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Further Results on Chromatic Number with Complementary Connected Perfect Domination Number of a Graph

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Abstract: *The concept of Complementary connected perfect domination number was introduced by G.Mahadevan et.alc., in [5]. A subset S of V of a non trivial graph G is said to be complementary connected perfect dominating set if S is a dominating and <V−S> is connected and has a perfect matching. The minimum cardinality taken over all complementary connected perfect dominating sets in G (CCPD-set) is called the complementary connected perfect domination number of G and is denoted by* γ_{ccp}*. In [6, the authors already characterized the extremal graphs whose sum of complementary connected domination number and chromatic number upto 2n-5. Since the characterization of extremal graphs whose sum of complementary connected domination number and chromatic number equals to 2n-6 for any n > 3*

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1. Introduction

By a graph G = (V, E) simple undirected connected graph. The concept of Complementary connected perfect domination number was introduced by G. Mahadevan et.al., in [5]. The A subset S of V of a non trivial graph G is said to be complementary connected perfect dominating set if S is a dominating set and <V−S> has a perfect matching and connected. The minimum cardinality taken over all Complementary connected perfect dominating sets in G (CCPD-set) is called the complementary connected perfect domination number of G and is denoted by γ_{ccp}. The minimum number of colours required to colour all the vertices in such a way that the adjacent vertices do not receive the same colour is called the chromatic number and is denoted by χ.

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We use the following notations in our further discussions.

Notation 1.1: Let H be a regular graph.

a) H (mP_k) is a graph obtained from H by attaching m times an end vertex of P_k to a vertex of H.

b) H $(m_1,m_2,\ldots,m_n)$ is a graph obtained from H by attaching m_i pendant edges to the vertex v_i, $1 \leq i \leq n$.

c) H $(m_1P_{k1},m_2P_{k2},\ldots,m_nP_{kn})$ is a graph obtained from H by attaching m_i times an end vertex of a path P_{ki} on k_i vertices to the vertex v_i, $1 \leq i \leq n$.

d) H $(u(P_n, P_m), m_2P_{k2},m_3P_{k3}, \ldots,m_nP_{kn})$ is the graph obtained from H by attaching an end vertex of P_n and an end vertex of P_m to a vertex $u=v_1$ of H and attaching the m_i times an end vertex of P_{ki} to the vertex v_i, $2 \leq i \leq n$.

Notation 1.2: P_k $(u(P_n, P_m))$ is the graph obtained from P_k by attaching an end vertex of P_n and an end vertex of P_m to an end vertex u of P_k.

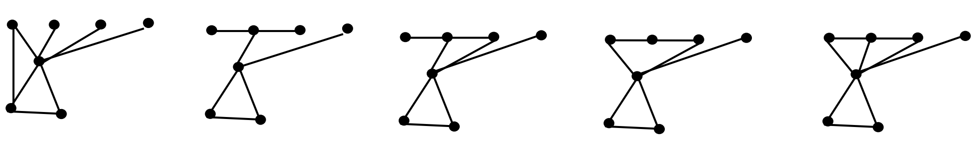
Notation 1.3: $P_k(mP_r, , nPs,)$ is the graph obtained from P_k by attaching m times an end vertex of P_r to an end vertex of P_n and by attaching n times an end vertex of Ps to the other end vertex of P_k

Notation 1.4: $P_n(C_r, C_s)$ is the graph by attaching one vertex of C_r and one vertex of C_s to the end vertices of P_n

Notation 1.5: P_n (mP_k, C_r) is the graph by attaching m times of P_k to an end vertex of P_n and attaching a vertex of C_r to other end vertex of $P_{n.}$

Theorem 1.6:[5] For any graph G, γ_{ccp} (G) = n if and only if G is a star.

Theorem 1.7:[6] Let G be a connected graph with γ_{ccp} = n − 2 and χ = n − 4. Then $\gamma_{ccp} + \chi$ = 2 n − 6, for any n > 3. if and only if G is isomorphic to G_i = { $C_4(P_3)$, $C_5(C_3)$ P_6, C_6, $P_3(u(P_2,P_3),0)$, $C_4(P_2,P_2,0,0)$,$C_3(P_5)$, $P_3(C_3,0,C_3)$, $C_4(2P_2)$, $P_2(2P_2,2P_2)$, $P_4(2P_2)$, $C_3(u(P_4,P_2),0,0)$, $C_3(P_4,P_2,0)$,$P_3(3P_3)$,$P_3(u(P_2,P_3),0)$, $C_3(u(P_3,2P_2),0,0)$, $C_3(u(P_3,P_2),0,0)$, $C_3(P_3,P_2,P_2)$, $P_3(2P_2)$, $P_2(3P_2,C_3)$, $P_3(3P_2,C_3)$, $K_4(4,0,0,0)$, $K_4(3,1,0,0)$, $K_4(2,2,0,0)$, $K_4(1,1,1,1)$, $K_3(4,0,0)$, $K_3(2,2,0)$, $K_3(3,1,0)$ and any one of the following figure 1.1}



G_1 G_2 G_3 G_4 G_5

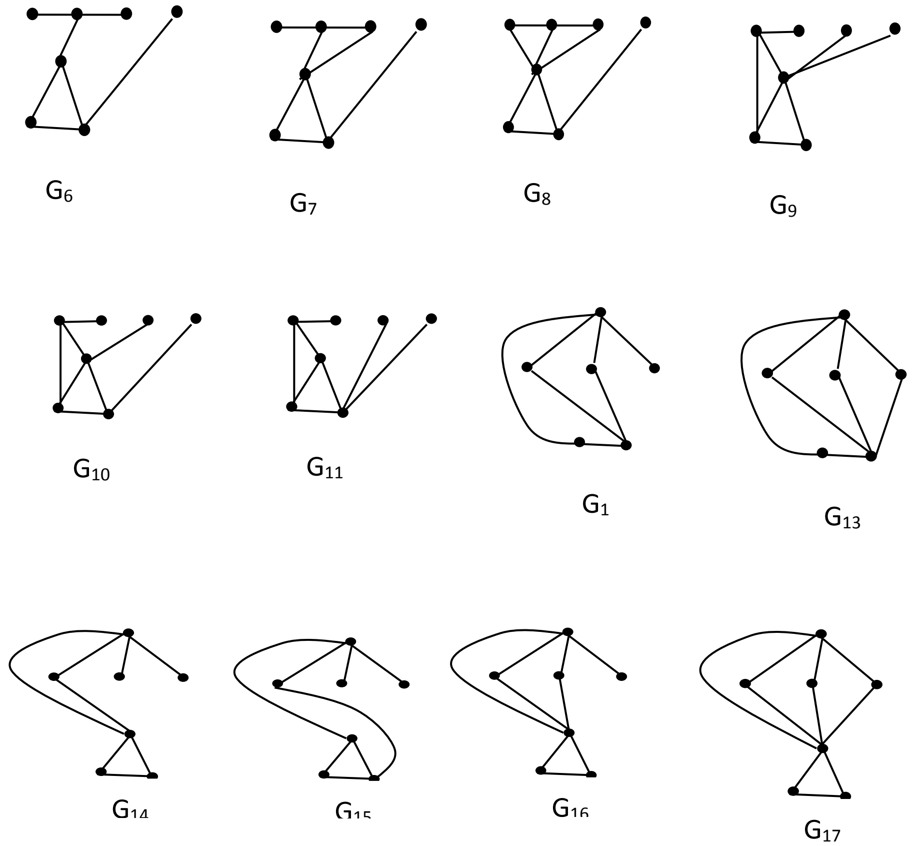
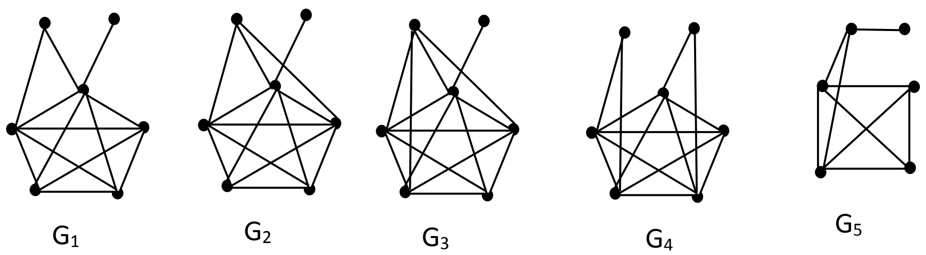


Figure 1.1

2. Main Result

In [6], It has been already characterized the extremal graphs whose sum of complementary connected domination number and chromatic number upto 2n-5. Since the characterization of graphs whose sum of complementary connected perfect domination number and chromatic number is equal to 2n-6

Theorem: 2.1 Let G be a connected graph with $\gamma_{ccp} = n - 4$ and $\chi = n - 2$. Then $\gamma_{ccp} + \chi = 2n - 6$, for any n >3 if and only if G is isomorphic to $K_6(2P_2)$, $K_6(1,1,0,0,0,0)$, $K_5(2P_2)$, $K_5(1,1,0,0,0)$, $K_4(P_3)$, G_i and any of the following graphs in figure 1.2.



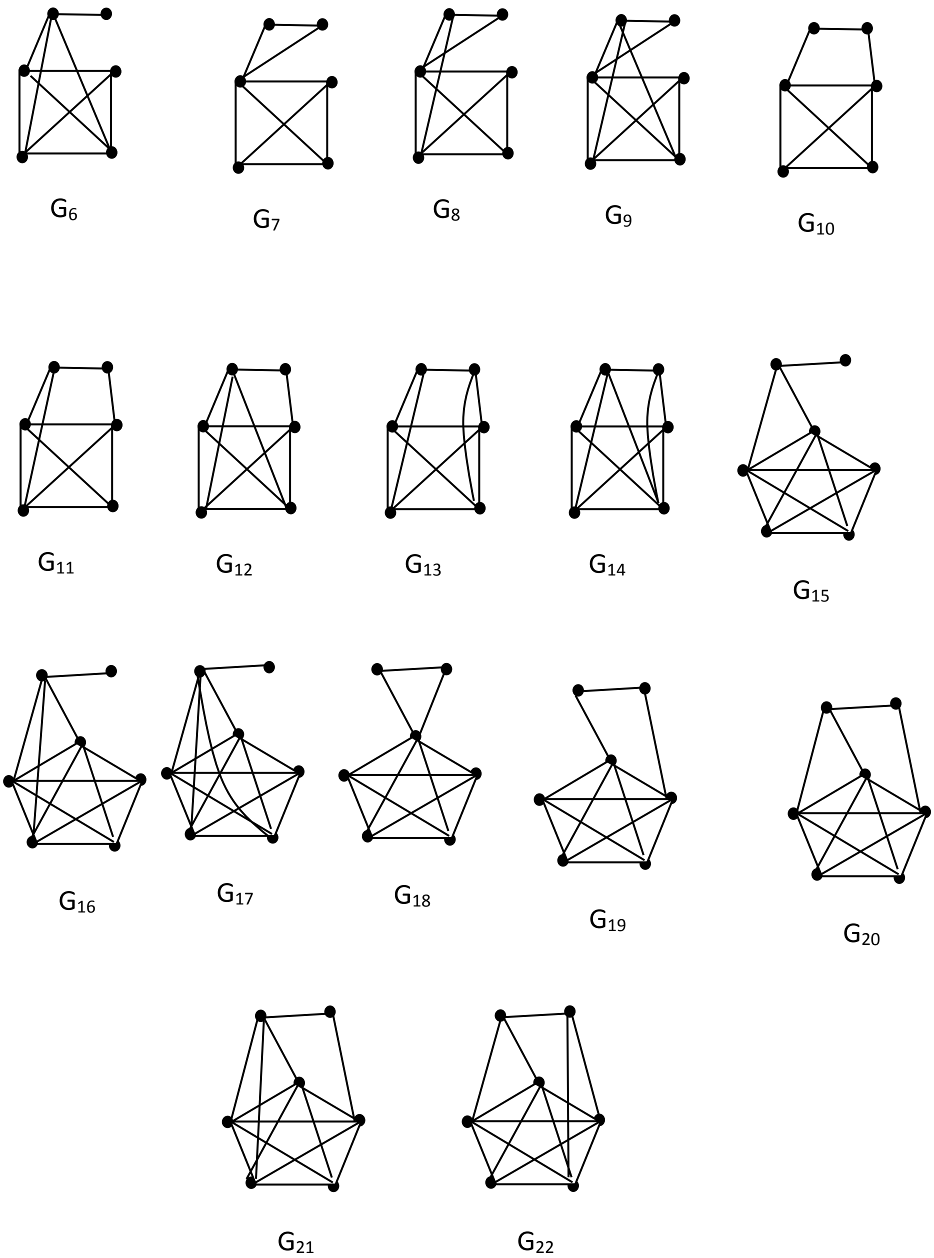


Figure 1.2

Proof: Let γ_{ccp} (G) + $\chi(G)$ = 2n−6, then $\gamma_{ccp} + \chi$ = 2 n− 6 for any n > 3. Then all the possible cases are (i) γ_{ccp} = n and χ = n−6 (ii) γ_{ccp} = n−1 and χ = n−5 , (iii) γ_{ccp} = n−2 and χ = n−4, (iv) γ_{ccp} = n−3 and χ = n−3, (v) γ_{ccp} = n−4 and χ = n−2, (vi) γ_{ccp} = n−5 and χ = n−1, (vii) γ_{ccp} = n−6 and χ = n.

The cases, (ii), (iv), (vi) < V –S > has odd number of vertices. Hence, it not possible to form a perfect matching. Hence in the all these cases, no graph exists. For the remaining cases, the graph exists. The case, (iii) already proved in [6].

Case (i) : γ_{ccp} = n and χ = n−6

Let γ_{ccp} = n, by theorem 1.3, G is a star. But for star χ = 2 so that n = 8. Hence G = $K_{1,7}$.

Case (ii): γ_{ccp} = n−4 and χ = n−2

Since χ = n − 2, G contains a clique K on n − 2 or does not contain a clique K on n − 2 vertices.. Let G contains a clique on K= K_{n-2} vertices and let S={v_1, v_2}∈V(G)-V(K). Then the induced sub graph<S> = $\overline{K_2}$, K_2

Sub case 1: <S> = $\overline{K_2}$

Let v_1 and v_2 be the vertices of $\overline{K_2}$ (i) If v_1 or v_2 is mapped to a single vertex say u_i of K_{n-2}.(ii) If v_1 and v_2 is mapped to different vertices of K_{n-2}.

a) Suppose K = K_{n-2} has even number of vertices, then{v_1,v_2,u_j,u_k} forms a γ_{ccp} set of G. Since $\gamma_{ccp} = n-2$ so that n = 8 and hence K = K_6. In this case the possible graphs are K_4(2,0,0,0,0,0), K_4(1,1,0,0,0,0). If d(v_1) > 1, then we get a contradiction to the hypothesis.

b) Suppose K= K_{n-2} has odd number of vertices {v_1,v_2 ,u_j} forms a γ_{ccp} set of G. Since $\gamma_{ccp} = n-2$ so that n = 7 and hence K= K_5. Let u_i adjacent to v_1 of K_3 then the possible graphs are K_5(2P_2), K_3(P_2,P_2,0,0,0). If v_1 and v_2 is adjacent to u_1 d(v_1) =2 and d(v_2) = 1 , then G ≅ G_1. If d(v_1) = 3 and d(v_2) = 1 , then G ≅ G_2. If d(v_1) = 4 and d(v_2) = 1 , then G ≅ G_3. If v_1 is adjacent to u_1 and v_2 is adjacent to u_2 d(v_1) = d(v_2) = 1,then G ≅ G_4. If d(v_1) = d(v_2) = 2,then G ≅ G_4.

Sub case2: <S> = K_2

Let v_1 and v_2 be the vertices of K_2 (i) If v_1 or v_2 is mapped to a single vertex say u_i of K_{n-2}. (or) (ii) If v_1 and v_2 is mapped to different vertices of K_{n-2}.

a) Suppose K = K_{n-2} has even number of vertices, then{v_1,u_j} forms a γ_{ccp} -set of G. Since $\gamma_{ccp} = n-2$, we have n = 6 and hence K = K_4. In case (i) the possible graphs are K_4(P_3), and if v_1 is adjacent to u_1 and d(v_1) = 3, d(v_2) = 1 , then G ≅ G_5. If d(v_1) = 4, d(v_2) = 1 ,then G ≅ G_6. If v_1 and v_2 is adjacent to u_1 and if d(v_1) = d(v_2) = 2 then

G $\cong$ G_7. If $d(v_1) = 3$, $d(v_2) = 2$, then G $\cong$ G_8. If $d(v_1) = 4$, $d(v_2) = 2$,then G $\cong$ G_9. If v_1 is adjacent to u_1 and v_2 is adjacent to u_2 $d(v_1) = d(v_2) = 2$,then G $\cong$ G_{10}. If $d(v_1) = 3$, $d(v_2) = 2$, then G $\cong$ G_{11}. If $d(v_1) = 4$, $d(v_2) = 2$,then G $\cong$ G_{12}. If $d(v_1) = d(v_2) = 3$, then G $\cong$ G_{13}. If $d(v_1) = 4$, $d(v_2) = 3$,then G $\cong$ G_{14}.

b) Suppose K= K_{n-2} has odd number of vertices, then $\{v_1,u_j,u_k\}$ forms a γ_{ccp} -set of G. Since $\gamma_{ccp} = n-2$, we have n = 7 and hence K= K_5 . Let u_1 be adjacent to v_1 of K_5 then the possible graph is $K_5(P_3)$. If v_1 is adjacent to u_1 and $d(v_1) = 3$, then G $\cong$ G_{15}. If $d(v_1) = 4$, then G $\cong$ G_{16}. If $d(v_1) = 5$, then G $\cong$ G_{17}. If v_1 is adjacent to u_1 and u_2 and $d(v_1) = d(v_2) = 2$, then G $\cong$ G_{18}. If v_1 is adjacent to u_1 and v_2 is adjacent to u_2 and $d(v_1) = d(v_2) = 2$,then G $\cong$ G_{19}. If $d(v_1) =3$ $d(v_2) = 2$,then G $\cong$ G_{20}. If $d(v_1) = 4$ $d(v_2) = 2$,then G $\cong$ G_{21}. If $d(v_1) = d(v_2) = 3$,then G $\cong$ G_{22}.

Case (iii): $\gamma_{ccp} = n-6$ and $\chi = n$

Since χ = n, G is a complete graph. If n is even then γ_{ccp} = 2 which gives n = 8 and hence G = K_8. If n is odd then γ_{ccp} = 1 which gives n = 7 and hence G = K_7. The converse is obvious.

3. Conclusion

In this paper, we characterized the concept of complementary connected perfect domination number *and chromatic number equals to 2n-6 for any n > 3* of a graph. The authors are also characterized the sum of complementary connected perfect domination number and chromatic number equals to 2p-7, 2p-8 which will be report in the subsequent papers.

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