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Odd Sum Labeling of Alternative Quadrilateral Snake

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Abstract: An injective function $f: V(G) \to \{0,1,2,\ldots,q\}$ is an odd sum labeling if the induced edge labeling f^* defined by $f^*(uv) = f(u) + f(v)$ for all $uv \in E(G)$ is a bijection and $f^*: E(G) \to \{1,3,5,\ldots,2q-1\}$. A graph is said to be an odd sum graph if it admits an odd sum labeling. In this paper we investigate odd sum labeling of Alternative quadrilateral snake.

Key words: *Odd sum Labeling, Odd Sum Graph. AMS Subject Classification (2010):05C78*

1. Introduction

Through out this paper, by a graph we mean a finite undirected simple graph. Let $G(V, E)$ be a graph with p vertices q edges. For notation and terminology, we follow [7]. For detailed survey of graph labeling we refer to Gallian[4].

In[9], the concept of mean labeling was introduced and further studied in [5, 6]. An injective function $f: V(G) \to \{0,1,2,\ldots,q\}$ is said to be a mean labeling if the induced edge labeling f^* defined by

$$f^*(uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

is injective and the resulting edge labels are distinct.

A graph G is said to be an odd mean graph if there exists an injective function $f: V(G) \to \{0,1,2,\ldots,2q-1\}$ such that the induced map $f^*: E(G) \to \{1,3,5,\ldots,2q-1\}$ defined by

$$f^*(uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

is a bijection[8]

In[1], the concept of odd sum labeling was introduced and studied[2,3]. An injective function $f: V(G) \to \{0,1,2,\ldots,q\}$ is an odd sum labeling if the induced edge labeling f^* defined by $f^*(uv) = f(u) + f(v)$ for all $uv \in E(G)$ is a bijection and

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$f^*: E(G) \to \{1,3,5,\ldots,2q-1\}$. A graph is said to be an odd sum graph if it admits an odd sum labeling. In this paper we investigate odd sum labeling of alternative quadrilateral snake.

2. Main Results

Definition: 2.1:

$A(D(Q_n)$ consist of two alternate quadrilateral snakes that have a common path.

Definition: 2.2:

An alternative quadrilateral snake $A(Q_n)$ is obtained from a path $u_1, u_2, ..u_n$ by joining u_i and u_{i+1} to a new vertices v_i,w_i respectively and then joining v_i and w_i. That is every alternative edge of a path replaced by a Cycle C_4.

Definition: 2.3:

The double quadrilateral snake $D(Q_n)$ consists of two Quadrilateral snakes that have a common path.

Theorem 2.4:

The graph $A(D(Q_n))(n \geq 4)$ is a odd sum graph.

Proof:

Case1: If the graph $A(D(Q_n))$ is start from u_1

Let $\{v_i, v_i', 1 \leq i \leq n-2, u_i, 1 \leq i \leq n\}$ be the vertices and $\{a_i, a_i', 1 \leq i \leq n-2, e_i, 1 \leq i \leq n-1\}$ be the edges which are denoted as in

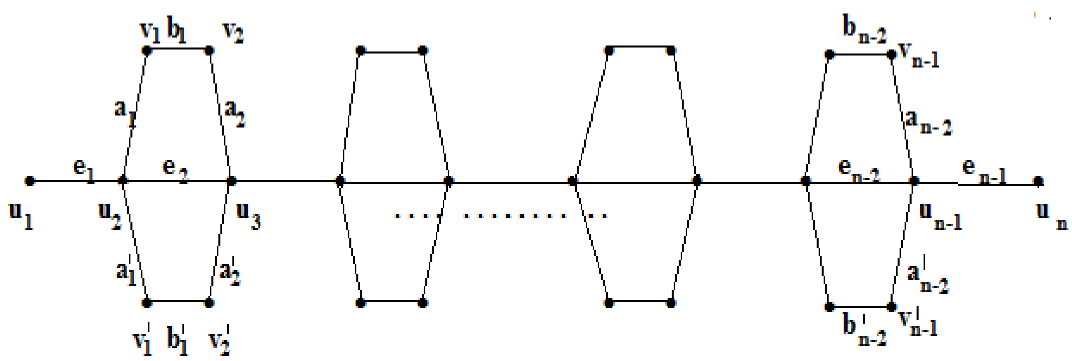

Figure 1.1: Ordinary labeling of $A(D(Q_n))$

First we label the vertices as follows

Define $f : V \rightarrow \{0,1,2,\ldots,q\}$ by

For $1 \le i \le n-2$ $\quad f(v_i) = \begin{cases} 2(2i-1) & i \text{ is odd} \\ 4i-3 & i \text{ is even} \end{cases}$

For $1 \le i \le n$ $\quad f(u_i) = \begin{cases} 4(i-1) & i \text{ is odd} \\ 4i-7 & i \text{ is even} \end{cases}$

For $1 \le i \le n-2$ $\quad f(v_i') = \begin{cases} 4i & i \text{ is odd} \\ 4i-1 & i \text{ is even} \end{cases}$

Then the induced edge labels are:

For $1 \le i \le n-1$ $\quad f^*(e_i) = 8i-7$

For $1 \le i \le n-2$ $\quad f^*(a_i) = \begin{cases} 8i-5 & i \text{ is odd} \\ 8i-3 & i \text{ is even} \end{cases}$

For $1 \le i \le n-2$ $\quad f^*(a_i') = \begin{cases} 8i-3 & i \text{ is odd} \\ 8i-1 & i \text{ is even} \end{cases}$

For $1 \le i \le n$ $\quad f^*(b_i) = 16i-9$, $f^*(b_i') = 16i-5$

Therefore $f^*(E) = \{1,3,5,\ldots\ldots,2q-1\}$ .So f is an odd sum and hence, the graph $A(D(Q_n))(n \ge 4)$ is an odd sum graph.

Odd sum labeling of the graph $A(D(Q_4))$ is shown in the Figure 1.2

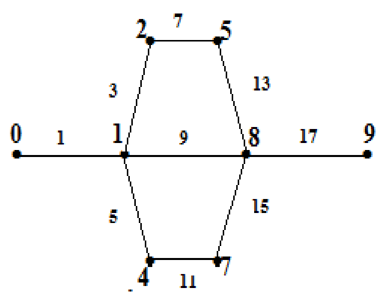

Figure 1.2: Odd sum labeling of $A(D(Q_4))$

Case2: If the graph $A(D(Q_n))$ is start from u_2

Let $\{v_i, u_i, v_i', 1 \le i \le n\}$ be the vertices and $\{b_i, b_i', e_i, 1 \le i \le n-1, a_i, a_i', 1 \le i \le n\}$ be the edges which are denoted as in figure 1.3

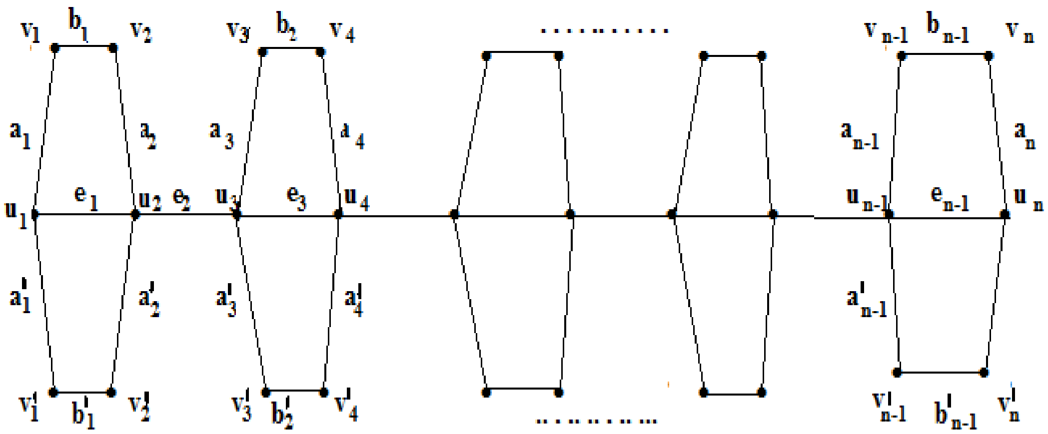

Figure 1.3: Odd sum labeling of $A(D(Q_n))$

First we label the vertices as follows

Define $f : V \rightarrow [0,1,2,...,q\}$ by

For $1 \le i \le n$ $\quad f(v_i) = \begin{cases} 4i-3 & i \text{ is odd} \\ 4(i-1) & i \text{ is even} \end{cases}$

For $1 \le i \le n$ $\quad f(u_i) = \begin{cases} 8(i-1) & i \text{ is odd} \\ 4i-1 & i \text{ is even} \end{cases}$

For $1 \le i \le n$ $\quad f(v_i') = \begin{cases} 4i-1 & i \text{ is odd} \\ 2(2i-1) & i \text{ is even} \end{cases}$

Then the induced edge labels are:

For $1 \le i \le n-1$ $\quad f^*(b_i) = 16i-11$

For $1 \le i \le n$ $\quad f^*(a_i) = \begin{cases} 8i-7\ , i \text{ is odd} \\ 8i-5\ , i \text{ is even} \end{cases}$

$$f^*(a_i') = \begin{cases} 16i-13, i \text{ is odd} & \text{for } 1 \le i \le n-1 \\ 8i-3,\ i \text{ is even} & \text{for } 2 \le i \le n-1 \end{cases}$$

For $1 \le i \le n-1$ $\quad f^*(b_i') = 16i-7$

Therefore $f^*(E) = \{1,3,5,.....,2q-1\}$. So f is an odd sum and hence, the graph $A(D(Q_n))(n \ge 4)$ is an odd sum graph.

Odd sum labeling of the graph $A(D(Q_6))$ is shown in Figure 1.4

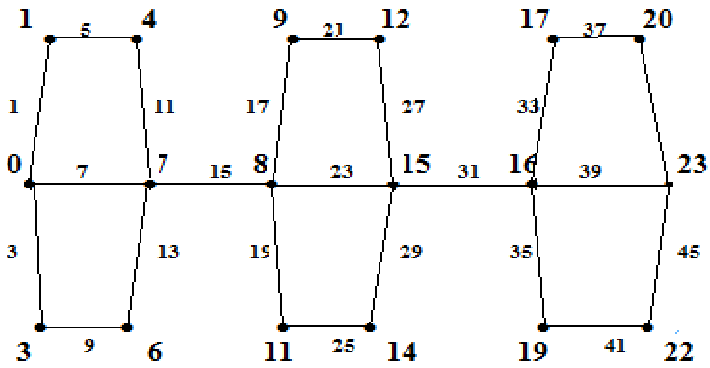

Figure 1.4: Odd sum labeling of $A(D(Q_6)$

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