and edges of a graph are called its elements.

The line graph of an undirected graph G is another graph L(G) that represents the adjacencies between edges of G. A Line graph of a simple graph is obtained by associating a vertex with each edge of the graph and connecting two vertices with an edge if and only if the corresponding edges of G have a vertex in common.

The concept of Zagreb eccentricity (E_1 and E_2) indices was introduced by Vukicevic and Gravoc in the chemical graph theory very recently [1-4]**.** The first Zagreb eccentricity (E1) and the second Zagreb eccentricity (E2) indices of a graph G are defined as $E_1(G) = \sum_{v_i \in V(G)} e_i^2$ and $E_2(G) = \sum_{v_i v_j \in E(G)} e_i e_j$ where E(G) is the edge set and

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e_i is the eccentricity of the vertex v_i in G. The multiplicative variant of Zagreb indices was introduced by Todeschini et. al [5]. They are defined as $\prod_1(G) = \prod_{v\epsilon V(G)}(deg_G(v))^2$and $\prod_2(G) = (deg_G(u))(deg_G(v))$. Also we recently defined Harmonic Eccentric index and it is defined as $HEI(G) = \sum_{uv\epsilon E\,(G)} = \frac{2}{e_u+e_v}$ where e_u is the eccentricity of the vertex u. In [6], Kulli introduced the first and second *K* Banhatti indices to take account of the contributions of pairs of incident elements, and it was defined as

$$B_1(G) = \sum_{ue}[d_G(u) + d_G(e)] \;,\; B_2(G) = \sum_{ue}[d_G(u)d_G(e)] \,.$$

In [7], Kulli introduced the first and second *K Hyper* Banhatti indices and it was defined as $HB_1(G) = \sum_{ue}[d_G(u) + d_G(e)]^2$, $HB_2(G) = \sum_{ue}[d_G(u)d_G(e)]^2$.

In [8, 9], he introduced the Multiplicative first and second *K* Banhatti indices and it was defined as $B\pi_1(G) = \prod_{ue}[d_G(u) + d_G(e)]$, $B\pi_2(G) = \prod_{ue}[d_G(u)d_G(e)]$.

Similarly in [10], Kulli introduced the Multiplicative first and second *K* Hyper Banhatti indices and it was defined as

$$HB\pi_1(G) = \prod_{ue}[d_G(u) + d_G(e)]^2, \quad HB\pi_2(G) = \prod_{ue}[d_G(u)d_G(e)]^2.$$

Here, we introduce some new Topological indices based on eccentricity as follows:

Let G be a graph with vertex set $V(G)$ and the edge set $E(G)$. Let $u \in V$(G) and $e \in E$(G). We define the first and second Multiplicative K Eccentric indices as,

$$B\pi_1E(G) = \prod_{ue}\left[e_G(u) + e_{L(G)}(e)\right]. \quad B\pi_1E(G) = \prod_{ue}\left[e_G(u)e_{L(G)}(e)\right],$$

and we define the first and second Multiplicative K Hyper Eccentric indices as,

$$HB\pi_1E(G) = \prod_{ue}\left[e_G(u) + e_{L(G)}(e)\right]^2,$$

$$HB\pi_2E(G) = \prod_{ue}\left[e_G(u)e_{L(G)}(e)\right]^2,$$

where in all the cases *ue* means that the vertex *u* and edge *e* are incident in *G* and $e_{L(G)}(e)$ is the eccentricity of e in the line graph $L(G)$ of G.

2. Multiplicative K-Eccentric Indices and Multiplicative K Hyper - Eccentric Indices of Some Special Graphs.

Theorem 2.1 : Let G be a complete graph K_n, then

$$(i) \;\; B\pi_1E(K_n) = 3^{n(n-1)}, \;\; B\pi_2E(K_n) = 2^{n(n-1)}.$$

$$(ii) \;\; HB\pi_1E(K_n) = 9^{n(n-1)}, \qquad HB\pi_2E(K_n) = 4^{n(n-1)}.$$

Proof: The complete graph K_n has n vertices , $m = \frac{n(n-1)}{2}$, edges and all the vertices have eccentricity 1. Also every vertex of K_n is incident with $(n-1)$ edges, and every edge of K_n is incident with exactly 2 vertices. Also $e_{L(K_n)}(e) = 2$ for all e in E(G).

$$(i) B\pi_1E(K_n) = \prod_{ue}\left[e_{K_n}(u) + e_{L(K_n)}(e)\right] = \prod_{e=uv}\left[e_{K_n}(u) + e_{L(K_n)}(e)\right]\left[e_{K_n}(v) + e_{L(K_n)}(e)\right]$$

$$= \prod_{m}\left[e_{K_n}(u) + e_{L(K_n)}(e)\right]^2 = \prod_{m}[[1+2]^2]^m = 9^{\frac{n(n-1)}{2}} = 3^{n(n-1)}$$

$$Also\ B\pi_2E(K_n) = \prod_{ue}[e_{K_n}(u)e_{L(K_n)}(e)]$$

$$= \prod_{m}[e_{K_n}(u) \times e_{L(K_n)}(e)]^2 = \prod_{m}[[1\times 2]^2]^m = 4^{\frac{n(n-1)}{2}} = 2^{n(n-1)}$$

$$(ii) HB\pi_1E(K_n) = \prod_{ue}[e_{K_n}(u) + e_{L(K_n)}(e)]^2$$

$$= \prod_{e=uv\in E(K_n)} [e_{K_n}(u) + e_{L(K_n)}(e)]^2 \times [e_{K_n}(v) + e_{L(K_n)}(e)]^2$$

$$= \prod_{e=uv\in E(K_n)} [[1+2]^4]^m = 9^{n(n-1)}$$

$$Also, HB\pi_2E(K_n) = \prod_{ue}[e_{K_n}(u) \times e_{L(K_n)}(e)]^2$$

$$= \prod_{e=uv\in E(K_n)} [e_{K_n}(u) + e_{L(K_n)}(e)]^2 \times [e_{K_n}(v) + e_{L(K_n)}(e)]^2$$

$$= \prod_{e=uv\in E(K_n)} [[1\times 2]^4]^m = 4^{n(n-1)}$$

Theorem 2.2 : Let G be a cycle graph C_n , then

$$\text{(i) } B\pi_1E(C_n) = \begin{cases} (n-1)^{2n}, n\ is\ odd \\ n^{2n}, n\ is\ even. \end{cases}$$

$$B\pi_2E(C_n) = \begin{cases} \left[\frac{(n-1)}{2}\right]^{4n}, n\ is\ odd \\ \left[\frac{n}{2}\right]^{4n}, \quad n\ is\ even. \end{cases}$$

$$(ii) HB\pi_1E(C_n) = \begin{cases} (n-1)^{4n}, n\ is\ odd \\ (n)^{4n}, n\ is\ even. \end{cases}$$

$$HB\pi_2E(C_n) = \begin{cases} \left[\frac{(n-1)}{2}\right]^{8n}, n\ is\ odd \\ \left[\frac{n}{2}\right]^{8n}, n\ is\ even. \end{cases}$$

Proof: Let $C_n$be a Cycle with $n \geq 3$ vertices. C_n has n edges. Every vertex of C_n is incident with exactly two edges, every edge of C_n is incident with exactly two vertices. Eccentricity of any vertex in

$C_n = \begin{cases} \frac{n-1}{2}, \forall n \geq 3 (\ n\ is\ odd) \\ \frac{n}{2}, \forall n \geq 2 (\ n\ is\ even) \end{cases}$. Also $e_{L(C_n)}(e) = \begin{cases} \frac{n-1}{2}, n\ is\ odd \\ \frac{n}{2}, n\ is\ even. \end{cases}$

Proof of (i):<u>Case 1:</u> If n is odd

$$(i)\quad B\pi_1 E(C_n) = \prod_{ue}\left[e_{C_n}(u) + e_{L(C_n)}(e)\right]$$

$$= \prod_{e=uv}\left[e_{C_n}(u) + e_{L(C_n)}(e)\right]\left[e_{C_n}(v) + e_{L(C_n)}(e)\right]$$

$$= \prod^{n}\left[e_{C_n}(u) + e_{L(C_n)}(e)\right]^2 = \prod^{n}\left[\frac{n-1}{2} + \frac{n-1}{2}\right]^2 = [n-1]^{2n}$$

$$Also,\ \ B\pi_2 E(C_n) = \prod_{ue}\left[e_{C_n}(u) e_{L(C_n)}(e)\right]$$

$$= \prod^{n}\left[e_{K_n}(u) \times e_{L(K_n)}(e)\right]^2 = \prod^{n}\left[\frac{n-1}{2} \times \frac{n-1}{2}\right]^2 = \left[\frac{(n-1)}{2}\right]^{4n}$$

<u>**Case 2:**</u> If n is even

$$(i)\quad B\pi_1 E(C_n) = \prod_{ue}\left[e_{C_n}(u) + e_{L(C_n)}(e)\right]$$

$$= \prod^{n}\left[e_{K_n}(u) + e_{L(K_n)}(e)\right]^2 = \prod^{n}\left[\frac{n}{2} + \frac{n}{2}\right]^2 = [\mathrm{n}]^{2n}$$

$$Also\ \ B\pi_2 E(C_n) = \prod_{ue}\left[e_{C_n}(u) e_{L(C_n)}(e)\right]$$

$$= \prod^{n}\left[e_{K_n}(u) \times e_{L(K_n)}(e)\right]^2 = \prod^{n}\left[\frac{n}{2} \times \frac{n}{2}\right]^2 = \left[\frac{\mathrm{n}}{2}\right]^{4n}$$

Proof of (ii): Case 1: If n is odd

$$HB\pi_1 E(C_n) = \prod_{ue}\left[e_{C_n}(u) + e_{L(C_n)}(e)\right]^2$$

$$= \prod_{\mathrm{e=uv \in E(C_n)}}\left[\mathrm{e_{C_n}(u) + e_{L(C_n)}(e)}\right]^2 \times \left[\mathrm{e_{C_n}(v) + e_{L(C_n)}(e)}\right]^2$$

$$= \prod_{\mathrm{e=uv \in E(C_n)}}\left[\frac{n-1}{2} + \frac{n-1}{2}\right]^2 \times \left[\frac{n-1}{2} + \frac{n-1}{2}\right]^2$$

$$= \prod_{\boldsymbol{e=uv \in E(G)}}\left[[n-1]^4\right]^n = (n-1)^{4n}$$

$$Also\ \ HB\pi_2 E(C_n) = \prod_{ue}\left[e_{C_n}(u) \times e_{L(C_n)}(e)\right]^2$$

$$= \prod_{\boldsymbol{e=uv \in E}(\mathrm{C_n})}\left[e_{C_n}(u) e_{L(C_n)}(e)\right]^2 \times \left[e_{C_n}(v) e_{L(C_n)}(e)\right]^2$$

$$= \prod_{\mathrm{e=uv \in E(C_n)}}\left[\frac{n-1}{2} \times \frac{n-1}{2}\right]^2 \times \left[\frac{n-1}{2} \times \frac{n-1}{2}\right]^2$$

$$= \prod_{\boldsymbol{e=uv \in E}(\mathrm{C_n})}\left[\left[\frac{(n-1)^2}{4}\right]^4\right]^n = \left[\frac{(n-1)}{2}\right]^{8n}$$

Case 2: If n is even

$$HB\pi_1E(C_n) = \prod_{ue}\left[e_{C_n}(u) + e_{L(C_n)}(e)\right]^2$$

$$= \prod_{e=uv\in E(C_n)}\left[e_{C_n}(u) + e_{L(C_n)}(e)\right]^2 \times \left[e_{C_n}(v) + e_{L(C_n)}(e)\right]^2$$

$$= \prod_{e=uv\in E(C_n)}\left[\frac{n}{2}+\frac{n}{2}\right]^2 \times \left[\frac{n}{2}+\frac{n}{2}\right]^2 = \prod_{e=uv\in E(C_n)}[[n]^4]^n = (n)^{4n}$$

$$Also\quad HB\pi_2E(C_n) = \prod_{ue}\left[e_{C_n}(u) \times e_{L(C_n)}(e)\right]^2$$

$$= \prod_{e=uv\in E(C_n)}\left[e_{C_n}(u)e_{L(C_n)}(e)\right]^2 \times \left[e_{C_n}(v)e_{L(C_n)}(e)\right]^2$$

$$= \prod_{e=uv\in E(C_n)}\left[\frac{n}{2}\times\frac{n}{2}\right]^2 \times \left[\frac{n}{2}\times\frac{n}{2}\right]^2 = \prod_{e=uv\in E(C_n)}\left[\left[\frac{n^2}{4}\right]^4\right]^n = \left[\frac{n}{2}\right]^{8n}$$

Theorem 2.3 : Let G be a wheel graph W_n, n ≥ 6. Then

$$(i)B\pi_1E(W_n) = 3^{n-1}2^{2(n-1)}5^{2(n-1)}, B\pi_2E(W_n) = = 2^{5(n-1)}3^{2(n-1)}$$

$$(ii)HB\pi_1E(W_n) = 3^{2(n-1)}2^{4(n-1)}5^{4(n-1)}, HB\pi_2E(W_n) = 2^{10(n-1)}3^{4(n-1)}$$

Proof: Let W_n be a wheel graph. The wheel graph W_n has n vertices and 2(n-1) edges. Every non-central vertex v_i, 1 ≤ i ≤ n-1, of W_n is incident with exactly three edges and every edge of W_n is incident with exactly two vertices. In $W_{n,}$ there are n-1 edges incident with the centre u and n-1 edges $e_i = v_iv_{i-1}$ which form a cycle C_{n-1}. The centre has eccentricity 1 and the remaining n-1 vertices on cycle have eccentricity 2. Also, $e_{L(W_n)}(e) = 2$ for all e which are incident with the centre u and $e_{L(W_n)}(e) = 3$ for all edges $e_i = v_iv_{i-1}$ which form a cycle C_{n-1}.

Hence, $(i)B\pi_1E(W_n) =$

$$\prod_{xe}\left[e_{W_n}(x) + e_{L(W_n)}(e)\right]\prod_{v_ie_i}\left[e_{W_n}(v_i) + e_{L(W_n)}(e_i)\right]$$

$$= \prod_{uv=e}\left[e_{W_n}(u) + e_{L(W_n)}(e)\right]\left[e_{W_n}(v) + e_{L(W_n)}(e)\right]\prod_{v_ie_i}\left[e_{W_n}(v_i) + e_{L(W_n)}(e_i)\right]\left[e_{W_n}(v_{i-1}) + e_{L(W_n)}(e_i)\right]$$

$$= \prod(1+2)(2+2)\prod(2+3)(2+3) = 12^{n-1}25^{n-1}$$

$$= 3^{n-1}2^{2(n-1)}5^{2(n-1)}$$

$Also, B\pi_2E(W_n) = \prod_{xe}[e_{W_n}(x)e_{L(W_n)}(e)] \prod_{v_ie_i}[e_{W_n}(v_i)e_{L(W_n)}(e_i)]$

$$= \prod (2\times 3)(2\times 3) = 8^{n-1}36^{n-1} = 2^{3(n-1)}2^{2(n-1)}3^{2(n-1)}$$

$$= 2^{5(n-1)}3^{2(n-1)}$$

$$(ii) HB\pi_1E(W_n) = \prod_{xe}[e_{W_n}(x) + e_{L(W_n)}(e)]^2 \prod_{v_ie_i}[e_{W_n}(v_i) + e_{L(W_n)}(e_i)]^2$$

$$= \prod_{uv=e}[e_{W_n}(u) + e_{L(W_n)}(e)]^2 [e_{W_n}(v) + e_{L(W_n)}(e)]^2 \prod_{v_ie_i}[e_{W_n}(v_i) + e_{L(W_n)}(e_i)]^2 [e_{W_n}(v_{i-1}) + e_{L(W_n)}(e_i)]^2$$

$$= 3^{2(n-1)}2^{4(n-1)}5^{4(n-1)}$$

$Also, HB\pi_2E(W_n) = 2^{10(n-1)}3^{4(n-1)}$

Theorem 2.4: Let G be a Star graphS_n, then

$$(i) B\pi_1E(S_n) = 6^{n-1}, B\pi_2E(S_n) = 2^{n-1}.$$

$$(ii) HB\pi_1E(S_n) = 6^{2(n-1)}, HB\pi_2E(S_n) = 2^{2(n-1)}.$$

Proof: The Star graph S_n has n vertices and n-1 edges. Every edge of $S_n$is incident with exactly two vertices. Let v be the central vertex of S_n and $u_i, i = 1,2,3,\dots,n-1$ be other pendant vertices. The vertex v is incident with $n-1$ edges, u_i is incident with only one edge such that $e(v) = 1, e(u_i) = 2$. Also, $e_{L(S_n)}(e) = 1$. Hence

$$(i) B\pi_1E(S_n) = \prod_{ue}[e_{S_n}(u) + e_{L(S_n)}(e)]$$

$$= \prod_{e=uv}[e_{S_n}(u) + e_{L(S_n)}(e)][e_{S_n}(v) + e_{L(S_n)}(e)]$$

$$= \prod_{e=uv}(1+1)(2+1) = 6^{n-1}.$$

$$B\pi_2E(S_n) = \prod_{ue}[e_{S_n}(u) \times e_{L(S_n)}(e)]$$

$$= \prod_{e=uv}[e_{S_n}(u) \times e_{L(S_n)}(e)][e_{S_n}(v) \times e_{L(S_n)}(e)]$$

$$= \prod_{e=uv}(1\times 1)(2\times 1) = 2^{n-1}$$

$$(ii) HB\pi_1E(S_n) = \prod_{ue}[e_{S_n}(u) + e_{L(S_n)}(e)]^2$$

$$= \prod_{e=uv} [e_{S_n}(u) + e_{L(S_n)}(e)]^2 [e_{S_n}(v) + e_{L(S_n)}(e)]^2$$

$$= \prod_{e=uv} [1 + 1]^2 [2 + 1]^2 = 6^{2(n-1)}$$

$$Also, HB\pi_2 E(S_n) = \prod_{ue} [e_{S_n}(u) \times e_{L(S_n)}(e)]^2$$

$$= \prod_{e=uv} [e_{S_n}(u) \times e_{L(S_n)}(e)]^2 [e_{S_n}(v) \times e_{L(S_n)}(e)]^2$$

$$= \prod_{e=uv} [1 \times 1]^2 [2 \times 1]^2 = 2^{2(n-1)}$$

Theorem 2.5 : Let G be a graph $K_{m,n}$ then

$$(i) B\pi_1 E(K_{m,n}) = 16^{mn}, B\pi_2 E(K_{m,n}) = 16^{mn}.$$

$$(ii) HB\pi_1 E(K_{m,n}) = 256^{mn}, HB\pi_2 E(K_{m,n}) = 256^{mn}.$$

Proof: Let $K_{m,n}$ be a complete bipartite graph with m+n vertices and mn edges, $|V1| = m, |V2| = n, V(K_{m,n}) = V_1 \cup V_2$. Every vertex of $K_{m,n}$ is incident with *one* vertex of V_1 and another vertex of V_2. Let $V_1 = \{v_1, v_2, v_3 \ldots\}$ and $V_2 = \{w_1, w_2, w_3 \ldots\}$. Clearly vertex of $v_i \; of \; V_1$ is incident with n edges $e_{ij}, j = 1, 2, 3, \ldots, n$, and every vertex w_j of V_2 is incident with m edges e_{ij} $i = 1, 2, 3, \ldots m$. Also, the eccentricity of all vertices are 2 and $e_{L(K_{m,n})}(e) = 2$. Hence

$$(i) B\pi_1 E(K_{m,n}) = \prod_{ue} [e_{K_{m,n}}(u) + e_{L(K_{m,n})}(e)]$$

$$= \prod_{v_j \in V_1} [e_{K_{m,n}}(v_j) + e_{L(K_{m,n})}(e_{ij})] \prod_{w_j \in V_2} [e_{K_{m,n}}(w_j) + e_{L(K_{m,n})}(e_{ij})]$$

$$= \prod^{mn} [2 + 2] \prod^{mn} [2 + 2] = 4^{2mn} = 16^{mn}$$

$$Also, B\pi_2 E(K_{m,n}) = \prod_{ue} [e_{K_{m,n}}(u) e_{L(K_{m,n})}(e)]$$

$$= \prod_{m} [e_{K_{m,n}}(u) \times e_{L(K_{m,n})}(e)]$$

$$= \prod_{v_j \in V_1} [e_{K_{m,n}}(v_j) \times e_{L(K_{m,n})}(e_{ij})] \prod_{w_j \in V_2} [e_{K_{m,n}}(w_j) \times e_{L(K_{m,n})}(e_{ij})]$$

$$= \prod^{mn} [2 \times 2] \prod^{mn} [2 \times 2] = 4^{2mn} = 16^{mn}$$

$$HB\pi_1E(K_{m,n}) = \prod_{ue}\left[e_{K_{m,n}}(u) + e_{L(K_{m,n})}(e)\right]^2$$

$$= \prod_{e=uv\in E(K_{m,n})}\left[e_{K_{m,n}}(u) + e_{L(K_{m,n})}(e)\right]^2 \times \left[e_{K_{m,n}}(u) + e_{L(K_{m,n})}(e)\right]^2$$

$$= \prod^{mn}[2+2]^2\prod^{mn}[2+2]^2 = 16^{2mn} = 256^{mn}$$

$$Also HB\pi_2E(K_{m,n}) = \prod_{ue}\left[e_{K_{m,n}}(u) \times e_{L(K_{m,n})}(e)\right]^2$$

$$= \prod_{e=uv\in E(K_{m,n})}\left[e_{K_{m,n}}(u) \times e_{L(K_{m,n})}(e)\right]^2 \times \left[e_{K_{m,n}}(u) \times e_{L(K_{m,n})}(e)\right]^2$$

$$= \prod^{mn}[2\times 2]^2\prod^{mn}[2\times 2]^2 = 16^{2mn} = 256^{mn}$$

Theorem 2.6 : Let G be a Path graph P_n with radius r. Then

$(i) B\pi_1E(P_n)$

$$= \begin{cases} (4r)^{n-1} \times [(4r+3)(4r+7)(4r+11) \times \ldots \times (8r-9) \times (4r-1)]^{2(n-1)}, & n \text{ is odd} \\ [(4r-2) \times (4r+3) \times (4r+7) \times (4r+11) \times \ldots \times (8r-9) \times (4r-3)]^{\mathbf{2(n-1)}}, & n \text{ is even.} \end{cases}$$

$B\pi_2E(P_n)$

$$= \begin{cases} \begin{bmatrix} 2r^2 + (2r^2+3r+1) + (2r^2+7r+6) + \cdots + \\ (8r^2-10r+3) + (8r^2-18r+10) + (4r^2-2r) \end{bmatrix}^{\mathbf{2(n-1)}}, & n \text{ is odd} \\ \begin{bmatrix} (4r^2-4r+1) \times (4r^2+6r+2) \times (4r^2+14r+12) \times \ldots \times \\ (16r^2-14r+6) \times (16r^2-36r+20) \times (4r^2-6r+2) \end{bmatrix}^{\mathbf{2(n-1)}}, & n \text{ is even.} \end{cases}$$

$(ii)\ HB\pi_1E(P_n)$

$$= \begin{cases} (4r)^{\mathbf{2(n-1)}} \times [(4r+3)(4r+7)(4r+11) \times \ldots \times (8r-9) \times (4r-1)]^{4(n-1)}, & n \text{ is odd .} \\ [(4r-2) \times (4r+3) \times (4r+7) \times (4r+11) \times \ldots \times (8r-9) \times (4r-3)]^{\mathbf{4(n-1)}}, & n \text{ is even} \end{cases}$$

$HB\pi_2E(P_n)$

$$= \begin{cases} \begin{bmatrix} 2r^2 + (2r^2+3r+1) + (2r^2+7r+6) + \cdots + \\ (8r^2-10r+3) + (8r^2-18r+10) + (4r^2-2r) \end{bmatrix}^{\mathbf{4(n-1)}}, & n is odd \\ \begin{bmatrix} (4r^2-4r+1) \times (4r^2+6r+2) \times (4r^2+14r+12) \times \ldots \times \\ (16r^2-14r+6) \times (16r^2-36r+20) \times (4r^2-6r+2) \end{bmatrix}^{\mathbf{4(n-1)}}, & n is even. \end{cases}$$

Proof: Proof of (i):

Case 1: If n is odd

Let r be the radius of P_n. Then $r = \frac{n-1}{2}$ and $n = 2r+1$. Then P_n has only one unicentral vertex with eccentricity r,P_n has two vertices with eccentricity r+1, …, P_n has two vertices with eccentricity 2r, In this case $L\,(P_n) = P_{n-1}, n-1 = 2r. P_{n-1}$ has 2 central vertices with eccentricity r, 2 vertices with eccentricity $r+1$,...... 2 vertices with eccentricity $2r-1$.G has only one vertex with$e(v) = r$ and it has two incident edges with $e_{L(G)}(e) = r$;2 vertices with eccentricity$e(v) = r+1$ and each vertex has 2 incident edges with $e_{L(G)}(e) = r \; and \; r+1$,......... and 2 peripheral vertices with eccentricity $2r$ with one incident edge having eccentricity $2r-1$.

$$(i)B\pi_1 E(P_n) = \prod_{ue} [e_{P_n}(u) + e_{L(P_n)}(e)]$$

$$= \prod_{uv\varepsilon E(G)} [e_{P_n}(u) + e_{L(P_n)}(e)] \prod_{uv\varepsilon E(G)} [e_{P_n}(v) + e_{L(P_n)}(e)] =$$
$$[(\mathrm{r}+\mathrm{r}) + (\mathrm{r}+\mathrm{r})]^{\boldsymbol{n-1}}$$
$$\times \begin{bmatrix} ((\mathrm{r}+1)+\mathrm{r}) + ((\mathrm{r}+1)+(\mathrm{r}+1)) \times ((\mathrm{r}+2)+(\mathrm{r}+1)) + ((\mathrm{r}+2)+(\mathrm{r}+2)) \times \\ \ldots + ((2\mathrm{r}-3)+(2\mathrm{r}-2)) + ((2\mathrm{r}-2)+(2\mathrm{r}-2)) \times (2\mathrm{r}+2\mathrm{r}-1) \end{bmatrix}^{\boldsymbol{2(n-1)}}$$
$$= (4r)^{n-1} \times [(4r+3)(4r+7)(4r+11) \times \ldots \times (8r-9) \times (4r-1)]^{2(n-1)}$$

$$Also, B\pi_2 E(P_n) = \prod_{ue} [e_{P_n}(u) e_{L(P_n)}(e)]$$

$$= \prod_{uv\varepsilon E(G)} [e_{P_n}(u) \times e_{L(P_n)}(e)] \prod_{uv\varepsilon E(G)} [e_{P_n}(v) \times e_{L(P_n)}(e)]$$
$$[(\mathrm{r}+\mathrm{r}) \times (\mathrm{r}+\mathrm{r})]^{\boldsymbol{n-1}}$$
$$\times \Big[[((\mathrm{r}+1)\times \mathrm{r}) + ((\mathrm{r}+1)\times(\mathrm{r}+1))]$$
$$\times [((\mathrm{r}+2)\times(\mathrm{r}+1)) + ((\mathrm{r}+2)\times(\mathrm{r}+2))] \times \ldots$$
$$\times [((2\mathrm{r}-1)\times(2\mathrm{r}-2)) + ((2\mathrm{r}-1)\times(2\mathrm{r}-1))]$$
$$\times [((2\mathrm{r}-3)\times(2\mathrm{r}-2)) + ((2\mathrm{r}-2)\times(2\mathrm{r}-2))]$$
$$\times [2\mathrm{r}\times 2\mathrm{r}-1] \Big]^{\boldsymbol{2(n-1)}}$$
$$= [2r^2 + (2r^2+3r+1) + (2r^2+7r+6) + \cdots + (8r^2-10r+3) + (8r^2-18r+10) + (4r^2-2r)]^{\boldsymbol{2(n-1)}}$$

Case 2: If n is even

Let r be the radius of P_n. Then $r = \frac{n}{2}$ and $n = 2r$. Then P_n has two central vertices with eccentricity r,$P_n$has two vertices with eccentricity r+1, ..., P_n has two vertices with eccentricity 2r-1,

In this case $L\,(P_n) = P_{n-1}, n-1 = 2r-1. P_{n-1}$ has only one vertex with eccentricity $r-1$, 2 vertices with eccentricity r, ..., 2 vertices with eccentricity $2r-2$.G has two vertices with $e(v) = r$ and each one has two incident edges such that one edge has eccentricity $r-1 \; in \; L(G)$ and another one has eccentricity $r \; in \; L(G)$

2 vertices with eccentricity $e(v) = r+1$ and each vertex has 2 incident edges with $e_{L(G)}(e) = r \; and \; r+1$, ..., and 2 peripheral vertices with eccentricity $2r-1$ with incident edges with $e_{L(G)}(e) = 2r-2$.

$$(i)B\pi_1E(P_n) = \prod_{ue}[e_{P_n}(u) + e_{L(P_n)}(e)]$$

$$= \prod_{uv\varepsilon E(G)}[e_{P_n}(u) + e_{L(P_n)}(e)] \prod_{uv\varepsilon E(G)}[e_{P_n}(v) + e_{L(P_n)}(e)]$$

$$\left[\begin{matrix}\left((r-1)+r\right)+\left((r-1)+(r)\right)\times\left((r+1)+(r)\right)+\left((r+1)+(r+1)\right)\times \\ \ldots+\left((2r-3)+(2r-2)\right)+\left((2r-2)+(2r-2)\right)\times\left((2r-2)+(2r-1)\right)\end{matrix}\right]^{2(n-1)}$$

$$= [(4r-2)\times(4r+3)\times(4r+7)\times(4r+11)\times\ldots\times(8r-9)\times(4r-3)]^{2(n-1)}$$

$$Also\ B\pi_2E(P_n) = \prod_{ue}[e_{P_n}(u)e_{L(P_n)}(e)]$$

$$= \prod_{uv\varepsilon E(G)}[e_{P_n}(u)\times e_{L(P_n)}(e)] \prod_{uv\varepsilon E(G)}[e_{P_n}(v)\times e_{L(P_n)}(e)]$$

$$= \Big[[(r+r-1)\times(r+r-1)]\times\left[\left((r+1)+r\right)\times\left((r+1)+(r+1)\right)\right]\times \left[\left((r+2)+(r+1)\right)\times\left((r+2)+(r+2)\right)\right]\times\ldots\times\left[\left((2r-1)+(2r-2)\right)\times \left((2r-1)+(2r-1)\right)\right]\times\left[\left((2r-3)+(2r-2)\right)\times\left((2r-2)+(2r-2)\right)\right]\times [2r-1\times 2r-2]\Big]^{2(n-1)}.$$

.

$$= [(4r^2-4r+1)\times(4r^2+6r+2)\times(4r^2+14r+12)\times\ldots\times(16r^2-14r+6) \times(16r^2-36r+20)\times(4r^2-6r+2)]^{2(n-1)}$$

Proof of (ii):

Case 1: If n is odd

Let r be the radius of P_n. Then $r = \frac{n-1}{2}$ and $n = 2r+1$. Then P_n has only one unicentral vertex with eccentricity r,P_n has two vertices with eccentricity r+1,….P_n has two vertices with eccentricity 2r,In this case $L\,(P_n) = P_{n-1}, n-1 = 2r. P_{n-1}$ has 2 central vertices with eccentricity r, 2 vertices with eccentricity $r+1$,… 2 vertices with eccentricity $2r-1$.G has only one vertex with $e(v) = r$ and it has two incident edges with $e_{L(G)}(e) = r;$

2 vertices with eccentricity $e(v) = r+1$ and each vertex has 2 incident edges with $e_{L(G)}(e) = r\ and\ r+1$,… and 2 peripheral vertices with eccentricity $2r$ with one incident edge having eccentricity $2r-1$.

$$HB\pi_1E(P_n) = \prod_{ue}[e_{P_n}(u) + e_{L(P_n)}(e)]^2$$

$$= \prod_{e=uv\in E(K_n)}[e_{P_n}(u) + e_{L(P_n)}(e)]^2 \times [e_{P_n}(u) + e_{L(P_n)}(e)]^2$$

$$[(\mathrm{r}+\mathrm{r})+(\mathrm{r}+\mathrm{r})]^{2(n-1)} \times \left[\begin{array}{c}((\mathrm{r}+1)+\mathrm{r})+((\mathrm{r}+1)+(\mathrm{r}+1))\times((\mathrm{r}+2)+(\mathrm{r}+1))+((\mathrm{r}+2)+(\mathrm{r}+2))\times \\ \ldots+((2\mathrm{r}-3)+(2\mathrm{r}-2))+((2\mathrm{r}-2)+(2\mathrm{r}-2))\times(2\mathrm{r}+2\mathrm{r}-1)\end{array}\right]^{4(n-1)}$$

$$=(4r)^{2(n-1)}\times[(4r+3)(4r+7)(4r+11)\times\ldots\times(8r-9)\times(4r-1)]^{4(n-1)}$$

$$Also\ HB\pi_2E(P_n)=\prod_{\mathrm{ue}}\left[\mathrm{e}_{P_n}(\mathrm{u})\times e_{L(P_n)}(e)\right]^2$$

$$=\prod_{\mathrm{e=uv}\in\mathrm{E}(K_{m,n})}\left[\mathrm{e}_{P_n}(\mathrm{u})\times e_{L(P_n)}(e)\right]^2\times\left[\mathrm{e}_{P_n}(\mathrm{u})\times e_{L(P_n)}(e)\right]^2$$

$$[(\mathrm{r}+\mathrm{r})\times(\mathrm{r}+\mathrm{r})]^{2(n-1)} \times\Big[[((\mathrm{r}+1)\times\mathrm{r})+((\mathrm{r}+1)\times(\mathrm{r}+1))] \times[((\mathrm{r}+2)\times(\mathrm{r}+1))+((\mathrm{r}+2)\times(\mathrm{r}+2))]\times\ldots \times[((2\mathrm{r}-1)\times(2\mathrm{r}-2))+((2\mathrm{r}-1)\times(2\mathrm{r}-1))] \times[((2\mathrm{r}-3)\times(2\mathrm{r}-2))+((2\mathrm{r}-2)\times(2\mathrm{r}-2))] \times[2\mathrm{r}\times2\mathrm{r}-1]\Big]^{4(n-1)}$$

$$=[2r^2+(2r^2+3r+1)+(2r^2+7r+6)+\cdots+(8r^2-10r+3)+(8r^2-18r+10)+(4r^2-2r)]^{4(n-1)}$$

Case 2: If n is even

Let r be the radius of P_n. Then $r=\frac{n}{2}$ and $n=2r$. Then P_n has two central vertices with eccentricity r,P_n has two vertices with eccentricity r+1,…………………………………P_n has two vertices with eccentricity 2r-1,

In this case $L\,(P_n)=P_{n-1}, n-1=2r-1. P_{n-1}$ has only one vertex with eccentricity $r-1$, 2 vertices with eccentricity r,… 2 vertices with eccentricity $2r-2$.G has two vertices with $e(v)=r$ and each one has two incident edges such that one edge has eccentricity $r-1\ in\ L(G)$ and another one has eccentricity $r\ in\ L(G)$.2 vertices with eccentricity $e(v)=r+1$ and each vertex has 2 incident edges with $e_{L(G)}(e)=r\ and\ r+1$,… and 2 peripheral vertices with eccentricity $2r-1$with incident edges with $e_{L(G)}(e)=2r-2$.

$$HB\pi_1E(P_n)=\prod_{\mathrm{ue}}\left[\mathrm{e}_{P_n}(\mathrm{u})+\mathrm{e}_{\mathrm{L(P_n)}}(\mathrm{e})\right]^2$$

$$=\prod_{\mathrm{e=uv}\in\mathrm{E(K_n)}}\left[\mathrm{e}_{\mathrm{P_n}}(\mathrm{u})+\mathrm{e}_{\mathrm{L(P_n)}}(\mathrm{e})\right]^2\times\left[\mathrm{e}_{\mathrm{P_n}}(\mathrm{u})+\mathrm{e}_{\mathrm{L(P_n)}}(\mathrm{e})\right]^2$$

$$\left[\begin{array}{c}((\mathrm{r}-1)+\mathrm{r})+((\mathrm{r}-1)+(\mathrm{r}))\times((\mathrm{r}+1)+(\mathrm{r}))+((\mathrm{r}+1)+(\mathrm{r}+1))\times \\ \ldots+((2\mathrm{r}-3)+(2\mathrm{r}-2))+((2\mathrm{r}-2)+(2\mathrm{r}-2))\times((2\mathrm{r}-2)+(2\mathrm{r}-1))\end{array}\right]^{4(n-1)}$$

$$= [(4r-2)\times(4r+3)\times(4r+7)\times(4r+11)\times \dots \times(8r-9)\times(4r-3)]^{\mathbf{4(n-1)}}$$

$$Also\ HB\pi_2E(P_n) = \prod_{ue}\left[\mathrm{e}_{P_n}(\mathrm{u})\times e_{L(P_n)}(e)\right]^2$$

$$= \prod_{\mathrm{e=uv\in E}(K_{m,n})}\left[\mathrm{e}_{P_n}(\mathrm{u})\times e_{L(P_n)}(e)\right]^2\times\left[\mathrm{e}_{P_n}(\mathrm{u})\times e_{L(P_n)}(e)\right]^2$$

$$= \Big[[(\mathrm{r}+\mathrm{r}-1)\times(\mathrm{r}+\mathrm{r}-1)]\times\big[((\mathrm{r}+1)+\mathrm{r})\times((\mathrm{r}+1)+(\mathrm{r}+1))\big]\times \big[((\mathrm{r}+2)+(\mathrm{r}+1))\times((\mathrm{r}+2)+(\mathrm{r}+2))\big]\times\dots\times\big[((2\mathrm{r}-1)+(2\mathrm{r}-2))\times ((2\mathrm{r}-1)+(2\mathrm{r}-1))\big]\times\big[((2\mathrm{r}-3)+(2\mathrm{r}-2))\times((2\mathrm{r}-2)+(2\mathrm{r}-2))\big]\times [2\mathrm{r}-1\times 2\mathrm{r}-2]\Big]^{\mathbf{4(n-1)}}.$$

$$= [(4r^2-4r+1)\times(4r^2+6r+2)\times(4r^2+14r+12)\times\dots\times(16r^2-14r+6) \times(16r^2-36r+20)\times(4r^2-6r+2)]^{\mathbf{4(n-1)}}$$

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