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Strong Co-Chromatic Number of Some Well Known Graphs

*M. Poobalaranjani[1] and R.Pichailakshmi[2]

[1,2] *Department of Mathematics*

Seethalakshmi Ramaswami College, Tiruchirappalli, Tamil Nadu, India.

Email: **mpranjani@hotmail.com, laxmirajesh1806@gmail.com*

Abstract: *A strong k-coloring is a proper k-coloring in which all the color classes are of same size. For sets to have equal size,* $k\left\lceil\frac{n}{k}\right\rceil - n$ *isolated vertices are added. An r co-coloring of a graph is a partition of the vertex set into r sets such that each set in the partition is either an independent set or induces a clique, where empty sets are permitted in the partition. Combining these two concepts, in this paper a new graph partition called strong co-coloring is defined. A strong r co-coloring is a partition of the vertex set into r sets of equal size in which each set in the partition is either an independent set or induces a clique and (if necessary)* $k\left\lceil\frac{n}{k}\right\rceil - n$ *isolated vertices are added. The new graph parameter called the strong co-chromatic number of a graph G is the least r for which G has a strong r co-coloring. In this paper, exact bounds of strong co-chromatic number of some well-known graphs are found.*

Key words: *proper coloring, co-coloring, strong co-coloring, co-chromatic number, strong co-chromatic number.*

1. Introduction

In this paper, only finite simple graphs are considered. For undefined terms and concepts, the reader is referred to [4]. A k-coloring is coloring the vertices of a graph using k colors such that no two adjacent vertices receive the same color. Equivalently, this can be defined as partitioning the vertex set into disjoint sets such that every partite set induces an independent set. This is called chromatic partition. Later the concept of partitioning took manifolds and different types of partitioning were introduced. Two of such partitions are strong coloring and co-coloring. Strong coloring imposes condition on the size of sets in the partition. A strong k-coloring is a proper k-coloring in which all the color classes are of same size. In this case, k must divide the order of the graph. Since, it is not possible for any graph of any order, for sets in the partition to have equal size, isolated vertices are added with the condition that in a strong k-coloring of a graph of order n, only $k\left\lceil\frac{n}{k}\right\rceil - n$ isolated vertices can be added. To avoid ambiguity, the graph G is denoted as G even after adding (if necessary) the isolated vertices. The strong chromatic number is the minimum k for which G has a strong k-coloring and is denoted by $\chi_s(G)$. Strong chromatic number was independently introduced by Alon [1] in 1988 and by Fellows [3] in 1990.

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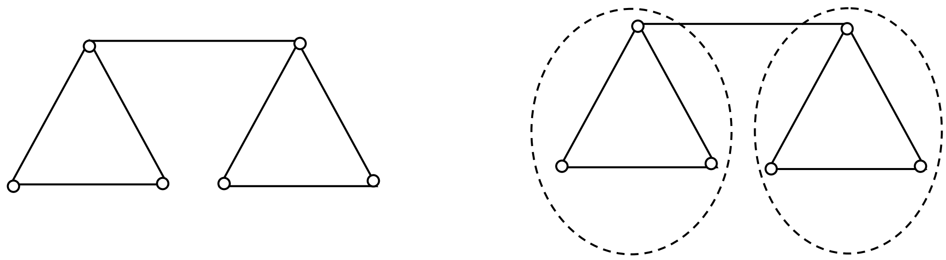
*Corresponding Author

On the other hand, co-coloring is about the nature of the set in the partition. It was introduced by Lesniak and Straight [5]. An *r* co-coloring of a graph is a partition of the vertex set into *r* sets in which each set in the partition is either an independent set or induces a clique, where empty sets are permitted in the partition. In 1994, Brandstadt [2] proved
whether the vertex set is partitioned into one or two independent sets and one or two cliques can be recognized in polynomial time. Since, in co-coloring there is no condition on the number of independent sets and cliques and empty sets are permitted, every proper coloring is also a co-coloring. The co-chromatic number of G, denoted by Z(G) is the minimum *r* such that G has an r co-coloring. The definitions of strong coloring and co-coloring motivated us to define the term "strong r co-coloring". A ***strong r co-coloring*** is an r co-coloring such that all the sets in the partition are of same size. Analogously, the strong co-chromatic number of G, denoted by $Z_s(G)$ is the minimum *r* such that G has strong *r* co-coloring. It is easy to see that $Z(G) \leq Z_s(G)$. As in the case of strong coloring, in a strong r co-coloring, $r\left\lceil \frac{n}{r} \right\rceil - n$ isolated vertices are added and the graph G is denoted as G after adding (if necessary) the isolated vertices.

For the graph given in Figure 1(a), a strong 2 co-coloring is shown in Figure 1 (b).

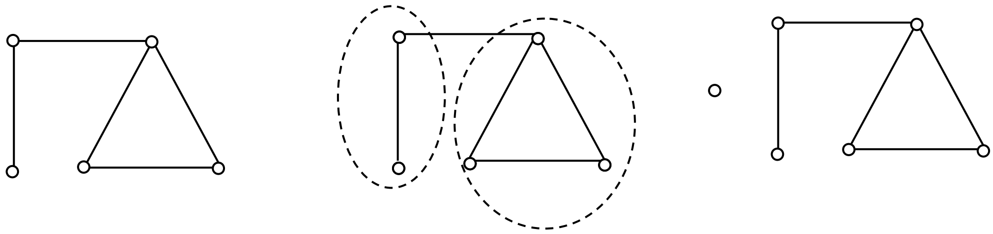


(a) (b)

Figure 1

Here, the vertex set is partitioned into two cliques of size 3. Since the order of the graph is 6, addition of isolated vertices is not needed for a strong 2 co-coloring. Now, consider the graph given in Figure 2(a). Figure 2(b) shows a 2 co-coloring which is not strong. If this graph were to possess a strong 2 co-coloring, one isolated vertex is added (refer Figure 2(c)) and each partite set is of cardinality 3. Since, ⊠(G) = 3, the vertex set cannot be partitioned into two independent sets. Clearly two cliques cannot cover G. Hence if G has a strong 2-cocoloring, then one partite set must induce a clique and other induce an independent set. In G, if one set induces K_3, then the other induces K_2UK_1 which is a contradiction. Hence G has no strong 2-cocolring and hence, $Z_s(G) \geq 3$. To have a strong

3 co-coloring, one isolated vertex is added (refer Figure 2(c)). Since $\chi(G) = 3$, a chromatic partition gives a strong 3 co-coloring



(a) (b) (c)

Figure 2

2. Prior Results

In this section, the existing results on co-chromatic number of some well-known graphs are given.

Proposition 2.1.

(i) For $n \geq 3$, $Z(P_n) = 2$

(ii) For $n \geq 1$, $Z(K_n) = 1$

(iii) For $n \geq 4$, $Z(C_n) = \begin{cases} 2, & if\ n\ is\ even \\ 3, & if\ n\ is\ odd \end{cases}$

(iv) For $m, n \geq 1$, $Z(K_{m,n}) = 2$

(v) For $m, n \geq 1$, $Z(B_{m,n}) = 2$

(vi) For $n \geq 5$, $Z(W_n) = \begin{cases} 1, & \text{if } n = 1 \\ 2, & if\ n = 2 \\ 3, & if\ n \geq 3 \end{cases}$

3. Strong co-chromatic number of some well known graphs

In this section, the exact bounds on strong co-chromatic number of P_n, K_n, C_n, $K_{1,n}$, and the friendship graph F_n are obtained.

Proposition 3.1.

(i) For $n \geq 3$, $Z_s(P_n) = 2$

(ii) For $n \geq 1$, $Z_s(K_n) = 1$

(iii) For $n \geq 4$, $Z_s(C_n) = \begin{cases} 2, & if\ n\ is\ even \\ 3, & if\ n\ is\ odd \end{cases}$

(iv) For $n \geq 2$, $Z_s(K_{1,n}) = \begin{cases} 2 & n = 2,3 \\ \frac{n-1}{3} + 2 & n \equiv 1 (\bmod 3) \\ \frac{n-2}{3} + 2 & n \equiv 2 (\bmod 3) \\ \frac{n}{3} + 1 & n \equiv 0 (\bmod 3) \end{cases}$

Proof:

Proof of (*i*): If n is even, then as P_n is a bipartite graph, the vertex set can be partitioned into two independent sets of size $\frac{n}{2}$. Suppose n is odd. Then in the vertex bi-partition, one set is of size one greater than the other. In this case, one isolated vertex is added in the set of smaller size and then the two sets are of same size. Hence, Z_s (P_n) =2 in both cases.

Proof of (ii): Trivial.

Proof of (*iii*): Suppose n is even, then as in the case of path, $Z_s(C_n) = 2$.

Now let n be odd. Let $V(C_n) = \{u_1, u_2, \ldots, u_n\}$.

First let us show that $Z_s(C_n) \geq 3$. On the contrary, suppose C_n has a strong 2 co-coloring. Let $P = \{S_1, S_2\}$ be the corresponding partition. If P is a clique partition, then P can cover only 4 vertices as ⊠$(C_n) = 2$, a contradiction. It P is a chromatic partition, then P must have atleast 3 sets as ⊠$(C_n) = 3$, a contradiction. Hence, one set in P must be an independent set and the other induces a clique. In this case also, P can cover only 4 vertices, a contradiction. Hence, $Z_s(C_n) \geq 3$. There are 3 cases to consider.

Case (i): $n \equiv 0$ (mod 3)

Let $n = 3r$ and let $S_1 = \{u_1, u_4, \ldots, u_{3r-2}\}$, $S_2 = \{u_2, u_5, \ldots, u_{3r-1}\}$and $S_3 = \{u_3, u_6, \ldots, u_{3r}\}$.

Clearly S_1, S_2 and S_3 are independent sets of size r.

Case (ii): $n \equiv 1$ (mod 3)

Let $n = 3r + 1$. Now add two isolated vertices w_1 and w_2. Let $S_1' = S_1 \cup \{w_1\}$, $S_2' = S_2 \cup \{u_{3r+1}\}$ and $S_3' = S_3 \cup \{w_2\}$. Then S_1', S_2' and S_3' are independent sets of size r+1.

Case (iii): $n \equiv 2$ (mod 3)

Let $n = 3r + 2$. Now add one isolated vertex w_1. Let $S_1' = S_1 \cup \{u_{3r+1}\}$, $S_2' = S_2 \cup \{u_{3r+2}\}$ and $S_3' = S_3 \cup \{w_1\}$. Then S_1', S_2' and S_3' are independent sets of size r + 1.

Hence, from all the three cases $Z_s(C_n) = 3$.

Proof of (*iv*): Throughout the proof u denotes the central vertex of $K_{1,n}$. There are 4 cases to consider.

Case (i): n = 2, 3

If n = 2, then $K_{1,n} = P_3$. Hence from (i), $Z_s(K_{1,2}) = 2$.

For n = 3, Figure 3 shows a strong 2 co-coloring into one clique and one independent set of size 2.

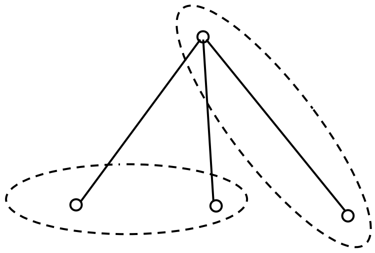


Figure 3

Case (ii): $n \equiv 1 \pmod 3$

Let n = 4. Clearly, $V(K_{1,4})$ cannot be partitioned into two sets with a newly added isolated vertex. But with one isolated vertex, it can be partitioned into three independent sets of size 2.

Hence, $Z_s(K_{1,4}) = 3 = \frac{4-1}{3} + 2$.

Again when n = 7, and if $K_{1,7}$ has a strong 3 co-coloring, then one isolated vertex is added and each set in the partition is of size 3. Then any set containing u can be of size atmost 2, a contradiction. Then $Z_s(K_{1,7}) \geq 4$. With 7 pendant vertices and one vertex of degree 7, $V(K_{1,7})$ can be partitioned into one clique and three independent sets, with each set is of size 2, which gives $Z_s(K_{1,7}) = 4$.

Now let $n = 3r + 1$, $r \geq 3$. We prove that Let us prove that $K_{1,n}$ has strong (r + 2) co-coloring and has no strong (r + 1) co-coloring. In a strong (r + 2) co-coloring, the number of newly added isolated vertices

$$= (r+2)\left\lceil \frac{3r+2}{r+2} \right\rceil - (3r+2)$$

$$= (r+2)\left\lceil \frac{3r+6-4}{r+2} \right\rceil - (3r+2)$$

$$= (r+2)\left\lceil 3 - \frac{4}{r+2} \right\rceil - (3r+2).$$

Since, $r \geq 3$, $\frac{4}{r+2} < 1$.

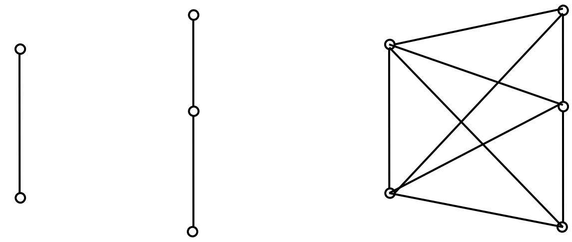
Thus, $\left\lceil 3 - \frac{4}{r+2} \right\rceil = 4$ and hence, the number of isolated vertices = (3r + 6)−(3r + 2) = 4.

Therefore the number vertices = n + 5 = 3r + 6 = 3(r + 2). Hence the vertex set is partitioned into r + 2 independent sets each of which is of size 3.

On the other hand, if $K_{1,n}$ has a strong (r + 1) co-coloring, then simple computation shows that the number of isolated vertices added = 1. Then the graph has 3r + 3 vertices. Hence vertex set can be partitioned into r + 1 sets of size 3. With one isolated vertex, in the partition, the set S containing u cannot be of size 3, a contradiction. Hence, $Z_s(K_{1,n}) = r + 2 = \frac{n-1}{3} + 2$.

By similar discussion as in case (ii), case (iii) and case (iv) can be proved.

Definition3.2. The join of two graphs G_1 and G_2 is the graph G whose vertex set is $V(G) = V_1 \cup V_2$ and the edge set E(G) consists of $E_1 \cup E_2$ together with all edges joining every vertex of V_1 with every vertex of V_2. It is denoted by $G_1 + G_2$.



(a) : G_1 (b) :G_2 (c) :G_1+G_2

Figure 4

For the graphs G_1 and G_2 given in Figure 4(a) and (b), the join $G_1 + G_2$ given in Figure 4(c).

Definition 3.3. The friendship graph F_n is constructed by joining n copies of the cycle C_3 with a common vertex.

It can also be defined as join of the graphs K_1 and nK_2 (n copies of K_2). Thus, $F_n = K_1 + nK_2$.

Notation 1: Throughout this paper, the following conventions are observed.

(i) $F_n = K_1 + nK_2$.

(ii) Vertex u corresponds to the vertex of K_1.

(iii) In any co-coloring partition, set S contains u.

(iv) The join of any graph G and K_1 of F_n is written as u + G instead of K_1+ G.

(v) u_iv_i ($1 \leq i \leq n$) denote the n edges of nK_2 in F_n.

(vi) w_i, i = 1, 2, … denote the newly added isolated vertices.

Observation 3.4.

If S is an independent set, then S cannot contain any vertex of F_n other than u.

If S induces a clique, then S cannot contain any newly added isolated vertex.

If S induces a clique of size 3, then S is the only set inducing clique in the partition.

⊠(F_n) = 3, ⊠(F_n) = n.

Any set in a co-coloring partition of size atleast 4 is an independent set.

In a strong co-coloring in w

Proposition 3.5. For $n \geq 2$, $Z_s(F_n) \geq 3$.

Proof. For $n \geq 2$, F_n is an incomplete connected graph and hence $Z_s(F_n) \geq 1$.

Suppose $Z_s(F_n) = 2$ and $\{S_1, S_2\}$ is a strong 2 co-chromatic partition of F_n. Since $|V(F_n)|$ is odd, only one isolated vertex is added. Let it be w. Since $n \geq 2$, $|V(F_n)| \geq 5$ and hence $|S_1|$, $|S_2| \geq 3$.

Suppose $u \in S_1$. If S_1 is an independent set, then each $v \in V(F_n) - u$, $v \notin \in S_1$. Hence, if $w \in S_1$, then $|S_1| = 2$ and if $w \notin S_1$, then $|S_1| = 1$, a contradiction. Therefore, S_1 must induce a clique. Since $n \geq 2$, $K_2 \cup K_1$ is an induced subgraph of $<S_2>$, again a contradiction.

Therefore, $Z_s(F_n) \geq 3$.

Proposition 3.6.

$$Z_s(F_n) = \begin{cases} 1, & if\ n = 1 \\ 3, & if\ 2 \leq n \leq 4 \\ \left\lceil \frac{n}{2} \right\rceil + 1, & if\ n \geq 5 \end{cases}$$

Proof. There are four cases to consider.

Case (i): For $n = 1$

$Z_s(F_n) = 1$ follows from the fact that $F_1 = K_1 + K_2 = K_3$.

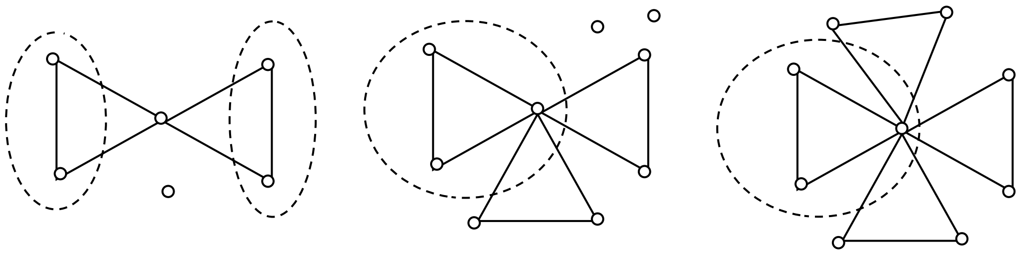
Case (ii): For $2 \leq n \leq 4$

From Proposition 3.3, $Z_s(F_n) \geq 3$. Suppose $Z_s(F_n) = 3$. Table 1 gives the number isolated vertices to be added to F_n to have a strong 3 co-coloring.

n	$\lvert V(F_n)\rvert$	No. of isolated vertices to be added
2	5	1
3	7	2
4	9	0

Table 1

For the graphs F_2, F_3, F_5, required number of isolated vertices are added and cliques are circled in Figure 5(a), 4 (b) and 4(c).



(a) : F_2 (b) : F_3 (c) : F_4

Figure 5

From Figure 4(a), it can be seen that $V(F_2)$ is partitioned into two cliques and one independent set and each of size 2. From Figure 4(b), removal of K_3 leaves $2K_2 \cup 2K_1$ as an induced sub graph, and this can be partitioned into 2 independent sets of size 3. From Figure 4(c),removal of K_3 leaves $3K_2$ as an induced sub graph, and this can be partitioned into 2 independent sets of size 3.

Case (iii): $5 \leq n \leq 7$

If $Z_s(F_5) = 3$, then as $|V(F_5)| = 11$, only one isolated vertex is added in any strong 3 co-coloring and each partite set is of size 4. Then from observation 3.3(iii), each set in the partition is an independent set. If $u \in S$ in the partition, then by observation 3.3, $|S| = 2$, a contradiction.

This leads to $Z_s(F_5) \geq 4$.

Suppose F_5 has a strong 4 co-coloring. Again only one isolated vertex is added in any strong 4 co-coloring and each partite set is of size 3. Hence the set S containing u must induce a clique. As before removal of K_3 leaves $4K_2 \cup K_1$ as an induced subgraph, and this can be partitioned into 3 independent sets of size 3. Hence, $Z_s(F_5) = 4 = \left\lceil \frac{5}{2} \right\rceil + 1$.

Now let us show that F_6 has a strong 4 co-coloring. For a strong 4 co-coloring, 3 isolated vertices are added and each set is of size 4. Hence each set is an independent set. If S is the set containing u in the partition, then S must contain all the newly added isolated vertices. Then the remaining graph is $6K_2$. If u_iv_i $(1 \leq i \leq 6)$ are the edges of $6K_2$, then the sets $S_1 = \{u_1, u_2, v_3, v_4\}$, $S_2 = \{u_3, u_4, v_5, v_6\}$ and $S_3 = \{u_5, u_6, v_1, v_2\}$ together S is a strong 4 co-coloring partition in which each set induces an independent set. Hence, $Z_s(F_6) = 4 = \left\lceil \frac{6}{2} \right\rceil + 1$.

Suppose $n = 7$. If $Z_s(F_7) = 4$, then only one isolated vertex can be added and each set is of size 4. Then each set in the partition is an independent. But the set S containing u is of size atmost 2, a contradiction. Hence, $Z_s(F_7) \geq 5$.

For a strong 5 co-coloring, isolated vertices are not added and each set is of size 3. Suppose $u \in S$. Hence S induces a clique and then remaining graph is $6K_2$.

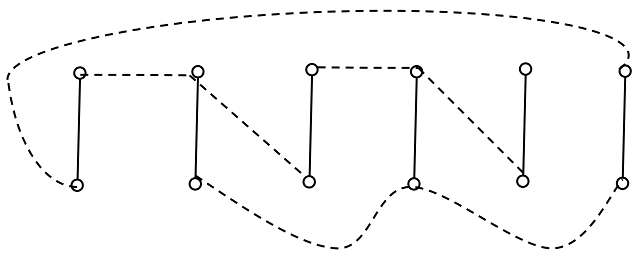


Figure 6

$V(6K_2)$ can be partitioned into four independent sets of size 3 as shown in Figure 6 (dashed lines show the independent sets of size 3). Hence, $Z_s(F_7) = 5 = \left\lceil \frac{7}{2} \right\rceil + 1$

Case(iv): $n \geq 8$ and n is odd

Let $n = 2r - 1$.Hence $r \geq 5$. Let us prove that F_n has strong $(r + 1)$ co-coloring and has no strong r co-coloring.

The number of isolated vertices to be added in a strong $(r + 1)$ co-coloring =

$$(r+1)\left\lceil \frac{2n+1}{r+1} \right\rceil - (2n+1) = (r+1)\left\lceil \frac{4r-1}{r+1} \right\rceil - (4r-1)$$

$$= (r+1)\left\lceil \frac{4r+4-5}{r+1} \right\rceil - (4r-1)$$

$$= (r+1)\left\lceil 4-\frac{5}{r+1} \right\rceil - (2n+1).$$

Since $r \geq 5$, $\frac{5}{r+1} < 1$, $\left\lceil 4-\frac{5}{r+1} \right\rceil = 4$.

Hence $(r+1)\left\lceil \frac{2n+1}{r+1} \right\rceil - (2n+1) = 5$. Hence the number of isolated vertices to be added is 5. Then the graph has $2n + 6 = 4r + 4$ vertices. Hence vertex set can be partitioned into r + 1 sets of size 4. Set S with u and 3 newly added isolated vertices is an independent set. The remaining vertices induce $K_2 \cup 2K_1$. Since n - 1 is even, there are 2 sub cases to consider.

Sub case (i): $n - 1 \equiv 0 \pmod 4$

Let $n - 1 = 4m$. Then $nK_2 \cup 2K_1 = (4m + 1)K_2 \cup 2K_1$. Hence $nK_2 \cup 2K_1$ can be decomposed into m + 1 groups such that each of the m groups consists of $4K_2$ and the $(m + 1)^{th}$ group consists of $K_2 \cup 2K_1$. $4K_2$ can be decomposed into two independent sets of size 4. Hence, each of the m groups gives rise to 2 independent sets. Consider the m^{th} and $(m + 1)^{th}$ groups. They induce $5K_2 \cup 2K_1$. If u_iv_i $(1 \leq i \leq 5)$ are the edges of $5K_2$, then the sets $S_1 = \{u_1, u_2, u_3, u_4\}$, $S_2 = \{u_5, v_1, v_2, w_1\}$ and $S_3 = \{v_3, v_4, v_5, w_2\}$ are independent sets. Hence the number of independent sets of size 4 = Number of groups formed of size 4 = $2(m - 1) + 1 + 3 = 2m + 2$

$= \frac{4m+4}{2} ==$ r + 1. Hence F_n has a strong $(r + 1)$ co-coloring. Thus, $Z_s (F_n) \leq r + 1$.

Suppose F_n has strong r co-coloring. Then number of isolated vertices to be added =

$r\left\lceil \frac{2n+1}{r} \right\rceil - (2n+1) = r\left\lceil \frac{4r-1}{r} \right\rceil - (4r-1) = r\left\lceil 4-\frac{1}{r} \right\rceil - (4r-1) = 1$. Then the graph has 4r vertices. In a strong r co-coloring, each partite set is of cardinality 4. Then by observation 3.3(e), each set in the partition is an independent set. By observation 3.3(a),

|S| can be atmost 2, a contradiction. Therefore, $Z_s(F_n) > r$ and hence the result follows. Hence, $Z_s(F_n) = r + 1 = \frac{n+1}{2} + 1 = \left\lceil \frac{n}{2} \right\rceil + 1$.

Subcase (ii): n $\equiv 2(\mathrm{mod}\ 4)$

Let n = 4m + 2. Then $nK_2 = (4m + 2)K_2$. They can be decomposed into m – 1 groups of $4K_2$ and a group consists of $6K_2$. These m – 1 groups can be partitioned into 2(m – 1) independent sets. give in sub case (i) $4mK_2$ gives 2m independent sets. The remaining graph $3K_2 \cup 2K_1$ gives 2 independent sets. Then the total number of independent sets =1 + 2m + 2 $= \frac{4m+6}{2} = \frac{n+3}{2}$ = r +1. By similar discussion as in sub case (i), F_n cannot have strong r co-coloring. Hence,

$$Z_s(F_n) = r + 1 = \frac{n+1}{2} + 1 = \left\lceil \frac{n}{2} \right\rceil + 1.$$

Case(v): n ≥ 8 and n is even

Let n = 2r, r ≥ 4. As in case (iv) we prove that F_n has a strong (r + 1) co-coloring and no strong r co-coloring. Number of isolated vertices to added for a strong (r +1) co-coloring

$$= (r + 1)\left\lceil \frac{2n+1}{r+1} \right\rceil - (2n + 1)$$

$$= (r + 1)\left\lceil \frac{4r+1}{r+1} \right\rceil - (4r + 1)$$

$$= (r + 1)\left\lceil 4 - \frac{3}{r+1} \right\rceil - (4r + 1).$$

Since r ≥ 4, $\left\lceil 4 - \frac{3}{r+1} \right\rceil = 4$. Hence, $(r + 1)\left\lceil \frac{2n+1}{r+1} \right\rceil - (2n + 1) = 3$

Then the graph has 2n + 4 = 4r + 4 vertices. Hence in a strong (r + 1) co-coloring, each partite set is of cardinality 4. Let S contain vertex u and the 3 isolated vertices. Then the remaining 4r vertices induce nK_2. Each $4K_2$ gives rise to two independent sets of size 4. Here again there are there are two sub cases to consider.

Subcase(i) n $\equiv 0(\mathrm{mod}\ 4)$

Let n = 4m. Hence $nK_2 = 4mK_2$ can be decomposed into m groups such that each group consists of $4K_2$. By similar discussion, number of independent sets of in the strong co-chromatic partition is 2m + 1 $= \frac{4m+2}{2} = \frac{n+2}{2} = \frac{n}{2} + 1 = \left\lceil \frac{n}{2} \right\rceil + 1$ = r + 1. Hence, $Z_s(F_n) \leq r + 1$. As in sub case (i) of case (v), F_n) cannot have a strong r co-coloring. Hence,

$$Z_s(F_n) = \left\lceil \frac{n}{2} \right\rceil + 1.$$

Subcase (ii): $n \equiv 2(\mathrm{mod}\,4)$

Let n = 4m + 2. Hence nK_2= (4m + 2)K_2. Now decompose into m groups such that m – 1 groups consist of $4K_2$ and one group consists $6K_2$. The m – 1 groups contribute 2(m – 1) independent sets of size 4 and from sub case (ii) of case (iv), $6K_2$ gives 3 independent sets of size 4. Hence, the total number of sets in the strong co-chromatic partition is 2(m – 1) + 3 + 1 = 2m + 2= $\frac{n}{2}$ + 1 = $\left\lceil \frac{n}{2} \right\rceil$ + 1.

Since, n = 2r = 4m + 2, r = 2m + 1. Hence, 2m + 2 = r + 1. Consequently, $Z_s(F_n) \le r + 1$. As in sub case (i) of case (v), F_n cannot have a strong r co-coloring. Hence, $Z_s(F_n) = \left\lceil \frac{n}{2} \right\rceil + 1$.

4. Conclusion

In this paper, the strong co-chromatic numbers of some well-known graphs are obtained. This can be extended to any general graphs having these well-known graphs as induced subgraphs.

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Authors' Profile:

Poobalaranjani Maharajan was born in Kovilpatty, Tirunelveli, India, in 1959. She received her B.Sc degree in Mathematics from the Madras University, India, in 1982, M.Sc, & M.Phil Mathematics from the Bharathidasan University, Tiruchirappalli , India in 1984 and 1990 respectively. In 2003, she received M.C.A degree from the Bharathidasan University. She did her research work in the National Institute of Technology, Tiruchirappalli (NITT) and received her Ph.D in 2006 from the Bharathidasan University. Her area of interest is Domination and coloring in Graph Theory. She joined as an Assistant Professor in the Department of Mathematics, Seethalakshmi Ramaswami College, Tiruchirappalli, India in 1986. Since then she was with the department. She became a Lecturer-Selection Grade in 2002 and in 2006, she became as a Reader. In 2015, she became the head of the Department of Mathematics.

R. Pichailakshmi was born in India in 1983. She received her M.Sc. and M.Phil degrees from Bharathidasan University, Trichy, Tamilnadu, India, in 2005, and 2006, respectively. She joined as a lecture in the Department of Mathematics, Seethalakshmi Ramaswami college, Trichy, Tamilnadu, India in 2009. Her main area of research interest is Graph Coloring. Currently she is doing her under the guidance of Dr. M. Poobalaranjani, Associate Professor & Head, PG and Research Department of Mathematics, Seethalakshmi Ramaswami college, Trichy, Tamilnadu,India,