International Journal of Engineering Science, Advanced Computing and Bio-Technology
Vol. 7, No. 4, October – December 2016, pp. 86 - 94

Medium domination Number of product of Path

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Abstract: *The medium domination number was introduced by Vargor D and Dundar P to find the number of vertices which protect the pairs of vertices in a graph. The total number of vertices that dominates all the pairs of vertices and evaluates the averages of this value is called medium domination number. In this chapter, we concentrate mainly to find the medium domination number of product of Path graphs such as Cardinal, Strong, Cartesian and equivalent paths graphs. For basic concepts and medium domination number of some standard graphs one can refer to [1].*

Key Words: *Cardinal, strong, Cartesian and equivalent product of path, medium domination Number of a graph.*

AMS Subject Classification: 05C69

1 Introduction

The authors Vargor D, Dundar P. discusses conceptually the Medium domination number of a graph and found MD number for some Standard Graphs. Motivated by the above, we obtain the MD number for product of path graphs. For G = (V, E) be a graph and for all u ,v ∈ V(G), if u and v are adjacent they dominate each other then dom(u,v)=1. The total number of vertices that dominate every pair of vertices is defined by ***total domination vertex*** and is denoted by TDV(G) = Σ dom(u,v), for all u,v ∈ V(G). For any connected graph G of order n, the ***medium domination number*** of G is defined as MD(G) = $\frac{TDV(G)}{nC_2}$.

Theorem 1.2[1]:

For G has p vertices, q edges and for deg(v_i) ≥ 2, TDV(G) = $q + \left\{\sum_{u_i \in V} \binom{\deg v_i}{2}\right\}$

For example, the Strong product of P_3 and P_3 is given by,

Received: 20 June, 2016; Revised: 02 August, 2016; Accepted: 19 September, 2016

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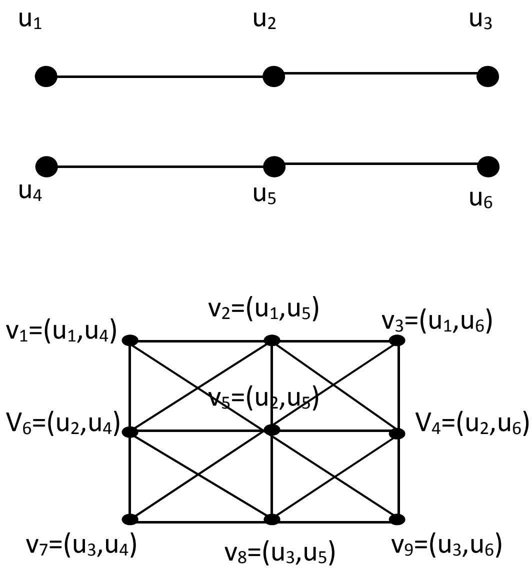

Figure 1.1

dom(v_1,v_2)=3, dom(v_1,v_3)=2, dom(v_1,v_4)=2, dom(v_1,v_5)=3, dom(v_1,v_6)=3, dom(v_1,v_7)=2, dom(v_1,v_8)=2, dom(v_1,v_9)=1. dom(v_2,v_3)=3, dom(v_2,v_4)=3, dom(v_2,v_5)=3, dom(v_2,v_6)=3. dom(v_2,v_7)=2, dom(v_2,v_8)=3, dom(v_2,v_9)=2. dom(v_3,v_4)=3, dom(v_3,v_5)=3, dom(v_3,v_6)=2, dom(v_3,v_7)=1, dom(v_3,v_8)=1, dom(v_3,v_9)=2. dom(v_4,v_5)=5, dom(v_4,v_6)=3, dom(v_4,v_7)=2, dom(v_4,v_8)=3, dom(v_4,v_9)=3. dom(v_5,v_6)=5, dom(v_5,v_7)=3, dom(v_5,v_8)=5, dom(v_5,v_9)=3. dom(v_6,v_7)=3, dom(v_6,v_8)=3, dom(v_6,v_9)=2, dom(v_7,v_8)=3, dom(v_7,v_9)=2, dom(v_8,v_9)=3.

TDV(G) = 97 , MDV(G) = TDV(G) /nC_2 = 97/9C_2

MD($P_3 \otimes P_3$) = 97/9C_2

2 Product of Path graphs

In [3], the concepts of product of path graphs were discussed. In this section we obtain the medium domination number of product of path graphs such as cardinal, strong, Cartesian and equivalent graphs.

Definitions 2.1 [3]:

On the ***strong product*** $G \otimes H$ of two paths G and H with the vertex set V(G) = (u_1,u_2,.....,u_m) and V(H) = (v_1,v_2,...v_n) then V($G \otimes H$) = V(G) $\otimes V(H)$ and $(u_1, v_1), (u_2, v_2) \in E(G \otimes H)$ if and only if (i) $(u_1, u_2) \in E(G)\ and\ (v_1, v_2) \in E(H)$ or (ii) $u_1 = u_2$ and $(v_1, v_2) \in E(H)$ or (iii) $v_1 = v_2$ and $(v_1, v_2) \in E(H)$. On the ***cardinal product*** $G \times H$ of two graphs G and H, with the vertex set V($G \times H$) = V(G) $\times V(H)$ and $(u_1, v_1), (u_2, v_2) \in E(G \times H\ )$ if and only if $(u_1, u_2) \in E(G)\ and\ (v_1, v_2) \in E(H)$. The ***Cartesian product*** of paths P_m and P_n with disjoint set of vertices V_m and V_n and edge sets E_m and $E_n$is the graph with vertex set $V(P_m \square P_n)$ and edge set $E(P_m \square P_n)$ such that any two vertices (u_1, u_2) and (v_1, v_2) are adjacent in $G \square H$ if and only if either (i) $u_1 = v_1$ and u_2 is adjacent to v_2 in H. (ii) $u_2 = v_2$ and u_1 is adjacent to v_1 in G, where $G \square H$ denotes the Cartesian product of graphs G and H. On the ***equivalent product*** $G \circ H$ of two graphs G and H, $(u_1, v_1), (u_2, v_2) \in E(G \circ H)$ if and only if (i) $(u_1, u_2) \in E(G)$ *and* $(v_1, v_2) \in E(H)$ (ii) $u_1 = u_2$ and $(v_1, v_2) \in E(H)$ (iii) $v_1 = v_2$ and $(v_1, v_2) \in E(H)$ (iv) $(u_1, u_2) \in E(G')$ *and* $(v_1, v_2) \in E(H')$.

Theorem 2.2:

The medium domination number of Strong product of path graph is $MD(P_p \otimes P_n) = \frac{2(16pn+23)-39(p+n)}{pnC_2}$

Proof:

Step 1: Let us prove this theorem by induction on p. Let p = 2, consider the strong product of path graph $P_2 \otimes P_n$. Consider $P_2 = \{u_1,u_2\}$ and $P_n = \{v_1,v_2,....,v_n\}$. Hence $V(P_2 \otimes P_n) = \{(u_1,v_1), (u_1,v_2)\ (u_1,v_3),.........\ (u_1,v_n)\}$ are the vertices in the first row and $\{ (u_2,v_1), (u_2,v_2)\ (u_2,v_3),.........\ (u_2,v_n)\}$ are the vertices in the second row of $P_2 \otimes P_n$. Thus $V(P_2 \otimes P_n) = 2n$. Out of all 2n vertices, the corner vertices such as $\{(u_1,v_1), (u_1,v_n)(u_2,v_n),(u_2,v_1)\}$ are of degree 3 and the remaining (2n – 4) vertices of degree 5. Thus, $E(P_p \otimes P_n) = (p - 1)\ n + (n - 1)\ p + 2(p - 1)(n - 1) = 2(2pn + 1) - 3(p + n) = 2(4n+1) - 3(2+n)$

We have, $TDV(G) = q + \left\{\sum_{u_i \in V} \binom{\deg u_i}{2}\right\}$ and $MD(G) = \frac{TDV(G)}{nC_2}$

$$MD(P_2 \otimes P_n) = \frac{q + \sum_{u_i \in V} \deg(u_i)\, C_2}{(2n)C_2}$$

$$= \frac{2(4n+1)-3(2+n)+\{4(3C2)+(2n-4)5C2\}}{(2n)C_2},$$

$$= \frac{(5n-4)+(12+20n-40)}{(2n)C_2}$$

$$= \frac{25n-32}{(2n)C_2}$$

Hence the result is true for p = 2.

Step 2 : Let us consider strong product of path graph on p – 1 vertices. Here $P_{p-1} \otimes P_n$ contains n (p – 1) vertices and {2(p – 2)(p – 1) + (p – 1) (p – 1) + p (p – 2)} edges. That is, $E(P_{p-1} \otimes P_n) = 4p^2 - 10p + 5$. Assume the result is true for $P_{p-1} \otimes P_n$.

$$MD(P_{p-1} \otimes P_n) = \frac{2(16(p-1)n+23)-39(p-1+n)}{(p-1)nC_2}$$

Step 3: To prove the result is true for $P_p \otimes P_n$. In $P_p \otimes P_n$, the total number of vertices dominate each pair of vertices by adding edges with the degrees, $4(3C_2) + \{2(p-2)+2(n-2)\}5C_2 + \{(p-2)(n-2)\}8C_2$.

To find the $MD(P_p \otimes P_n)$, the remaining vertices and edges such as, $2(5C_2)+(n-2)8C_2$ and (n +n – 1 +2(n – 1)) are added to $P_{p-1} \otimes P_n$.

Thus, $q + \sum_{u_i \in V} \deg(u_i)\, C_2 = 2(5C_2)+(n-2)8C_2 + n + n - 1 + 2(n-1) = 2(5C_2) + (n-2)8C_2 + 4n - 3$ is added to $P_{p-1} \otimes P_n$ to get the MD number of $P_p \otimes P_n$.

Hence $MD(P_p \otimes P_n) = MD(P_{p-1} \otimes P_n) + \frac{TD(V(G))}{nC_2}$

$$= \frac{2(16(p-1)n+23)-39(p-1+n)}{((p-1)n)C_2} + \frac{2(5C_2)+(n-2)8C_2+4n-3}{nC_2}$$

$$= \frac{32pn-32n+46-39p-39n+39+28n-36+4n-3}{(pn)C_2}$$

$$= \frac{32pn-39n-39p+46}{(pn)C_2}$$

$$MD(P_p \otimes P_n) = \frac{2(16pn+23)-39(p+n)}{(pn)C_2}$$

Hence the result is true for p.

Example 2.2:

Consider $P_4 \otimes P_3$ having $P_4 = \{u_1,u_2,u_3,u_4\}$ and $P_3 = \{u_5,u_6,u_7\}$

$V(P_4 \otimes P_3) = \{v_1 = (u_1,u_5), v_2 = (u_1,u_6), v_3 = (u_1,u_7), v_4 = (u_2,u_5), v_5 = (u_2,u_6), v_6 = (u_1,u_7), v_7 = (u_3,u_5), v_8 = (u_3,u_6), v_9 = (u_3,u_7), v_{10} = (u_4,u_5), v_{11} = (u_4,u_6), v_{12} = (u_4,u_7)\}$

$\Sigma dom(u_1,u_2) = dom(v_1,v_2) = 3$, $dom(v_1,v_3) = 2$, $dom(v_1,v_4) = 3$, $dom(v_1,v_5) = 3$, $dom(v_1,v_6) = 2$, $dom(v_1,v_7) = 2$, $dom(v_1,v_8) = 2$, $dom(v_1,v_9) = 1$, $dom(v_1,v_{10}) = 0$, $dom(v_1,v_{11}) = 0$, $dom(v_1,v_{12}) = 0$, $dom(v_2,v_3) = 3$, $dom(v_2,v_4) = 3$, $dom(v_2,v_5) = 5$, $dom(v_2,v_6) = 3$, $dom(v_2,v_7) = 2$, $dom(v_2,v_8) = 3$, $dom(v_2,v_9) = 2$, $dom(v_2,v_{10}) = 0$, $dom(v_2,v_{11}) = 0$, $dom(v_2,v_{12}) = 0$, $dom(v_3,v_4) = 2$, $dom(v_3,v_5) = 3$, $dom(v_3,v_6) = 3$, $dom(v_3,v_7) = 1$, $dom(v_3,v_8) = 2$, $dom(v_3,v_9) = 2$, $dom(v_3,v_{10}) = 0$, $dom(v_3,v_{11}) = 0$, $dom(v_3,v_{12}) = 0$, $dom(v_4,v_5) = 5$, $dom(v_4,v_6) = 3$, $dom(v_4,v_7) = 3$, $dom(v_4,v_8) = 3$, $dom(v_4,v_9) = 2$, $dom(v_4,v_{10}) = 2$, $dom(v_4,v_{11}) = 2$, $dom(v_4,v_{12}) = 1$, $dom(v_5,v_6) = 5$, $dom(v_5,v_7) = 3$, $dom(v_5,v_8) = 5$, $dom(v_5,v_9) = 3$, $dom(v_5,v_{10}) = 2$, $dom(v_5,v_{11}) = 3$, $dom(v_5,v_{12}) = 2$, $dom(v_6,v_7) = 2$, $dom(v_6,v_8) = 3$, $dom(v_6,v_9) = 3$, $dom(v_6,v_{10}) = 1$, $dom(v_6,v_{11}) = 2$, $dom(v_6,v_{12}) = 2$, $dom(v_7,v_8) = 5$, $dom(v_7,v_9) = 3$, $dom(v_7,v_{10}) = 3$, $dom(v_7,v_{11}) = 3$, $dom(v_7,v_{12}) = 2$, $dom(v_8,v_9) = 5$, $dom(v_8,v_{10}) = 3$, $dom(v_8,v_{11}) = 5$, $dom(v_8,v_{12}) = 3$, $dom(v_9,v_{10}) = 2$, $dom(v_9,v_{11}) = 3$, $dom(v_9,v_{12}) = 3$, $dom(v_{10},v_{11}) = 3$, $dom(v_{10},v_{12}) = 2$, $dom(v_{11},v_{12}) = 3$

$\Sigma dom(u_1,u_2) = 157$ and $MD(P_4 \otimes P_3) = \frac{TDV(G)}{nC_2} = \frac{157}{12C_2}$.

Using formula, $MD(P_4 \otimes P_3) = \frac{2(16pn+23)-39(p+n)}{(pn)C_2} = \frac{157}{12C_2}$.

Theorem 2.3:

The medium domination number of Cardinal product of path graph is $MD(P_p \times P_n) = \frac{2(4pn+9)-12(p+n)}{pnC_2}$.

Proof:

Step 1: Let us prove this theorem by induction on p. Let p = 2, consider the cardinal product of path graph $P_2 \times P_n$ having $P_2 = \{u_1,u_2\}$ and $P_n = \{v_1,v_2,....,v_n\}$ vertices. Hence $V(P_2 \times P_n) = \{(u_1,v_1), (u_1,v_2)\ (u_1,v_3),.........\ (u_1,v_n)\}$ are the vertices in the first row and $\{(u_2,v_1), (u_2,v_2)\ (u_2,v_3),.........\ (u_2,v_n)\}$ are the vertices of second row of $P_2 \times P_n$. Thus $V(P_2 \times P_n) = 2n$. Out of all 2n vertices, the corner vertices such as $\{(u_1,v_1), (u_1,v_n)(u_2,v_n),(u_2,v_1)\}$ are of degree 1 and the remaining (2n – 4) vertices of degree 2. Thus, $E(P_2 \times P_n) = 2n - 2$.

We have, $TDV(G)= q + \left\{\sum_{u_i \in V} \binom{\deg u_i}{2}\right\}$ and $MD(G) = \frac{TDV(G)}{nC_2}$

$$MD(P_2 \times P_n) = \frac{q + \sum_{u_i \in V} \deg(u_i)\, C_2}{2nC_2}$$
$$= \frac{(2n-2) + [4(1C_2) + 2(n-2)2C_2]}{2nC_2}$$
$$= \frac{[2n-2] + [2n-4]}{2nC_2}$$
$$= \frac{2(2n-3)}{2nC_2}$$

Hence the result is true for p=2.

Step 2 : Consider cardinal product of path graph on p – 1 vertices.

Assume the result is true for $P_{p-1} \times P_n$ then

$$MD(P_{p-1} \times P_n) = \frac{2(4(p-1)n+9)-12(p-1+n)}{((p-1)n)C_2}$$

In $P_p \times P_n$, the total number of vertices dominate each pair of vertices by adding edges with the degrees of $P_p \times P_n$. It contains $\{2(p-2)+2(n-2)\}\, 2C_2 + \{(p-2)(n-2)\}4C_2$ degrees. To find the $MD(P_p \times P_n)$, the remaining degrees of vertices and edges such as, $2(2C_2)+(n-2)4C_2$ and $2(n-1)$ are added to $P_{p-1} \times P_n$.

Thus, $q + \sum_{u_i \in V} d\,eg\,(u_i)\, C_2 = 2(2C_2)+(n-2)4C_2 + 2(n-1)$ is added to $P_{p-1} \times P_n$ to get $P_p \times P_n$

$$\text{Hence } MD(P_p \times P_n) = MD(P_{p-1} \times P_n) + \frac{2(2C2)+(n-2)4C2+2(n-1)}{(pn)C_2}$$
$$= \frac{2(4(p-1)n+9)-12(p-1+n)}{(pn)C_2}$$
$$= \frac{8pn-8n+18-12p-12n+12+2n-2+2+6n-12}{(pn)C_2}$$
$$= \frac{8pn-12p-12n+18}{pnC_2}$$
$$MD(P_p \times P_n) = \frac{2(4pn+9)-12(p+n)}{pnC_2}$$

Hence the result is true for p.

Example 2.3:

Consider the cardinal product of P_2 and P_4. Let $P_2 = \{u_1,u_2\}$ and $P_4 = \{u_4, u_5, u_6, u_7\}$ and $V(P_p \times P_n) = \{v_1 (= u_1,u_4), v_2 (= u_1,u_5), v_3 (= u_1,u_6), v_4 (= u_1,u_7), v_5 (= u_2,u_4), v_6 (= u_2,u_5), v_7 (= u_2,u_6), v_8 (= u_2,u_7)\}$

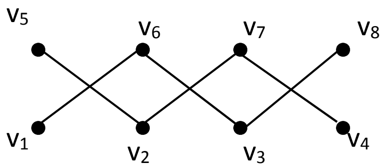

Figure 1.2 Cardinal product of P_2 and P_4 : $P_2 \times P_4$

$dom(v_1, v_3) = 1$, $dom(v_1, v_6) = 1$, $dom(v_2, v_4) = 1$, $dom(v_2, v_5) = 1$, $dom(v_2, v_7) = 1$, $dom(v_3, v_6) = 1$, $dom(v_3, v_8) = 1$, $dom(v_4, v_7) = 1$, $dom(v_5, v_7) = 1$, $dom(v_6, v_8) = 1$.

$TDV(G) = \sum dom(u_1,u_2) = 10$ and $MD(P_2 \times P_4) = \frac{TDV(G)}{nC_2} = \frac{10}{8C_2}$.

$MD(P_p \times P_n) = \frac{2(4pn+9)-12(p+n)}{pnC_2} = \frac{2(32+9)-12(6)}{8C_2} = \frac{10}{8C_2}$.

Theorem 2.4:

The medium domination number of Cartesian product of path graph is $MD(P_p \square P_n) = \frac{4(2pn+1)-7(p+n)}{pnC_2}$.

Proof:

Step 1: Let us prove this theorem by induction on n, let n = 2, In $P_p \square P_2$, Let $V(P_2) = \{u_1,u_2\}$ and $V(P_p) = \{v_1,v_2,\ldots,v_p\}$ then $V(P_p \square P_2) = 2p$ and $E(P_p \square P_n) = n(p-1) + p(n-1) = 2np - n - p$. Thus $E(P_p \square P_2) = 3p - 2$. The corner vertices $\{u_1v_1,u_2v_1,v_pu_1,v_pu_2\}$ are of degree 2 and the remaining vertices $\{u_1v_2,\ldots,u_1v_{p-1},u_2v_2,\ldots,u_2v_{p-1}\}$ of degree 3. Hence $\deg(P_2 \square P_n) = 4(2C_2)+2(n-2)3C_2$.

$$MD(P_p \square P_2) = \frac{q + \sum_{u_i \in V} \deg(u_i)\, C_2}{2pC_2}$$
$$= \frac{(3p-2)+\{4(2C2)+2(p-2)3C2\}}{2pC_2}$$
$$= \frac{(3p-2)+4+6p-12}{2pC_2}$$
$$= \frac{9p-10}{2pC_2}$$

Hence the result is true for n=2.

Step 2: Consider the Cartesian product of P_p and P_{n-1} has $n(p-1)$ vertices and $2n(p-1)$ edges. The corner vertices $\{u_1v_1,u_1v_{n-1},u_pv_{n-1},u_pv_{n-1}\}$ are of degree 2 and the outer horizontal and vertical degrees such as, $\{u_1v_2,u_1v_3,\ldots,u_1v_{n-2}, u_2v_1,u_3v_1,\ldots,u_{p-1}v_1,\ldots, u_pv_2, u_pv_3\ldots,u_pv_{n-2}, u_2v_{n-1},u_3v_{n-1},\ldots,u_{p-1}v_{n-1}\}$ of degree 3 and the remaining inner vertices $(p-3)(n-2)$ are of degree 4.

Suppose the result is true for $MD(P_p \square P_{n-1}) = \frac{q+2(3p(n-1)-3p-3(n-1)+2)}{pnC_2}$
$$= \frac{q+2(3pn-3p-3n+3-3p+2)}{pnC_2}$$
$$= \frac{q+2(3pn-6p-3n+5)}{pnC_2}$$

The total number of vertices that dominate each pair of vertices by adding, the total number of edges, vertices of degree 2, vertices of degree 3, vertices of degree 4. In $P_p \square P_n$, the additional edges and vertices will be $2(3C_2)+(p-2)4C_2+2(2C_2)-(p-2)3C_2-2(2C_2)+(p-2)3C_2 = 2(3C_2)+(p-2)4C_2$. Hence $MD(P_p \square P_{n-1})$ is added to $TD(V(G)) = 2(3C_2)+(p-2)4C_2$ to get $P_p \square P_n$.

Hence $MD(P_p \square P_n) = MD(P_p \square P_{n-1}) + \frac{TDV(G)}{pnC_2}$

$$MD(P_p \square P_n) = \frac{q+6pn-12p-6n+10+6+6p-12}{pnC_2}$$

$$= \frac{2pn-n-p+6pn-6p-6n+4}{pnC_2}$$
$$= \frac{8pn-7p-7n+4}{pnC_2}$$
$$= \frac{4(2pn+1)-7(p+n)}{pnC_2}$$

Hence the result is true for n.

Example 2.4: consider the Cartesian product P_2 and P_5, Let $P_2 = \{u_1,u_2\}$ and $P_4 = \{u_4, u_5, u_6, u_7,u_8\}$ and $V(P_p \times P_n) = \{v_1 (= u_1,u_4), v_2 (= u_1,u_5), v_3 (= u_1,u_6), v_4 (= u_1,u_7), v_5 (= u_1,u_8), v_6 (= u_2,u_8), v_7 (= u_2,u_7), v_8 (= u_2,u_6), v_9 (= u_2,u_5)\ v_{10} (= u_2,u_4)\}$.

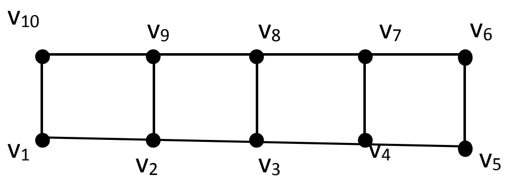

Figure 1.3 Cartesian product of P_2 and P_5 : $P_2 \square P_5$

dom(v_1, v_2) = 1, dom(v_1, v_3) = 1, dom(v_1, v_9) = 2, dom(v_1, v_{10}) = 1, dom(v_2, v_3) = 1, dom(v_2, v_4) = 1, dom(v_2, v_8) = 2, dom(v_2, v_9) = 1, dom(v_2, v_{10}) = 2, dom(v_3, v_4) = 1, dom(v_3, v_5) = 1, dom(v_3, v_7) = 2, dom(v_3, v_8) = 1, dom(v_3, v_9) = 2, dom(v_4,v_5) = 1, dom(v_4, v_6) = 2, dom(v_4, v_7) = 1, dom(v_4, v_8) = 2, dom(v_5, v_6) = 1, dom(v_5, v_7) = 2, dom(v_6, v_7) = 1, dom (v_6,v_8) = 1, dom(v_7, v_8) = 1, dom(v_7, v_9) = 1, dom(v_8, v_9) =1, dom(v_8, v_{10}) = 1, dom(v_9, v_{10}) = 1

$TDV(G) = \Sigma \text{dom}\ (u_1,u_2) = 35$ and $MD(P_2 \square P_5) = \frac{TDV(G)}{nC_2} = \frac{35}{10C_2}$.

$MD\ (P_2 \square P_5) = \frac{4(2pn+1)-7(p+n)}{pnC_2} = \frac{84-49}{10C_2} = \frac{35}{8C_2}$.

Theorem 2.5:

The medium domination number of Equivalent product of path graph is

$$MD(P_p \circ P_n) = \frac{\begin{array}{c}4pn - 3p - 3n + 2 + (n-1)(n-2)[(p-2)+(p-3)+ \ldots\ldots + (p-1)\text{times}]\\ +(n-2)(8C_2)+2(5C_2)+2(n-2)[(p-2)(n-3)+5]C_2+4[(n-2)+3)C_2]\end{array}}{pnC_2}$$

Proof:

In $P_p \circ P_n$, Let $V(P_n) = \{u_1\ldots\ldots u_n\}$ and $V(P_p)=\{v_1,v_2,\ldots\ldots,v_p\}$ then

$V(P_p \circ P_n)$= np and $E(P_p \circ P_n) = 4pn - 3p - 3n + 2 + (n - 1)(n - 2)\{(p - 2) +(p - 3) + \ldots\ldots + (p - 1)$ times$\}$.

$P_p \circ P_n$ contains (n – 2) times of vertices contains a degree 8, two times of vertices contains a degree 5, 2(n – 2) times of vertices contains a degree [(p – 2) (n – 3) + 5] and four times of vertices contains a degree (n + 1) .

Hence Medium domination number of $P_p \circ P_n$ can be written as,

$$MD(P_m \odot P_n)= \frac{4pn - 3p - 3n + 2 + (n-1)(n-2)[(p-2)+(p-3)+ \ldots\ldots + (p-1)\text{times}] + (n-2)(8C_2)+2(5C_2)+2(n-2)[(p-2)(n-3)+5]C_2+4[(n-2)+3)C_2]}{pnC_2}$$

We can verify the result for p = 3,

Consider $P_3 \odot P_n$, let $V(P_3) = \{u_1,u_2,u_3\}$ and $V(P_n)=\{v_1,v_2,\ldots,v_n\}$ then $V(P_3 \odot P_n)=3n$ and $E(P_3 \odot P_n) = 12n - 9 - 3n + 2 + (n-1)(n-2)(1) = n^2 + 6n - 5$

Sum of the degrees of $P_3 \odot P_n$ are $4(n+1)C_2 + 2(5C_2) + (n-2)8C_2 + 2(n-2)\{(n+2)C_2\}$.

To find the total number of vertices that dominates each pair of vertices, sum up the edges and degrees of $MD(P_p \odot P_n)$ using the formula,

$$TD(MD(P_p \odot P_n))= q + \sum_{u_i \in V} deg\,(u_i)\, C_2$$

$$TD(V(P_3 \odot P_n)) = q + \left\{\sum_{u_i \in V} \binom{\deg v_i}{2}\right\}$$

$$= n^2 + 6n - 5 + \left[4.n.\frac{n+1}{2}\right] + 20 + (n-2)28 + \frac{2(n-2)(n+2)(n+1)}{2}$$

$$= n^2 + 6n - 5 + 2n^2 + 2n + 20 + 28n - 56 + n^3 + n^2 - 4n - 4$$

$$= n^2 + 6n - 5 + n^3 + 3n^2 + 26n - 40$$

$$= n^3 + 4n^2 + 32n - 45$$

$$MD(P_3 \odot P_n)= \frac{q+\left\{\sum_{u_i \in V}\binom{\deg v_i}{2}\right\}}{3nC_2}$$

$$= \frac{n^3+4n^2+32n-45}{3nC_2}$$

For example, the equivalent product of P_3 and P_3 is given by,

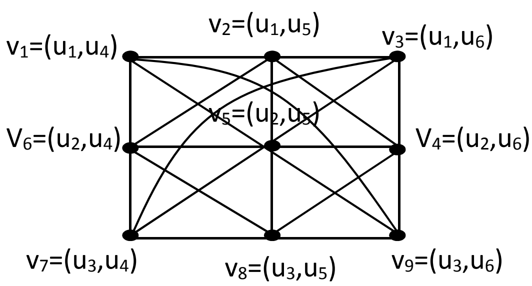

Figure 1.4 Equivalent product of P_3 and P_3: $P_3 \odot P_3$

$dom(v_1,v_2)=3$, $dom(v_1,v_3)=3$, $dom(v_1,v_4)=3$, $dom(v_1,v_5)=4$, $dom(v_1,v_6)=2$, $dom(v_1,v_7)=2$, $dom(v_1,v_8)=3$, $dom(v_1,v_9)=2$, $dom(v_2,v_3)=3$, $dom(v_2,v_4)=3$, $dom(v_2,v_5)=5$, $dom(v_2,v_6)=3$. $dom(v_2,v_7)=3$, $dom(v_2,v_8)=3$, $dom(v_2,v_9)=2$. $dom(v_3,v_4)=2$, $dom(v_3,v_5)=4$, $dom(v_3,v_6)=3$, $dom(v_3,v_7)=3$, $dom(v_3,v_8)=3$, $dom(v_3,v_9)=2$. $dom(v_4,v_5)=5$, $dom(v_4,v_6)=3$, $dom(v_4,v_7)=3$, $dom(v_4,v_8)=4$, $dom(v_4,v_9)=3$. $dom(v_5,v_6)=5$, $dom(v_5,v_7)=4$, $dom(v_5,v_8)=5$, $dom(v_5,v_9)=4$. $dom(v_6,v_7)=3$, $dom(v_6,v_8)=3$, $dom(v_6,v_9)=3$, $dom(v_7,v_8)=3$, $dom(v_7,v_9)=2$, $dom(v_8,v_9)=3$.

$TDV(G) = 114$, $MDV(G) = TDV(G) / nC_2 = 114/9C_2$

$MD(P_3 \odot P_3) = 114/9C_2$

Using formula, $MD(P_m \bigcirc P_n) = \frac{4pn - 3p - 3n + 2 + (n-1)(n-2)[(p-2)+(p-3)+ \ldots + (p-1)\text{times}] + (n-2)(8C_2)+2(5C_2)+2(n-2)[(p-2)(n-3)+5]C_2+4[(n-2)+3)C_2]}{pnC_2}$

$MD(P_3 \bigcirc P_3) = \frac{36-9-9+2+2+8C_2+2(5C_2)+2(5C_2)+4(4C_2)}{9C_2}$

$= 114/9C_2$

Hence verified.

Acknowledgement:

This Research work was supported by University Grants Commission, New Delhi under Departmental Special Assistance, GRI-DU.

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