International Journal of Engineering Science, Advanced Computing and Bio-Technology
Vol. 7, No. 4, October – December 2016, pp. 95 - 103

Odd Sum Labeling of Some More Graphs

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Abstract: *An injective function $f: V(G) \to \{0,1,2,\ldots,q\}$ is an odd sum labeling if the induced edge labeling f^* defined by $f^*(uv) = f(u)+f(v)$ for all $uv \in E(G)$ is a bijective and $f^*: E(G) \to \{1,3,5,\ldots,2q-1\}$. A graph is said to be an odd sum graph if it admits an odd sum labeling. In this paper we investigate odd sum labeling of some more graphs.*

Key words: *Odd sum Labeling, Odd Sum Graph, Slanting Ladder, shadow graph.*
AMS Subject Classification (2010):05C78

1. Introduction

Through out this paper, by a graph we mean a finite undirected simple graph. Let $G(V, E)$ be a graph with p vertices q edges. For notation and terminology, we follow [7]. For detailed survey of graph labeling we refer to Gallian[4].

In[9], the concept of mean labeling was introduced and further studied in [5, 6]. An injective function $f: V(G) \to \{0,1,2,\ldots,q\}$ is said to be a mean labeling if the induced edge labeling f^* defined by

$$f^*(uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

is injective and the resulting edge labels are distinct.

A graph G is said to be an odd mean graph if there exists an injective function $f: V(G) \to \{0,1,2,\ldots,2q-1\}$ such that the induced map $f^*(E(G)) = \{1,3,5,\ldots,2q-1\}$ defined by

$$f^*(uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

is a bijection[8].

In[1], the concept of odd sum labeling was introduced and studied[2,3]. An injective function $f: V(G) \to \{0,1,2,\ldots,q\}$ is an odd sum labeling if the induced edge labeling f^* defined by $f^*(uv) = f(u)+f(v)$ for all $uv \in E(G)$ is a bijection and

Received: 20 June, 2016; Revised: 07 September, 2016; Accepted: 02 November, 2016
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$f^*: E(G) \rightarrow \{1,3,5,\ldots,2q-1\}$. A graph is said to be an odd sum graph if it admits an odd sum labeling. In this paper we investigate odd sum labeling of some more graphs.

2. Main Results

Definition 2.1:

The slanting ladder SL_n is a graph obtained from two paths $u_1, u_2, \ldots\ldots\ldots u_n$ and $v_1, v_2, \ldots\ldots, v_n$ by joining each u_i with $v_{i+1}, 1 \le i \le n-1$.

Theorem 2.2:

The graph $SL_n\ (n \ge 2)$ is an odd sum graph.

Proof:

Let $\{u_i, v_i\ 1 \le i \le n\}$ be the vertices and $\{a_i, b_i, c_i, 1 \le i \le n-1\}$ be the edges which are denoted as in Figure 1.1

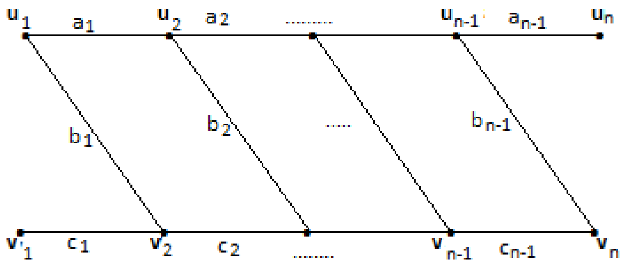

Figure 1.1: Ordinary labeling of SL_n

First we label the vertices as follows

Define $f: V \rightarrow \{0,1,2,\ldots.,q\}$ by

For $1 \le i \le n-1$ $\quad f(u_i) = 3i-1$

$f(u_n) = 3(n-1)$ $\quad f(v_1) = 0$

For $2 \le i \le n$ $\quad f(v_i) = 3i-5$

Then the induced edge labels are:

For $1 \le i \le n-1$ $\quad f^*(a_i) = 6i+1$

For $1 \le i \le n-1$ $\quad f^*(b_i) = 6i-3$ $\quad f^*(c_1) = 1$

For $2 \le i \le n-1$ $\quad f^*(c_i) = 6i-7$

Therefore $f^*(E) = \{1,3,5,\ldots.,2q-1\}$. So f is an odd sum labeling and hence, the graph $SL_n\ (n \ge 2)$ is an odd sum graph.

Odd sum labeling of SL_6 is shown in the Figure 1.2

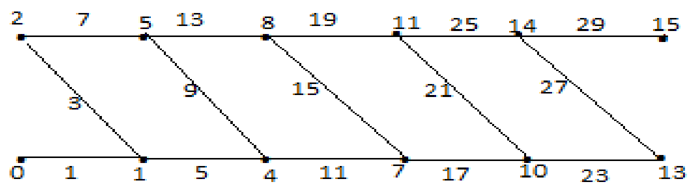

Figure 1.2: Odd sum labeling of SL_6

Definition 2.3:

The shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of G say G' & G'' join vertex u' in G' to the neighbors of corresponding vertex v' in G''.

Theorem 2.4:

The graph $D_2(K_{1,n})(n \geq 2)$ is an odd sum graph.

Proof:

Let $\{v, v_i, 1 \leq i \leq n,\ u, u_i, 1 \leq i \leq n\}$ be the vertices and $\{a_i, 1 \leq i \leq 4n\}$ be the edges which are denoted as in Figure 1.3

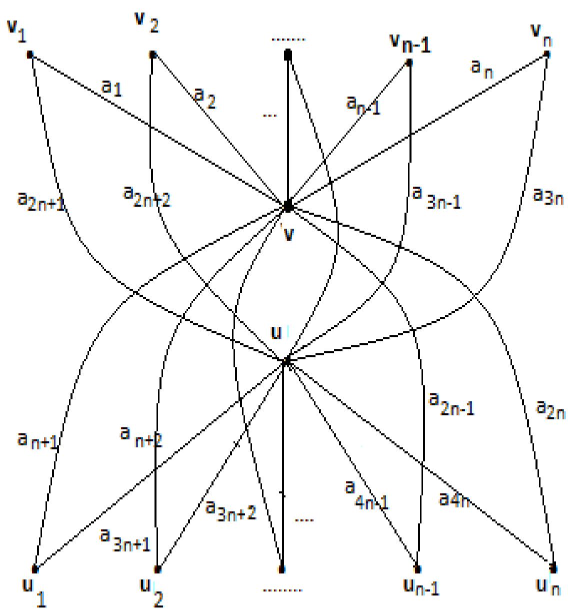

Figure 1.3: Ordinary labeling of $D_2(K_{1,n})$

First we labels the vertices as follows

Define $f: V \to \{0,1,2,....,q\}$ by

$f(v)=0$

For $1 \le i \le n$ $f(v_i) = 2i-1$ $f(u) = 4n$

For $1 \le i \le n$ $f(u_i) = 2n+2i-1$

Then the induced edge labels are:

For $1 \le i \le 4n$ $f^*(a_i) = 2i-1$

Therefore $f^*(E) = \{1,3,5......2q-1\}$. So f is an odd sum labeling and hence, the graph $D_2(K_{1,n})(n \ge 2)$ is an odd sum graph.

Odd sum labeling of $D_2(K_{1,5})$ shown in Figure 1.4

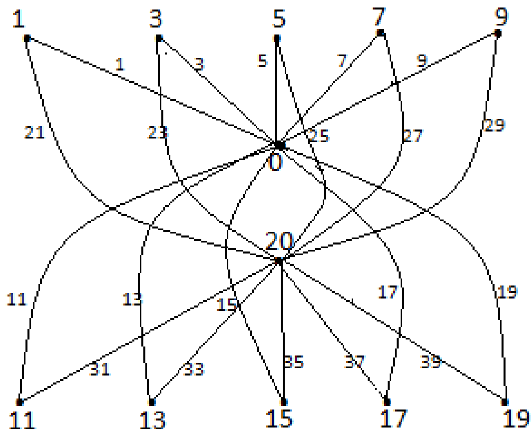

Figure 1.4: Odd sum labeling of $D_2(K_{1,5})$

Definition 2.5:

The Bistar graph $B_{m,n}$ is the graph obtained from K_2 by joining an m pendent edges to one end and n pendent edge to another end of K_2.

Theorem 2.6:

The graph $D_2(B_{n,n})(n \ge 3)$ is an odd sum graph.

Proof:

Let$\{ v, v_i, 1 \le i \le n,\ \ u, u_i, 1 \le i \le n,\ \ x, x_i, 1 \le i \le n,\ \ w, w_i, 1 \le i \le n\}$ be the vertices an $\{a_i, 1 \le i \le 2n,\ a'_i, 1 \le i \le n,\ a, a_i,\ 1 \le i \le n-1,\ b, 1 \le i \le 2n,\ b'_i\ 1 \le i \le n\}$ be the edges which are denoted as in Figure 1.5

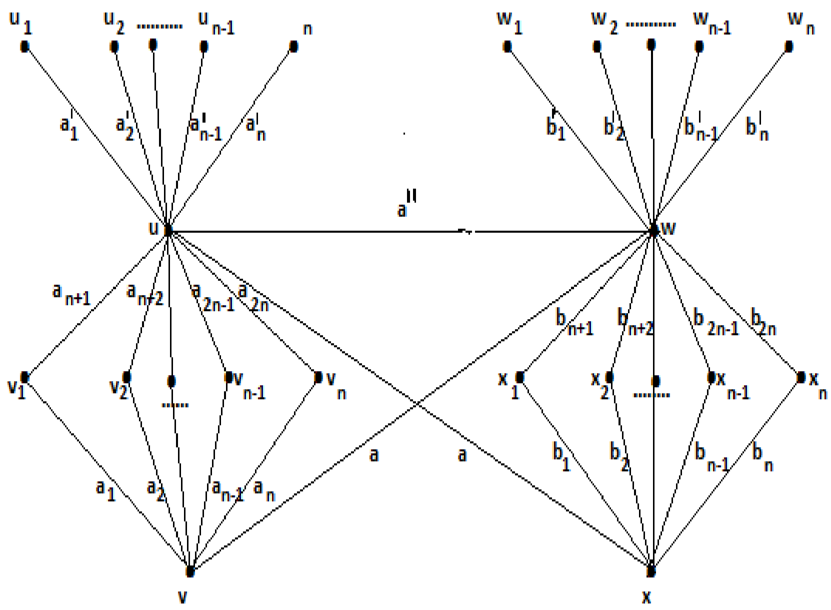

Figure 1.5: Ordinary labeling of $D_2(B_{n,n})$

First we label the vertices as follows

Define $f: V \to \{0,1,2,\ldots\ldots,q\}$

$f(v)=1$

For $1 \le i \le n$ $\quad f(v_i)=2(i-1)$ $\quad f(u)=2n+1$

For $1 \le i \le n$ $\quad f(u_i)=2n+2(i-1)$ $\quad f(x)=4n$

For $1 \le i \le n-1$ $\quad f(x_i)=2n+2i+3$

$f(x_n)=4n+5$ $\quad f(w)=6n+2$ $\quad f(w_1)=4n+3$

For $2 \le i \le n$ $\quad f(w_i)=4n+2i+3$

Then the induced edges labels are:

For $1 \le i \le 2n$ $\quad f^*(a_i)=2i-1$

For $1 \le i \le n$ $\quad f^*(a_i')=4n+2i-1$

$f^*(a)=6n+1$ $\quad f^*(a')=6n+3$ $\quad f^*(a'')=8n+3$

For $1 \le i \le n-1$ $\quad f^*(b_i)=6n+2i+3$

For $n+1 \le i \le 2n-1$ $\quad f^*(b_i)=6n+2i+5$

$f^*(b_{2n})=10n+7$ $\quad f^*(b_1')=10n+5$

For $2 \le i \le n$ $\quad f^*(b_i')=10n+2i+5$

Therefore $f^*(E) = \{1,3,5,....,2q-1\}$. So f is an odd sum labeling and hence, the graph $D_2(B_{n,n})(n \geq 3)$ is an odd sum graph.

Odd sum labeling of $D_2(B_{4,4})$ is shown in the Figure 1.6

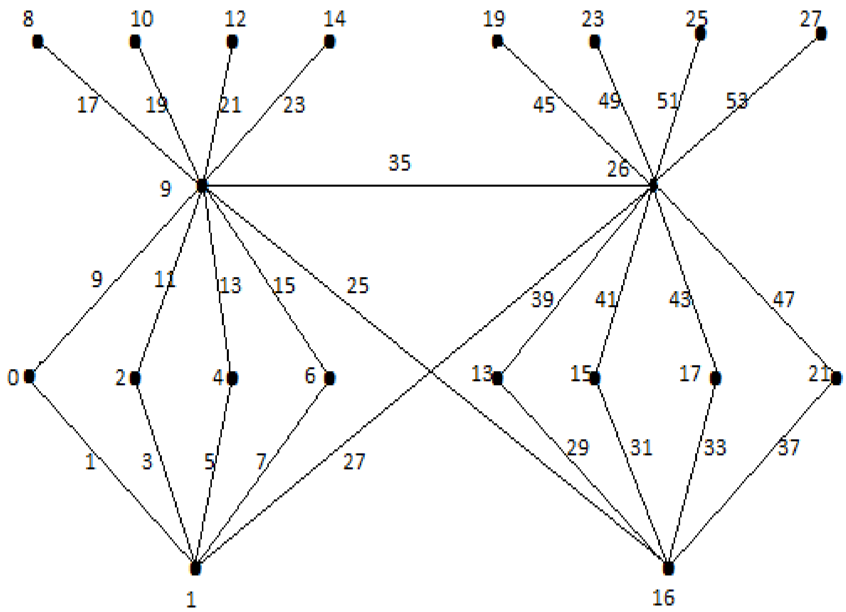

Figure 1.6: Odd sum labeling of $D_2(B_{4,4})$

Definition 2.7:

For a bipartite graph G with partite sets v_1 and v_2. Let G' be the copy of G and v_1' and v_2' be the copies of v_1 and v_2. The mirror graph $M(G)$ of g is obtained from G and G' by joining v_2 to its corresponding vertex in v_2' by an edge.

Theorem 2.8:

The mirror graph $M(P_n)(n \geq 4)$ is an odd sum graph.

Proof:

Let $\{v_i, v_i', 1 \leq i \leq n\}$ be the vertices and $\{e_i, e_i', 1 \leq i \leq n-1, a_i, 1 \leq i \leq n\}$ be the edges which are denoted as in Figure 1.7

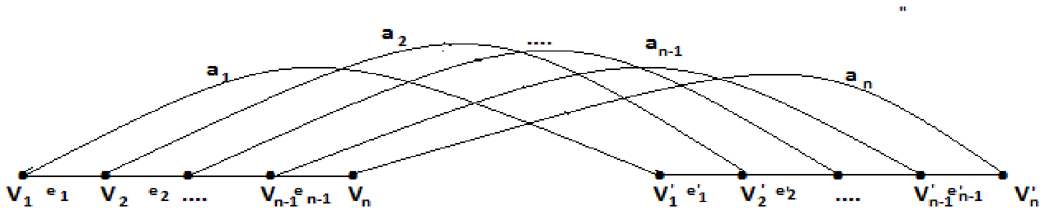

Figure 1.7: Ordinary labeling of $M(P_n)$

First we label the vertices as follows

Define $f : V \to \{0,1,2,\ldots,q\}$ by

For $1 \le i \le n$ $\quad f(v_i) = 3(i-1)$

For $1 \le i \le n$ $\quad f(v_i') = 3i-2$

Then the induced edges labels are :

For $1 \le i \le n$ $\quad f^*(a_i) = 6i-5$

For $1 \le i \le n-1$ $\quad f^*(e_i) = 6i-3$

For $1 \le i \le n-1$ $\quad f^*(e_i') = 6i-1$

Therefore $f^*(E) = \{1,3,5........2q-1\}$. So f is an odd sum labeling and hence, the graph $M(P_n)(n \ge 4)$ is an odd sum graph.

Odd sum labeling of $M(P_5)$ is shown in Figure 1.8

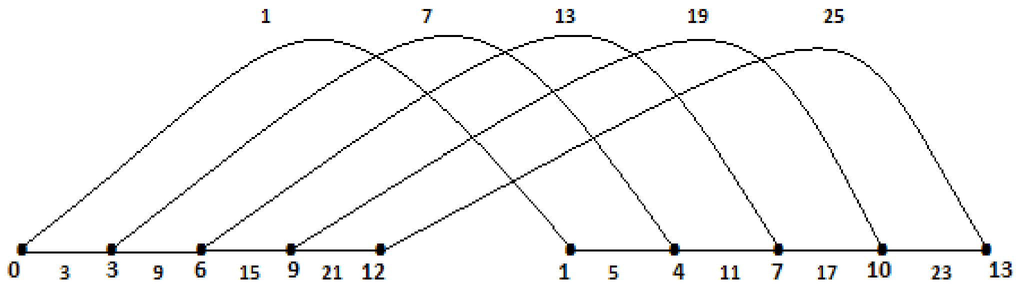

Figure 1.8: Odd sum labeling of $M(P_5)$

Definition 2.9:

Duplication of a vertex v of a graph G produces a new graph G' by adding a new vertex v' such that $N(v') = N(v)$. In other words a vertex v' is said to be vertex duplication of v if all the vertices of v in to G are also adjacent to v' and G'. It is denoted by $VD(G)$.

Theorem 2.10:

The graph $VD(P_n)(n \geq 4)$ is an odd sum graph.

Proof:

Let $\{v_1, v_i, 1 \leq i \leq n\}$ be the vertices and $\{e_1, e_2, e_i, 1 \leq i \leq n-1\}$ be the edges which are denoted as in Figure 1.9

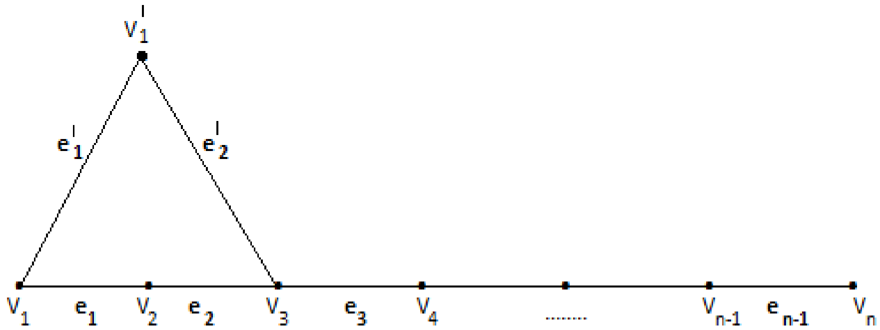

Figure 1.9: Ordinary labeling of $VD(P_n)$

First we label the vertices as follows

Define $f : V \to \{0, 1, 2, ..., q\}$ by

$f(v_1) = n+1$ $f(v_2) = n,$ $f(v_1') = n-2$

For $3 \leq i \leq n$ $f(v_i') = n-i$

Then the induced edge labels are:

$f^*(e_1') = 2n-1$ $f^{*}(e_2') = 2n-5$

$f^*(e_1) = 2n+1$ $f^*(e_2) = 2n-3$

For $3 \leq i \leq n-1$ $f^*(e_i) = 2n-2i-1$

Therefore $f^*(E) == \{1, 3, 5, ..., 2q-1\}$. So f is an odd sum labeling and hence, the graph $VD(P_n)(n \geq 4)$ is an odd sum graph.

Odd sum labeling of $VD(P_7)$ is shown in Figure 1.10

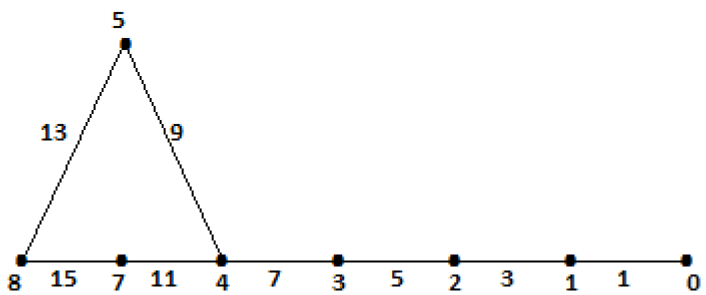

Figure 1.10: Odd sum labeling of $VD(P_7)$

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