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Radio Number for PVB-tree

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Abstract: *Let G be a connected graph with diameter diam(G). The PVB - tree is a tree consisting of b branches at each vertex of all the p copies of the path V on v vertices and these paths are joined to the vertices of a path P on p vertices. This PVB tree contains pvb + pv + p vertices and pvb + pv + p – 1 edges. The radio number for G, denoted by rn(G), is the smallest integer k such that there exists a function* $\lambda : V(G) \to \{0, 1, 2, \cdots, k\}$ *with* $|\lambda(u)-\lambda(v)| \geq diam(G)-d(u, v)+1$ *for all vertices u and v, where d(u, v) is the distance between u and v. We prove that the PVB - tree admits a radio labeling for all positive integers* $p \geq 3$, $v \geq 1$ *and* $b \geq 1$.

Keywords: *Diameter, Radio Number, PVB-tree*

1. Introduction

Radio labeling can be regarded as an extension of distance-two labeling and both of them are motivated by the channel assignment problem. Given a set of stations (or transmitters), a valid channel assignment is a function that assigns to each station with a channel (nonnegative integer) such that interference is avoided. The task is to find a valid channel assignment with the minimum span of the channels used. The degree (or level) of interference is related to the locations of the stations – the closer of two stations, the stronger interference that might occur. In order to avoid interference, the separation between the channels assigned to a pair of near-by stations must be large enough; the amount of the required separation depends on the distance between the two stations.

A graph model for this problem is to represent each station by a vertex, and connect every pair of close stations by an edge. Let G be a connected graph. We denote the distance between two vertices u and v by d(u, v). Motivated by the channel assignment problem with two levels of interference, a distance-two labeling for G is a function $\lambda: V \to \{0, 1, 2, 3, \cdots\}$ such that $|\lambda(u)-\lambda(v)| \geq 2$ if $d(u, v) = 1$; and $\lambda(u) \neq \lambda(v)$ if $d(u, v) = 2$. The span of f is defined as $\max \{ |\lambda(u)-\lambda(v)| / u, v \in V(G) \}$. The ⍰-number for a graph G, denoted by ⍰(G), is the minimum span of a distance-two labeling for G. Motivated by the channel assignment problem with diam(G) levels of interference, a multi-level distance labeling (or radio labeling) is a function $\lambda: V(G) \to \{0, 1, 2, 3, \cdots\}$ so that the following is satisfied: $|\lambda(u) - \lambda(v)| \geq diam(G) - d(u, v) + 1$, for $u, v \in V(G)$ where diam(G) is the diameter of G (the maximum distance over all pairs of vertices). The radio number (as suggested by the FM radio channel assignment for a graph G, denoted by rn(G), is the minimum span of a

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radio labeling for G. Note that when diam(G) = 2, distance-two labeling coincides with radio labeling, and in this case, λ(G)=rn(G). Finding the radio number for a graph is an interesting yet challenging task.So far, the value is known only to very limited families of graphs.

For paths and cycles, it was studied by Charchand el al. [3,2,9], while the exact value remained open until lately solved by Liu and Zhu [6]. The radio number for square paths (adding edges between vertices of distance two apart) was determined by Liu and Xie [7] who also studied the problem for square cycles [8].The aim of this article is to extend the study to a special tree called PVB-tree. We focus our study on the PVB-tree which is a generalized form of the familiar trees like spiders, ladders etc.

2. Basic Definitions

Definition 1:

The diameter of a graph G, diam(G) is the maximum distance between two vertices, i.e., the largest number of edges which must be traversed in order to travel from one vertex to another, following the shortest path between any two vertices.

Definition 2:

Let G be a connected graph with the set of all vertices V(G). A radio labeling of G is an injective function $\lambda : V(G) \rightarrow \mathbb{Z}^+$ satisfying the radio condition;

$$|\lambda(u)-\lambda(v)| \geq \text{diam}(G) - d(u, v) + 1$$

for any two vertices u, v $\in$ V (G)

Definition 3:

The variation of a radio labeling f, called span(f), is the maximum value that f can take. We call S(G,f) the set of consecutive numbers {m,m+1,…..M}

Where

$$m = \min_{u \in V(G)} f(u)$$

and

$$M = \max_{u \in V(G)} f(u)$$

Then

$$\text{Span}(f) = M - m$$

The Radio Number of G, written as rn(G), is the smallest variation of all radio labelings of G

$$rn(G) = \min_{f} \text{span } f$$

Definition 4:

A radio labeling for G with span equal to rn (G) is called an optimal radio labeling.

Definition 5:

Let T be a tree rooted at a vertex w. We define level function on V(T) by

$L_w(u) = d(w,u)$ for any $u \in V(T)$

The weight of T (rooted) at w is defined by

$$w_T(w) = \sum_{u \in V(T)} L_w(u)$$

The weight of T is the smallest weight among all vertices of T

$$w(T) = \min\{ \mathbf{w_T}(w) : w \in V(T)\}$$

3. Construction Of PVB-Tee

Consider a path P (main path) on p vertices . Take p copies of another path V (sub path) on v vertices. At each vertex of each copy of these sub paths attach b branches. These paths are joined to the main path P one at a vertex of the main path. The vertices of P are denoted by $x_1, x_2, \ldots, x_p$. The vertices of the sub paths V are denoted by x_{ij}, $i = 1,2,\ldots,p$, $j = 1,2,\ldots,v$. The vertices of the branches are denoted by x_{ij}^k, $i = 1,2,\ldots,p$, $j = 1,2,\ldots,v$ and $k = 1,2,\ldots,b$. The tree thus obtained is the PVB tree. The number of vertices of the PVB tree is pvb + p, the number of edges is pvb + pv + p – 1 and the diameter this PVB-tree diam (T) =p+2v +1.

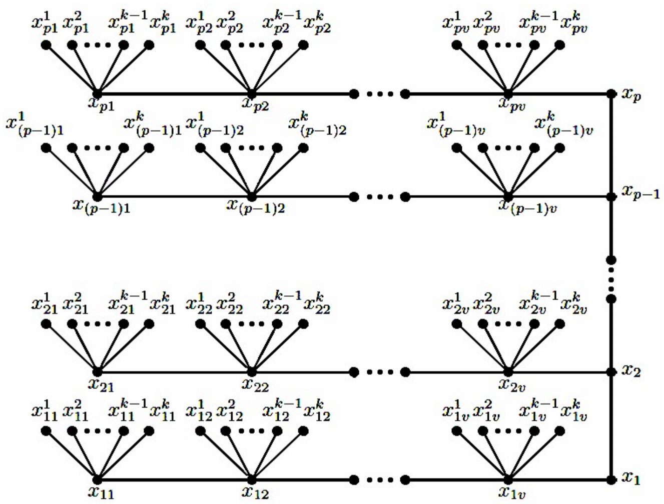

Figure 1. Generalized PVB-tree

Theorem :

Let T be a PVB-tree on p-odd vertices on P, v vertices on V and b branches, then weight of T is given by

$$w(T) = \frac{p}{2}\left[v(3b+1)+v^2(b+1)\right] + \frac{p^2-1}{4}\left[v(b+1)+1\right]$$

Proof :

The total weight of all vertices of a single path is $\frac{v(3b+1)+v^2(b+1)}{2} = x(\text{say})$ and the number of vertices in each path is $v(b+1)+1 = y(\text{say})$, then

$$w(T) = x + (p-1)x + p-1\left(y + \frac{p-3}{4}y\right)$$

$$= px + y\left(\frac{(p-1)(p+1)}{4}\right)$$

$$= px + y\left(\frac{p^2-1}{4}\right)$$

$$= \frac{p}{2}\left[v(3b+1)+v^2(b+1)\right] + \frac{p^2-1}{4}\left[v(b+1)+1\right]$$

Theorem 2:

Let T be a PVB-tree on p-even vertices on P, v vertices on V and b branches, then weight of T is given by

$$w(T) = \frac{p}{2}\left[v(3b+1)+v^2(b+1)\right] + \frac{p^2}{4}\left[v(b+1)+1\right]$$

Proof :

The total weight of all vertices of a single path is $\frac{v(3b+1)+v^2(b+1)}{2} = x(\text{say})$ an d the number of vertices in each path is $v(b+1)+1 = y(\text{say})$, then

$$w(T) = 2x + \frac{p}{2}y + p-2\left(x + 2y + \frac{p-4}{4}y\right)$$

$$= 2x + (p-2)x + \frac{p-2}{2}\left(2y + \frac{p-4}{4}y\right) + \frac{p}{2}y$$

$$= px + y\left(\frac{2p-4}{2} + \frac{(p-2)(p-4)}{2} + \frac{p}{2}\right)$$

$$= px + \frac{p^2}{4}y$$

$$= \frac{p}{2}\left[v(3b+1)+v^2(b+1)\right] + \frac{p^2}{4}\left[v(b+1)+1\right]$$

4. Main Theorems

In this section, we present the radio labeling and find the radio number of the newly constructed tree, called PVB-tree

Theorem 3:

Let T be a PVB-tree with pv(b+1)+p vertices having diameter p+2v+1 and p is odd,then rn(T)= (pvb+pv+p-1)(p+2v+2)+1-2w(T)

Proof:

Case 1: Suppose p = 3 and v = 1

For i = 1,2,3 and j=1

$$\lambda(x_{ij}^{k}) = \frac{p+2(2i+5k)-9}{2} \text{ if } k = 1,2,\ldots,b$$

$$\lambda(x_{ij}) = \frac{5(p+2b)+4(2i-j)-9}{2}$$

$$\lambda(x_i) = \frac{5(p+2b+i)+14}{2} \text{ for } i = 1 \text{ and } 3$$

$$\lambda(x_2) = 0$$

In this case rn(T) is obtained in x_p which is $\frac{5(p+2b)+29}{2}$ independent of v

Case 2: Suppose p = 3 and v = 2

For i = 1,2,3 and k = 1,2,…,b

$$\lambda(x_{ij}^{k}) = \begin{cases} \frac{p+4(i+8k)-29}{2} & \text{if j=1;} \\ \frac{p+8(i+4k)-21}{2} & \text{if j=2.} \end{cases}$$

For i = 1,2,3

$$\lambda(x_{ij}) = \begin{cases} \frac{3p+8(i+4b)-5}{2} & \text{if } j=1; \\ \frac{3(p+4j+5)+32b}{2} & \text{if } j=2. \end{cases}$$

$$\lambda(x_i) = \frac{3p+32b+7(i+8)}{2} \text{ for } i=1 \text{ and } 3$$

In this case rn(T) is obtained in x_p which is $\frac{3p+32b+77}{2}$, independent of v

Case 3: Suppose p > 3 is odd and v is either odd or even, where v < p

For j = 1 and k = 1,2,…,b

$$\lambda(x_{ij}^{k}) = \begin{cases} \frac{p(kp+1)+2(v+1)+k-p^2}{2} & \text{if i = 1;} \\ \frac{p^2(k-1)+p(2i-1)+2(v+1)+k}{2} & \text{if i = 2,}\ldots,\frac{p+1}{2}; \\ \frac{k(p^2+1)+2(v+p)-p^2+1}{2} & \text{if i = } \frac{p+3}{2}; \\ \frac{k(p^2+1)+2(v+ip)-p(2p+1)+1}{2} & \text{if i = } \frac{p+5}{2},\ldots,p. \end{cases}$$

For j = 1

$$\lambda(x_{ij}) = \begin{cases} \dfrac{b(p^2+1)+2i(p+4)+2v-p-3}{2} & \text{if } i=1,2,\ldots,\frac{p+1}{2}; \\ \dfrac{b(p^2+1)+2(p+4)+2v}{2} & \text{if } i=\frac{p+3}{2}; \\ \dfrac{b(p^2+1)+(p+4)[2(i+j)-p-3]+2v}{2} & \text{if } i=\frac{p+5}{2},\ldots,p. \end{cases}$$

Repeat the following equations (1),(2) and (3) in this order cyclically for j = 2,3,…,v

For k = 1

$$\lambda(x_{ij}^k) = \begin{cases} \lambda(x_{\frac{p+1}{2}\,j\text{-}1}) + \dfrac{p+4j-3}{2} & \text{if } i=1; \\ \lambda(x_{1j}^k) + \dfrac{2(i-1)(p+4j-4)}{2} & \text{if } i=2,3,\ldots,\frac{p+1}{2}; \\ \lambda(x_{1j}^k) + \dfrac{p+4j-5}{2} & \text{if } i=\frac{p+3}{2}; \\ \lambda(x_{\frac{p+3}{2}\,j}^k) + \dfrac{(2i-p-3)(p+4j-4)}{2} & \text{if } i=\frac{p+5}{2},\ldots,p. \end{cases} \quad (1)$$

For k = 2,3,…,b

$$\lambda(x_{ij}^k) = \begin{cases} \lambda(x_{ij}^{k-1}) + \dfrac{p^2+4p(j-1)+1}{2} & \text{if } i=1; \\ \lambda(x_{1j}^k) + \dfrac{2p(i-1)+8(ij-i-j+1)}{2} & \text{if } i=2,3,\ldots,\frac{p+1}{2}; \\ \lambda(x_{1j}^k) + \dfrac{p+4j-5}{2} & \text{if } i=\frac{p+3}{2}; \\ \lambda(x_{1j}^k) + \dfrac{(p+4j-5)(2i-p-2)+2i-p-3}{2} & \text{if } i=\frac{p+5}{2},\ldots,p. \end{cases} \quad (2)$$

$$\lambda(x_{ij}) = \begin{cases} \lambda(x_{\frac{p+1}{2}\,j}^b) + \dfrac{2i(p+4j)-p-4j-1}{2} & \text{if } i=1,2,\ldots,\frac{p+1}{2}; \\ \lambda(x_{1j}) + \dfrac{p+4j-1}{2} & \text{if } i=\frac{p+3}{2}; \\ \lambda(x_{\frac{p+3}{2}\,j}) + \dfrac{(2i-p-3)(p+4j)}{2} & \text{if } i=\frac{p+5}{2},\ldots,p \end{cases} \quad (3)$$

For p ≥ 7 consider the sequence 3,4, $\frac{p+3}{2}$, p-1, $\frac{p+5}{2}$, $\frac{p+3}{2}$ ($\frac{p-7}{2}$ pairs),…,4,2. This sequence contains q = p-1 terms. These terms are denoted by d_i,i=1,2,…,q. Transform this d_i to D_i by the formula $D_i = d - d_i + 1$,i=1,2,…,q, where d is the diam(T) Define the sequence $< N_i >$ by

$$N_i = \begin{cases} \lambda(x_{\frac{p+1}{2}v}) + D_i & \text{if } i=1; \\ N_{i-1} + D_i & \text{if } i=2,3,\ldots,q. \end{cases}$$

Rearrange the terms of sequence $< N_i >$ in the order $N_3, N_{q-3}, N_{q-5}, \ldots, N_5, N_1, N_{q-1}, N_q, N_2, N_{q-2}, N_{q-4}, \ldots, N_6, N_4$. Identify this new sequence as $\lambda(x_1), \lambda(x_2), \ldots, \lambda(x_{\frac{p-1}{2}}), \lambda(x_{\frac{p+3}{2}}), \ldots, \lambda(x_p)$ and take $\lambda(x_{\frac{p+1}{2}}) = 0$.

For p = 5 repeat the above procedure by defining d_1, d_2, d_3, d_4 to be 3,4,3,2 respectively. Arrange the sequence $< N_i >$ in the order $N_1, N_3, 0, N_4, N_2$ and identify with $\lambda(x_1), \lambda(x_2), \ldots, \lambda(x_5)$ .In this case, rn(T) is obtained in $x_{\frac{p+3}{2}}$.

Figure 2(a) and 2(b) are examples for the radio labeling of PVB-tree with p = 5, v = 3,b = 3 and p = 5,v = 2, b = 3

Example 2:

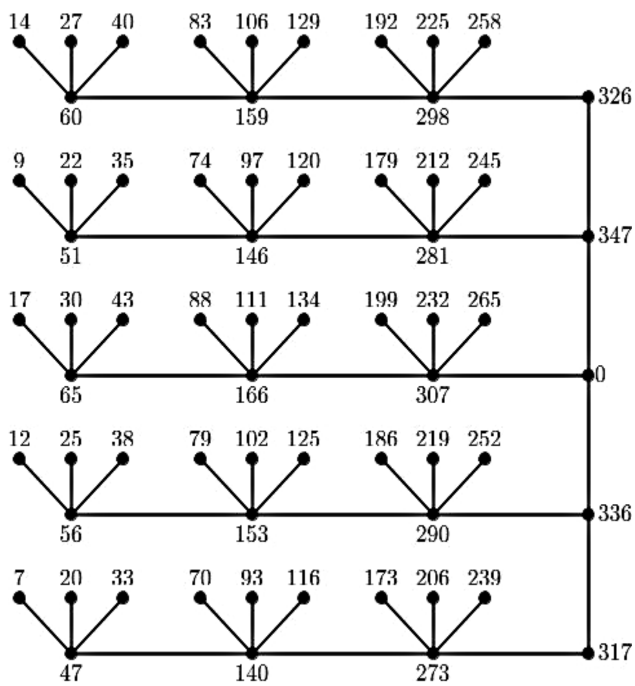

Figure 2(a)

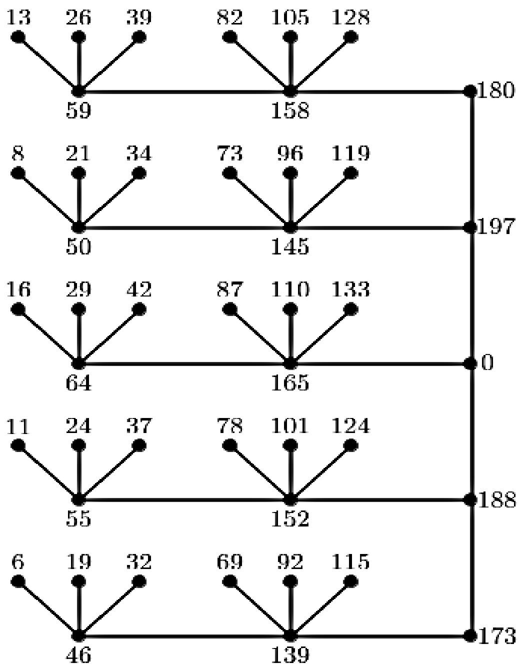

Figure 2(b)

Case 4: Suppose p and v are odd where $v \geq p$

For $i = 1,2,\ldots,\frac{p+1}{2}$, $j = 1$ and $k = 1$

$$\lambda(x_{ij}^{k}) = \frac{p+2v+3}{2} + (i-1)p$$

For $i = 1,2,\ldots,\frac{p+1}{2}$ and $k = 1$ repeat the equations (4a) and (4b) alternatively

For $j = \frac{v+3}{2},\ldots,v$

$$\lambda(x_{ij}^{k}) = \lambda(x^{k}_{\frac{p+1}{2}\,\frac{2j-v-1}{2}}) + \frac{(2i-1)(p+4j-4)-v}{2} \qquad (4a)$$

For $j = 2,3,\ldots,\frac{v+1}{2}$

$$\lambda(x_{ij}^{k}) = \lambda(x^{k}_{\frac{p+1}{2}\,\frac{v+2j-1}{2}}) + \frac{(2i-1)(p+4j-4)+v}{2} \qquad (4b)$$

For $k = 2,3,\ldots b$ also repeat the equations (5) and (4) (4a and 4b) alternatively

For $j = 1$

$$\lambda(x_{ij}^{k}) = \lambda(x^{k-1}_{\frac{p+1}{2}\,\frac{v+1}{2}}) + \frac{2pi-p+v}{2} \qquad (5)$$

For $j = 1,2,\ldots,v$ and $k = 1,2,\ldots,b$

$$\lambda(x_{ij}^{k}) = \begin{cases} \lambda(x_{1j}^{k}) + \dfrac{p+4j-5}{2} & \text{if } i = \frac{p+3}{2}; \\ \lambda(x_{1j}^{k}) + \dfrac{p(2i-p-2)+12j-13}{2} & \text{if } i = \frac{p+5}{2},\dots,p \end{cases}$$

For $i = 1,2,\dots,\frac{p+1}{2}$ and $j = 1$

$$\lambda(x_{ij}) = \lambda(x^{b}_{\frac{p+1}{2}\frac{v+1}{2}}) + \frac{p(2i-1)+8i+v-6}{2}$$

Apply the following equations (6) and (7) alternatively

For $i=1,2,\dots,\frac{p+1}{2}$ and $j=\frac{v+3}{2},\dots,v$

$$\lambda(x_{ij}) = \lambda(x_{\frac{p+1}{2}\frac{2j-v-1}{2}}) + \frac{(2i-1)(p+4j)-v}{2} \qquad (6)$$

For $i=1,2,\dots,\frac{p+1}{2}$ and $j=2,3,\dots,\frac{v+1}{2}$.

$$\lambda(x_{ij}) = \lambda(x_{\frac{p+1}{2}\frac{v+2j-1}{2}}) + \frac{(2i-1)(p+4j)+v}{2} \qquad (7)$$

For $j = 1,2,\dots,v$

$$\lambda(x_{ij}) = \begin{cases} \lambda(x_{1j}) + \dfrac{p+4j-1}{2} & \text{if } i = \frac{p+3}{2} \\ \lambda(x_{\frac{p+3}{2}j}) + \dfrac{2i(p+4j)-(p^2+3p)-4j(p+3)}{2} & \text{if } i=\frac{p+5}{2},\dots,p \end{cases}$$

For $p \geq 7$ consider the sequence $\frac{v+5}{2}$,4, $\frac{p+3}{2}$, p-1, $\frac{p+5}{2}$, $\frac{p+3}{2}$ ($\frac{p-7}{2}$ pairs),…,4,2. This sequence contains q = p-1 terms. These terms are denoted by d_i,i=1,2,…,q. Transform this d_i to D_i by the formula $D_i = d - d_i + 1, i=1,2,\dots,q$. Define the sequence $< N_i >$ by

$$N_i = \begin{cases} \lambda(x_{\frac{p+1}{2}\frac{v+1}{2}}) + D_i & \text{if } i=1; \\ N_{i-1} + D_i & \text{if } i=2,3,\dots,q. \end{cases}$$

Rearrange the terms of sequence $< N_i >$ in the order $N_3, N_{q-3}, N_{q-5},\dots, N_5, N_1, N_{q-1}, N_q, N_2, N_{q-2}, N_{q-4},\dots, N_6, N_4$. Identify this new sequence as $\lambda(x_1),\lambda(x_2),\dots,\lambda(x_{\frac{p-1}{2}}),\lambda(x_{\frac{p+3}{2}}),\ \dots,\lambda(x_p)$ and take $\lambda(x_{\frac{p+1}{2}}) = 0$.

For p = 5 take $\frac{v+5}{2}$,4,3,2 to be d_1,d_2,d_3,d_4 and take $\frac{v+3}{2}$,2 to be d_1,d_2 for p =3 respectively. Rearrange the terms of the sequence $< N_i >$ in the order $N_1,N_3,0,N_4,N_2$ for p = 5 and $N_1,0,N_2$ for p = 3 identify with $\lambda(x_1),\lambda(x_2),\dots,\lambda(x_p)$.

In this case also rn(T) is obtained in $x_{\frac{p+3}{2}}$.

Figure 3 is an examples for the radio labeling of PVB-tree with p = 5, v = 5, b = 3

Example 3:

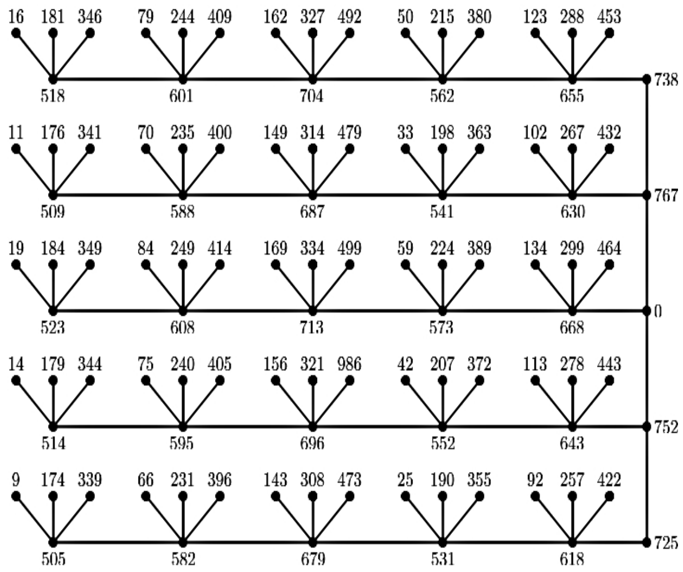

Figure 3

Case 5: Suppose p is odd and v is even where v ≥ p

For i= 1,2,..., $\frac{p+1}{2}$, j = 1 and k=1

$$\lambda(x_{ij}^{k}) = \frac{p+2v+3}{2} + (i-1)p$$

For i= 1,2,..., $\frac{p+1}{2}$, j = 1 and k=1 repeat the equations (8) and (9) alternatively followed by equations (10) and (11)

For j = $\frac{v+4}{2}$,..., v

$$\lambda(x_{ij}^{k}) = \lambda(x^{k}_{\frac{p+1}{2}\,\frac{2j-v-2}{2}}) + \frac{p(2i-1)+8ij-8i-4j-v+3}{2} \qquad (8)$$

For j = 2,3,..., $\frac{v}{2}$

$$\lambda(x_{ij}^{k}) = \lambda(x^{k}_{\frac{p+1}{2}\,\frac{2j+v}{2}}) + \frac{p(2i-1)+8ij-8i-4j+v+5}{2} \qquad (9)$$

For i= 1,2,..., $\frac{p+1}{2}$, j = $\frac{v+2}{2}$ and k=1,2,...,b

$$\lambda(x_{ij}^{k}) = \lambda(x^{k}_{\frac{p+1}{2}\,\frac{v}{2}}) + \frac{p(2i-1)+8(ij-i-j)+v+7}{2} \qquad (10)$$

For i= 1,2,..., $\frac{p+1}{2}$, j = 1 and k=2,...,b

$$\lambda(x_{ij}^{k}) = \lambda(x^{k-1}_{\frac{p+1}{2}\,\frac{v+2}{2}}) + \frac{p(2i-1)+v+1}{2} \qquad (11)$$

Repeat the above procedure, starting with the equation (8) for k = 2,3,…,b

For j = 1,2,…,v and k = 1,2,..,b

$$\lambda(x_{ij}^{k}) = \begin{cases} \lambda(x_{1j}^{k}) + \dfrac{p+4j-5}{2} & \text{if } i = \frac{p+3}{2}; \\ \lambda(x_{1j}^{k}) + \dfrac{p(2i-p-2)+12j-13}{2} & \text{if } i = \frac{p+5}{2},\dots,p \end{cases}$$

For $i=1,2,\dots,\frac{p+1}{2}$ and $j=1$

$$\lambda(x_{ij}) = \lambda(x^{b}_{\frac{p+1}{2}\frac{v+2}{2}}) + \frac{p(2i-1)+8i+v-5}{2}$$

Apply the following equations (12) and (13) alternatively

For $i=1,2,\dots,\frac{p+1}{2}$ and $j=\frac{v+4}{2},\dots,v$

$$\lambda(x_{ij}) = \lambda(x_{\frac{p+1}{2}\frac{2j-v-2}{2}}) + \frac{(2i-1)(p+4j)-v-1}{2} \qquad (12)$$

For $i=1,2,\dots,\frac{p+1}{2}$ and $j=2,3,\dots,\frac{v+1}{2}$

$$\lambda(x_{ij}) = \lambda(x_{\frac{p+1}{2}\frac{2j+v}{2}}) + \frac{(2i-1)(p+4j)+v+1}{2} \qquad (13)$$

For $i=1,2,\dots,\frac{p+1}{2}$ and $j=\frac{v+2}{2}$

$$\lambda(x_{ij}) = \lambda(x_{\frac{p+1}{2}\frac{v}{2}}) + \frac{p(2i-1)+8j(i-1)+2v+3}{2}$$

For $j = 1,2,\dots,v$

$$\lambda(x_{ij}) = \begin{cases} \lambda(x_{1j}) + \dfrac{p+4j-1}{2} & \text{if } i = \frac{p+3}{2} \\ \lambda(x_{\frac{p+3}{2}j}) + \\ \dfrac{2i(p+4j)-(p^2+3p)-4j(p+3)}{2} & \text{if } i=\frac{p+5}{2},\dots,p \end{cases}$$

For $p \geq 7$ consider the sequence $\frac{v+4}{2}$,4, $\frac{p+3}{2}$, p-1, $\frac{p+5}{2}$, $\frac{p+3}{2}$ ($\frac{p-7}{2}$ pairs),…,4,2. This sequence contains $q = p-1$ terms. These terms are denoted by d_i, $i=1,2,\dots,q$. Transform this d_i to D_i by the formulae $D_i = d - d_i + 1, i=1,2,\dots,q$. Define the sequence $< N_i >$ by

$$N_i = \begin{cases} \lambda(x_{\frac{p+1}{2}\frac{v+2}{2}}) + D_i & \text{if } i=1; \\ N_{i-1} + D_i & \text{if } i=2,3,\dots,q. \end{cases}$$

Rearrange the terms of sequence $< N_i >$ in the order $N_3, N_{q-3}, N_{q-5}, \dots, N_5, N_1, N_{q-1}, N_q, N_2, N_{q-2}, N_{q-4}, \dots, N_6, N_4$. Identify this new sequence as $\lambda(x_1), \lambda(x_2), \dots, \lambda(x_{\frac{p-1}{2}}), \lambda(x_{\frac{p+3}{2}}), \dots, \lambda(x_p)$ and take $\lambda(x_{\frac{p+1}{2}}) = 0$.

For $p = 5$ take $\frac{v+4}{2}$,4,3,2 to be d_1, d_2, d_3, d_4 and take $\frac{v+2}{2}$,2 to be d_1, d_2 for $p = 3$ respectively. rearrange the terms of the sequence $< N_i >$ in the order $N_1, N_3, 0, N_4, N_2$ for $p = 5$ and $N_1, 0, N_2$ for $p = 3$ identify with $\lambda(x_1), \lambda(x_2), \dots, \lambda(x_p)$.

In this case also rn(T) is obtained in $x_{\frac{p+3}{2}}$.

Figure 4 is examples for the radio labeling of PVB-tree with p = 5, v = 6, b = 3

Example 4:

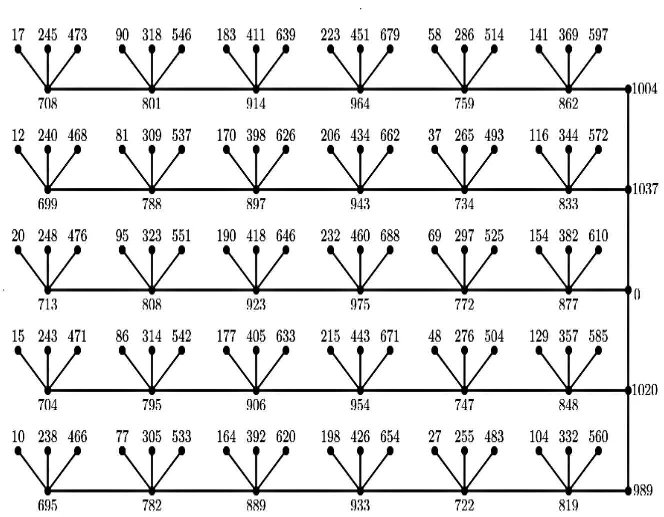

Figure 4

Theorem 4: Let T be a PVB-tree with pv(b+1)+p vertices having diameter p+2v+1 and p is even, then rn(T)= (pvb+pv+p-1)(p+2v+2)+1-2w(T)

Proof:

Case 1: If p is even and v is either odd or even where v-2 < p

For j = 1 and k = 1,2,…,b.

$$\lambda(x_{ij}^{k}) = \begin{cases} \dfrac{(p-1)(k(p+1)-p)+2(v+1)+k}{2} & \text{if } i=1; \\ \dfrac{(p-1)(k(p+1)-p+2(i-1))+2(v+1)+k}{2} & \text{if } i=2,3,\dots,\frac{p+2}{2}; \\ \dfrac{(p-1)(k(p+1)-p)+p+2v+k}{2} & \text{if } i=\frac{p+4}{2}; \\ \dfrac{(p-1)(k(p+1)-2(p+2-i))+p+2v+k}{2} & \text{if } i=\frac{p+6}{2},\dots,p. \end{cases}$$

For j=1

$$\lambda(x_{ij}) = \begin{cases} \dfrac{b(p^2-1)+2v+b-p+2i(p+3)+2}{2} & \text{if } i=1,2,\dots,\frac{p}{2}; \\ \dfrac{b(p^2-1)+p(p+4)+2v+b+2(2j-3)+2}{2} & \text{if } i=\frac{p+2}{2}; \\ \dfrac{b(p^2-1)+(2i-p)(p+3)+2(v-p-3)+b}{2} & \text{if } i=\frac{p+4}{2},\dots,p. \end{cases}$$

Repeat the following equations (14) (15) and (16) in this order cyclically for j=2,3,..,v

For k=1

$$\lambda(x_{ij}^k)=\begin{cases}\lambda(x_{\frac{p+2}{2}\,j-1})+\dfrac{p+4j-4}{2} & \text{if } i=1;\\ \lambda(x_{1j}^1)+\dfrac{2(i-1)(p+4j-5)}{2} & \text{if } i=2,3,\ldots,\frac{p}{2};\\ \lambda(x_{1j}^1)+\dfrac{p^2+4j(p-1)-5p+4}{2} & \text{if } i=\frac{p+2}{2};\\ \lambda(x_{1j}^1)+\dfrac{p+4j-6}{2} & \text{if } i=\frac{p+4}{2};\\ \lambda(x_{1j}^1)+\dfrac{(p+4j-5)(2i-p-3)-1}{2} & \text{if } i=\frac{p+6}{2},\ldots,p.\end{cases} \quad (14)$$

For k=2,3,….,b

$$\lambda(x_{ij}^k)=\begin{cases}\lambda(x_{1j}^{k-1})+\dfrac{p^2+4p(j-1)}{2} & \text{if } i=1;\\ \lambda(x_{1j}^k)+\dfrac{2(i-1)(p+4j-5)}{2} & \text{if } i=2,3,\ldots,\frac{p}{2};\\ \lambda(x_{1j}^k)+\dfrac{p(p+4j-5)-4(j-1)}{2} & \text{if } i=\frac{p+2}{2};\\ \lambda(x_{1j}^k)+\dfrac{2p(i+1)-4j(p+3)+14}{2}+\\ \dfrac{2i(4j-5)-p^2}{2} & \text{if } i=\frac{p+6}{2},\ldots,p.\end{cases} \quad (15)$$

$$\lambda(x_{ij})=\begin{cases}\lambda(x_{pj}^b)+2p+6(j-1)+\\ (i-1)(p+4j-1) & \text{if } i=1;\\ \lambda(x_{pj}^b)+2(p+2j-3)+\\ \dfrac{p(p+4j-1)}{2} & \text{if } i=2,3,\ldots,\frac{p}{2};\\ \lambda(x_{pj}^b)+\dfrac{5p+16j-14}{2} & \text{if } i=\frac{p+2}{2};\\ \lambda(x_{pj}^b)+\dfrac{(5p+16j-14)}{2}+\\ \dfrac{(2i-p-4)(p+4j-1)}{2} & \text{if } i=\frac{p+6}{2},\ldots,p.\end{cases} \quad (16)$$

For $p\geq 8$ consider the sequence $3,4,\frac{p+4}{2},p-1,\frac{p+4}{2},\frac{p+2}{2}(\frac{p-8}{2}\text{pairs}),\ldots,\frac{p+4}{2},\frac{p}{2},2$. This sequence contains q = p-1 terms these from are denoted by d_i,i=1,2,…,q. Transform this d_i to D_i by the formulae $D_i=d-d_i+1$,i=1,2,…,q. Define the sequence $<N_i>$ by

$$N_i=\begin{cases}\lambda(x_{\frac{p+2}{2}\,v})+D_i & \text{if } i=1;\\ N_{i-1}+D_i & \text{if } i=2,3,\ldots,q.\end{cases}$$

Rearrange the terms of the sequence $< N_i >$ in the order $N_3, N_{q-3}, N_{q-5}, \ldots, N_5, N_1, N_q, N_{q-1}, N_2, N_{q-3}, \ldots, N_6, N_4$. Identify this new sequence as $\lambda(x_1), \lambda(x_2), \ldots, \lambda(x_p)$ and take $\lambda(x_{\frac{p+2}{2}}) = 0$.

For p = 6 take $3, p-2, p-1, p-2, 2.$ to be d_1, d_2, d_3, d_4, d_5 and take 3,2,2 to be d_1, d_2, d_3 for p =4. Rearrange the terms of the sequence $< N_i >$ in the order $N_3, N_1, N_5, 0,\ N_4, N_2$ for p = 6 and $N_1, N_3, 0, N_2$ for p = 4 identify with $\lambda(x_1), \lambda(x_2), \ldots, \lambda(x_p)$.

In this case, rn(T) is obtained in $x_{\frac{p+4}{2}}$.

Figure 5(a) and 5(b) are examples for the radio labeling of PVB-tree with p = 6, v = 3, b = 3 and p = 6, v = 4, b = 3

Example 5:

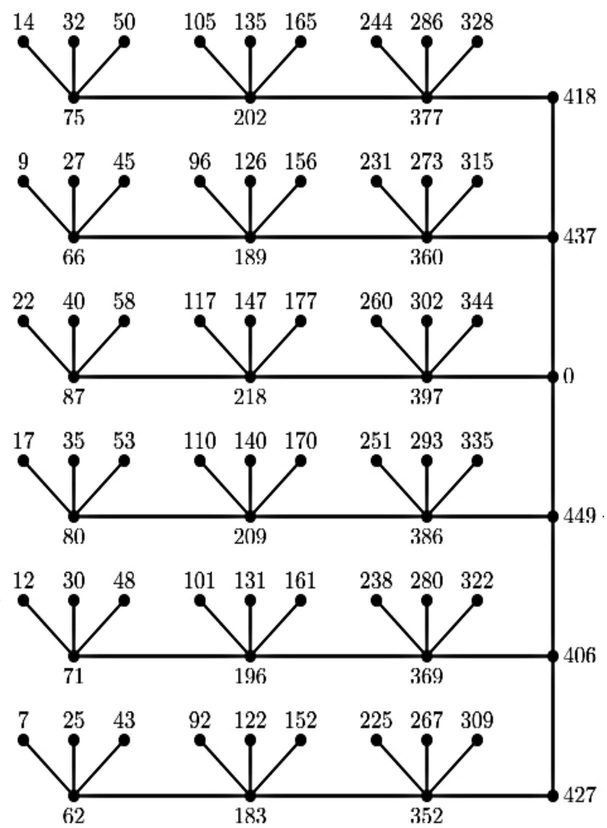

Figure 5(a)

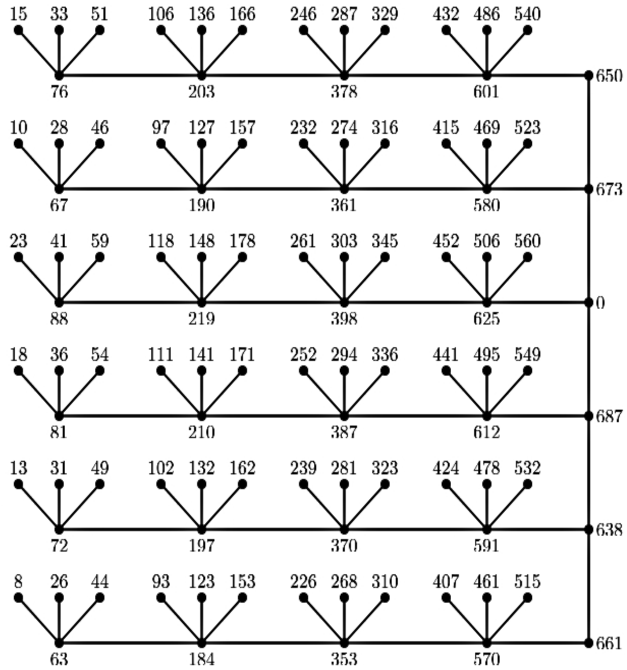

Figure 5(b)

Case 2: Suppose p is even and v is odd where $v \geq p-1$

For $i = 1,2,\ldots,\frac{p}{2}$, j=1 and k=1

$$\lambda(x_{ij}^{k}) = \frac{p+2v+2}{2} + (i-1)(p-1)$$

$i = 1,2,\ldots,\frac{p}{2}$ and k=1 repeat the following equations (17a) and (17b) alternatively

For $j = \frac{v+3}{2},\ldots,v$

$$\lambda(x_{ij}^{k}) = \lambda(x^{k}_{\frac{p+2}{2}\,\frac{2j-v-1}{2}}) + \frac{(2i-1)(p+4j-5)-v}{2} \qquad (17a)$$

For $j = 2,3,\ldots,\frac{v+1}{2}$

$$\lambda(x_{ij}^{k}) = \lambda(x^{k}_{\frac{p+2}{2}\,j+2}) + \frac{(2i-1)(p+4j-5)+v}{2} \qquad (17b)$$

For k = 2,3,…,b also repeat the equations (18) and (17) (17a and 17b) alternatively

For j= 1

$$\lambda(x_{ij}^{k}) = \lambda(x^{k}_{\frac{p+2}{2}\,\frac{v+1}{2}}) + \frac{p(2i-1)+2(i-1)(5-4j)+v-1}{2} \qquad (18)$$

For $i = \frac{p+2}{2}$, $j = 1,2,\ldots v$ and k=1,2,...,b

$$\lambda(x_{ij}^{k}) = \lambda(x_{1j}^{k}) + \frac{p(p-5)+4(j(p-1)+1)}{2}$$

For $i = \frac{p+2}{2}$, $j = 1,2,\ldots v$

$$\lambda(x_{ij}) = \lambda(x_{1j}) + \frac{(p-1)(p+4j)}{2}$$

For $i=1,2,\ldots,\frac{p}{2}$ and $j=1$

$$\lambda(x_{ij}) = \lambda(x^{b}_{\frac{p+2}{2}\frac{v+1}{2}}) + \frac{p(2i-1)+2(i-1)(4j-1)+v+1}{2}$$

Apply the following equations (19) and (20) alternatively for $i=1,2,\ldots,\frac{p}{2}$

$$\lambda(x_{ij}) = \begin{cases} \lambda(x_{\frac{p+2}{2}\frac{2j-v-1}{2}}) + \frac{(2i-1)(p+4j-1)-v}{2} & \text{if } j=\frac{v+3}{2},\ldots,v; \quad (19) \\ \lambda(x_{\frac{p+2}{2}\frac{v+2j-1}{2}}) + \frac{(2i-1)(p+4j-1)+v}{2} & \text{if } j=2,3,\ldots,\frac{v+1}{2}. \quad (20) \end{cases}$$

For j = 1,2,…,v and k =1,2,…,b

$$\lambda(x^{k}_{ij}) = \begin{cases} \lambda(x^{1}_{1j}) + \frac{p+4j-6}{2} & \text{if } i=\frac{p+4}{2}; \\ \lambda(x^{1}_{1j}) + \frac{(p+4j-5)(2i-p-3)-1}{2} & \text{if } i=\frac{p+6}{2},\ldots,p. \end{cases}$$

For j = 1,2,…,v

$$\lambda(x_{ij}) = \begin{cases} \lambda(x_{1j}) + \frac{p+4j-2}{2} & \text{if } i=\frac{p+4}{2}; \\ \lambda(x_{1j}) + \frac{3p+4(3j-1)}{2} & \text{if } i=\frac{p+6}{2},\ldots,p. \end{cases}$$

For $p \geq 8$ consider the sequence $\frac{v+5}{2}$,4, $\frac{p+4}{2}$,p-1, $\frac{p+4}{2}$, $\frac{p+2}{2}$($\frac{p-8}{2}$ pairs) ,…, $\frac{p+4}{2}$, $\frac{p}{2}$, 2. . This sequence contains q = p-1 terms these terms are denoted by d_i ,i=1,2,…,q. Transform this d_i to D_i by the formula $D_i = d - d_i + 1$,i=1,2,…,q. Define the sequence $< N_i >$ by

$$N_i = \begin{cases} \lambda(x_{\frac{p+2}{2}\frac{v+1}{2}}) + D_i & \text{if } i=1; \\ N_{i-1} + D_i & \text{if } i=2,3,\ldots,q. \end{cases}$$

Rearrange the terms of the sequence $< N_i >$ in the order $N_3, N_{q-3}, N_{q-5},\ldots, N_5, N_1, N_q, N_{q-1}, N_2, N_{q-3},\ldots, N_6, N_4$. Identify this new sequence as $\lambda(x_1),\lambda(x_2),...,\lambda(x_p)$ and take $\lambda(x_{\frac{p+2}{2}})=0$.

For p = 6 take $\frac{v+5}{2}, p-2, p-1, p-2, 2.$ to be d_1,d_2,d_3,d_4,d_5 and take $\frac{v+5}{2}$,3,2 to be d_1,d_2,d_3 for p =4 Rearrange the terms of the sequence $<N_i>$ in the order $N_3,N_1,N_5,0,N_4,N_2$ for p=6 and $N_1,N_3,0,N_2$ for p = 4 identify with $\lambda(x_1),\lambda(x_2),...,\lambda(x_p)$. In this case also rn(T) is obtained in $x_{\frac{p+4}{2}}$.

Figure 6 is an example for the radio labeling of PVB-tree with p = 6, v = 5, b = 3

Example 6:

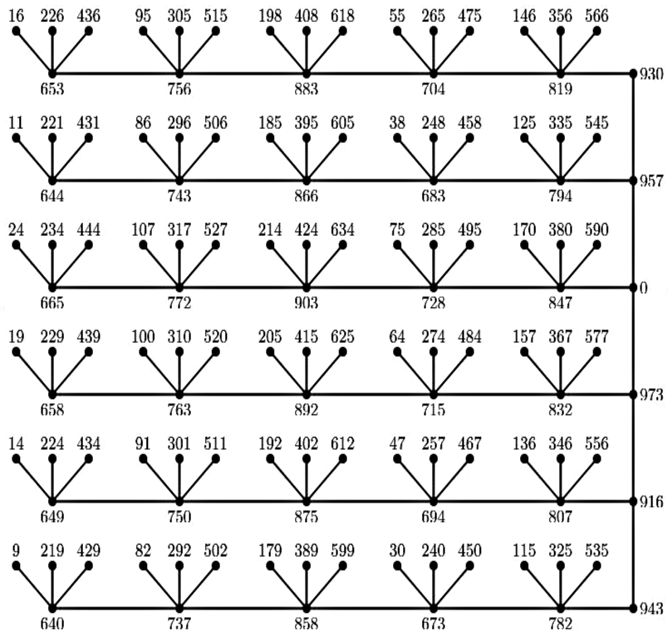

Figure 6

Case 3: Suppose p is even and v is even where v ≥ p

For i= 1,2,..., $\frac{p}{2}$,j=1 and k=1

$$\lambda(x_{ij}^k)=\frac{p+2v+2}{2}+(i-1)(p-1)$$

For i= 1,2,..., $\frac{p}{2}$ and k=1 repeat the equations (21) and (22) alternatively followed by equations (23) and (24)

For j= $\frac{v+4}{2}$,..., v

$$\lambda(x_{ij}^k)=\lambda(x^k_{\frac{p+2}{2}\frac{2j-v-2}{2}})+\frac{(2i-1)(p+4j)-2(5i-2)-v}{2} \qquad (21)$$

For j=2,3,...., $\frac{v}{2}$

$$\lambda(x_{ij}^k) = \lambda(x^k_{\frac{p+2}{2}\,\frac{2j+v}{2}}) + \frac{(2i-1)(p+4j)-2(5i-2)+v}{2} \quad (22)$$

For $j=\frac{v+2}{2}$

$$\lambda(x_{ij}^k) = \lambda(x^k_{\frac{p+2}{2}\,\frac{v}{2}}) + \frac{p(2i-1)+2[(i-1)(4j-5)+v-1]}{2} \quad (23)$$

For $j=1$

$$\lambda(x_{ij}^k) = \lambda(x^{k-1}_{\frac{p+2}{2}\,\frac{v+2}{2}}) + \frac{p+v}{2} \quad (24)$$

Repeat the above procedure starting with the equation (21) for k = 2,3,…,b

For $i=\frac{p+2}{2}$, $j=1,2,...v$ and $k=1,2,...,b$

$$\lambda(x_{ij}^k) = \lambda(x_{1j}^k) + \frac{p(p-5)+4(j(p-1)+1)}{2}$$

For $i=\frac{p+2}{2}$, $j=1,2,...v$

$$\lambda(x_{ij}) = \lambda(x_{1j}) + \frac{(p-1)(p+4j)}{2}$$

For $i=1,2,...,\frac{p}{2}$ and $j=1$

$$\lambda(x_{ij}) = \lambda(x^b_{\frac{p+2}{2}\,\frac{v+2}{2}}) + \frac{p(2i-1)+2(i-1)(4j-1)+v+2}{2}$$

Apply the following equations (25) and (26) alternatively for $i=1,2,...,\frac{p}{2}$

$$\lambda(x_{ij}) = \begin{cases} \lambda(x_{\frac{p+2}{2}\,\frac{2j-v-2}{2}}) + \dfrac{2i(p+4j-1)-p-4j-v}{2} & \text{if } j=\frac{v+4}{2},...,v; \quad (25) \\ \lambda(x_{\frac{p+2}{2}\,\frac{2j+v}{2}}) + \dfrac{2i(p+4j-1)-p-4j+v+2}{2} & \text{if } j=2,3,...,\frac{v}{2}; \quad (26) \end{cases}$$

For $i=1,2,...,\frac{p}{2}$ and $j=\frac{v+2}{2}$

$$\lambda(x_{ij}) = \lambda(x_{\frac{p+2}{2}\,\frac{v}{2}}) + \frac{2i(p+4j-1)-p-8j+2v+4}{2}$$

For j=1,2,…,v and k=1,2,…,b

$$\lambda(x_{ij}^k) = \begin{cases} \lambda(x_{1j}^1) + \dfrac{p+4j-6}{2} & \text{if } i=\frac{p+4}{2}; \\ \lambda(x_{1j}^1) + \dfrac{(p+4j-6)+(2i-p-4)(p+4j-5)}{2} & \text{if } i=\frac{p+6}{2},...,p. \end{cases}$$

For j=1,2,…,v

$$\lambda(x_{ij}) = \begin{cases} \lambda(x_{1j}) + \dfrac{p+4j-2}{2} & \text{if } i=\frac{p+4}{2}; \\ \lambda(x_{1j}) + \dfrac{3p+4(3j-1)}{2} & \text{if } i=\frac{p+6}{2},\ldots,p. \end{cases}$$

For p ≥ 8 consider the sequence $\frac{v+4}{2}$,4, $\frac{p+4}{2}$,p-1, $\frac{p+4}{2}$, $\frac{p+2}{2}$ ($\frac{p-8}{2}$ pairs) ,…, $\frac{p+4}{2}$, $\frac{p}{2}$, 2. . This sequence contains q = p-1 terms these terms are denoted by d_i ,i=1,2,…,q. Transform this d_i to D_i by the formulae $D_i = d - d_i + 1$,i=1,2,…,q. Define the sequence $< N_i >$ by

$$N_i = \begin{cases} \lambda(x_{\frac{p+2}{2}\frac{v+2}{2}}) + D_i & \text{if } i=1; \\ N_{i-1} + D_i & \text{if } i=2,3,\ldots,q. \end{cases}$$

Rearrange the terms of the sequence $< N_i >$ in the order $N_3, N_{q-3}, N_{q-5}, \ldots, N_5, N_1, N_q, N_{q-1}, N_2, N_{q-3}, \ldots, N_6, N_4$. Identify this new sequence as $\lambda(x_1), \lambda(x_2), \ldots, \lambda(x_p)$ and take $\lambda(x_{\frac{p+2}{2}}) = 0$.

For p = 6 take $\frac{v+4}{2}$, $p-2, p-1, p-2, 2$. to be d_1, d_2, d_3, d_4, d_5 and take $\frac{v+4}{2}$,3,2 to be d_1, d_2, d_3 for p =4. Rearrange the terms of the sequence $< N_i >$ in the order $N_3, N_1, N_5, 0, N_4, N_2$ for p = 6 and $N_1, N_3, 0, N_2$. for p = 4 identify with $\lambda(x_1), \lambda(x_2), \ldots, \lambda(x_p)$.

In this case also rn(T) is obtained in $x_{\frac{p+4}{2}}$.

Figure 7 is an example for the radio labeling of PVB-tree with p = 6, v = 6, b = 3

Example 7:

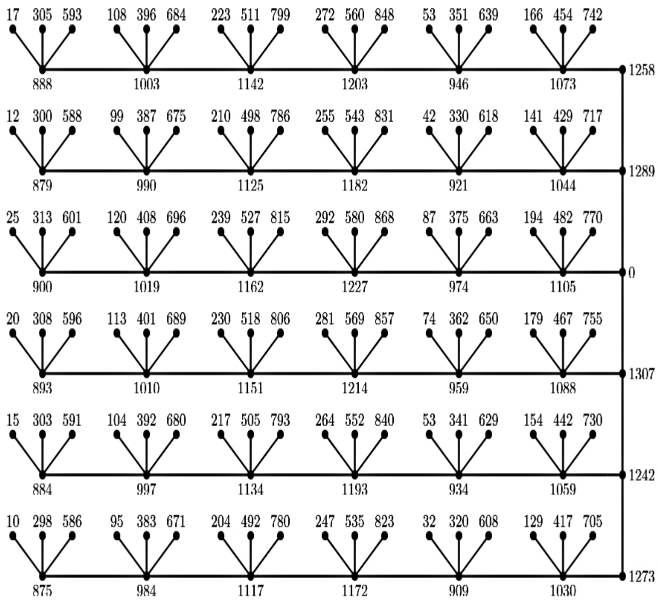

Figure 7

5. Conclusion

Graphs are used to represent networks of communications, data organization, computational device etc. The main objective of this paper is to familiarize the reader with the recently developed and interesting subject of Radio Labeling. Further studies on this PVB-tree may be done by giving variations to v and b.

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