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Domination parameters of the Boolean graph $BG_1(G)$ and its complement

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Abstract: *Let G be a simple (p, q) graph with vertex set V(G) and edge set E(G). $B_{G,\ NINC,\ \bar{K}_q}(G)$ is a graph with vertex set V(G) ∪ E(G) and two vertices are adjacent if and only if they correspond to two adjacent vertices of G or to a vertex and an edge not incident to it in G. For simplicity, denote this graph by $BG_1(G)$, Boolean graph of G-first kind. In this paper, domination parameters of $BG_1(G)$ and its complement are studied.*

Key words: *Boolean graph $BG_1(G)$.*

1. Introduction

Let G be a finite, simple, undirected (p, q) graph with vertex set V(G) and edge set E(G). For graph theoretic terminology refer to Harary [6], Buckley and Harary [4].

Definition 1.1 [7]:A set S ⊆ V is said to be a *dominating set* in G, if every vertex in V−S is adjacent to some vertex in S. A dominating set D is an *independent dominating set*, if no two vertices in D are adjacent that is D is an independent set. A dominating set D is a *connected dominating set*, if < D > is a connected subgraph of G. A dominating set D is a *perfect dominating set*, if for every vertex u ∈ V(G)−D, |N(u)∩ D|= 1. A dominating set D is a *total dominating set*, if < D > has no isolated vertices. A dominating set D is called an *efficient dominating set*, if the distance between any two vertices in D is at least three. A cycle C of a graph G is called a *dominating cycle* of C, if every vertex in V−C is adjacent to some vertex in C. A set D ⊆ V(G) is a *global dominating set*, if D is a dominating set in G and $\overline{G}$. A total dominating set D of a graph is a *total global dominating set*, if D is also a total dominating set of $\overline{G}$. A set D ⊆ V(G) is a *restrained dominating set*, if every vertex in V−S is adjacent to a vertex in S and other vertex in V−S.

Definition 1.2 [11]: A set S of vertices is said to be *irredundant*, if for every vertex v ∈ S, $p_n[v, S] = N[v]−N[S−\{v\}] \neq \phi$, that is, every vertex v ∈ S has a private neighbor.

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The *irredundance number* $ir(G)$ is the minimum cardinality of a maximal irredundant set in G.

Definition 1.3 [12]:A set S of vertices is called a *neighborhood set* provided G is the union of the subgraphs induced by the closed neighborhoods of the vertices in S; that is, G = $\cup$<N[v]>. The *neighborhood number* $n_o(G)$ of a graph G equals the minimum cardinality of a neighborhood set.

Definition 1.4 [7]:The *domination number* γ of G is defined to be the minimum cardinality of a dominating set in G. Similarly, one can define the perfect domination number γ_p, connected domination number γ_c, total domination number γ_t, independent domination number γ_i, efficient domination number γ_e, cycle domination number γ_o, global domination number γ_g, total global domination number γ_{tg}, restrained domination number γ_r.

An edge uv$\in$E(G) is a *dominating edge* of G, if all the vertices of G other than u and v are adjacent to either u or v.

Definition 1.5:Cockayne and Hedetniemi [5] defined the *domatic number* $d(G)$ of a graph to be the maximum number of elements in a partition of V(G) into dominating sets. G is *domatically full* if d(G) = 1+δ(G).

Definition 1.6:A vertex (point) and an edge are said to ***cover***each other, if they are incident. A set of vertices, which cover all the edges of a graph G is called a (*vertex*) *point cover of* G, while a set of lines (edges), which covers all the vertices is a *line cover*. The smallest number of points in any point cover for G is called its *point covering number* and is denoted by $\alpha_o(G)$ or α_o. A set of points in G is *independent*, if no two of them are adjacent. The largest number of points in such a set is called the *pointindependence number* of G and is denoted by $\beta_o(G)$ or β_o. Similarly, $\alpha_1(G)$ or α_1 is the smallest number of lines in any line cover of G and is called its *line covering number*. A point cover is called *minimum*, if it contains α_o points. A line cover is called *minimum*, if it contains $\alpha_1$points.Analogously, an independent set of lines (matching) of G has no two of its lines adjacent and the maximum cardinality of such a set is the *line independence number* $\beta_1(G)$ or β_1. A set of independent edges covering all the vertices of a graph G is called a *1-factor* or a *perfect matching* of G.

Let G be a simple (p, q) graph with vertex set V(G) and edge set E(G). In [3, 8], it is defined that $B_{G,\, NINC,\, \overline{K_q}}(G)$ is a graph with vertex set $V(G) \cup E(G)$ and two vertices are adjacent if and only if they correspond to two adjacent vertices of G or to a vertex and an edge not incident to it in G. For simplicity, denote this graph by $BG_1(G)$, Boolean graph of G-first kind. In this paper, domination parameters of $BG_1(G)$ and its complement are studied.

We need the following theorems to study the domination parameters of$BG_1(G)$ and its complement.

Theorem 1.1 [7]:$ir(G) \leq \gamma(G) \leq \gamma_i(G) \leq \beta_o(G)$. If $\gamma(G) \geq 2$, $\gamma(G) \leq \gamma_t(G) \leq \gamma_c(G)$.

Theorem 1.2[7]:$\gamma_{tg}(G) < \gamma_t(G)+2$.

Theorem 1.3[7]: Let D be a γ_g-set of G such that < D > has no isolates and diam(G) = 3, then $\gamma_{tg}(G) \leq \gamma_g(G)+2$.

Theorem 1.4[12]:$\gamma(G) \leq n_o(G) \leq \alpha_o(G)$.

Theorem 1.5 [9]: $\overline{BG_1(G)}$ is self-centered with diameter two if and any if $G \neq K_2$.

2. Domination Parameters of $BG_1(G)$ and $\overline{BG_1(G)}$

In this section, domination parameters of $BG_1(G)$ and $\overline{BG_1(G)}$ are studied. First domination parameters of $BG_1(G)$ are studied.

Let G be a given graph. G is non-trivial. Therefore, G has at least one edge $e = uv \in E(G)$, where $u, v \in V(G)$. Consider D = {u, v, e}. D is a dominating set of $BG_1(G)$. Also, radius of $BG_1(G)$ is always greater than one. Hence, $1 < \gamma(BG_1(G)) \leq 3$.

If G has a pendant vertex $u \in V(G)$ and $e \in E(G)$ is incident with u in G, then D = {u, e} dominates all the vertices of $BG_1(G)$. Hence, $\gamma(BG_1(G)) = 2$. This set {u, e} is independent in $BG_1(G)$. It is also an irredundant set (maximal) in $BG_1(G)$. Again, every element in V−D has adjacent elements in V−D. Hence, $\gamma_i(BG_1(G)) = \gamma_r(BG_1(G)) = 2$. D is also an efficient dominating set for $BG_1(G)$.

Now, assume that G has no pendant vertices.

(1) **Let G be a graph with $\gamma(G) = 1$:** Then the radius of G is one. If diam(G) = 1, that is $G = K_n$, $n \geq 3$, take $e \in E(G)$, e = uv, where $u, v \in V(G)$. The set D = {u, v, e} dominates $BG_1(G)$ and D is not independent. Hence, $\gamma(BG_1(K_n)) = 3$. If $n \geq 4$, D is restrained. If n = 3, D = E(G) is restrained and independent. Hence, $\gamma_i(BG_1(K_3)) = \gamma_r(BG_1(K_3)) = 3$.

When $n \geq 4$, let $S = \{e \in E(G) : e$ is incident with u in G$\}$. Take $D = \{u\} \cup S$. Then D is a minimal independent dominating set of $BG_1(K_n)$, and is also restrained. Hence, $\gamma_i(BG_1(G)) \leq p$. $D = \{u, v, e\}$, where $e \in E(G)$ is a restrained dominating set of $BG_1(G)$ and hence $\gamma_r(BG_1(K_n)) = 3$.

Next, if $r(G) = 1$ and $diam(G) = 2$, there exists non-adjacent vertices u and v in G. If there exists non-adjacent vertices u and v in G, which dominates G, then $\{u, v\}$ dominates $BG_1(G)$. Hence, $\gamma(BG_1(G)) = 2 = \gamma_i(BG_1(G))$, otherwise, $\gamma(BG_1(G)) = 3 = \gamma_r(BG_1(G)) = ir(BG_1(G))$. ($\{u, v, x\}$, where $e_G(x) = 1$ and u and v not adjacent in G is irredundant in $BG_1(G)$ and hence $ir(BG_1(G)) = 3$).

If $u \in V(G)$, $S = \{e \in E(G)$:e is incident with u in G$\}$, then $S \cup \{u\}$ is an independent dominating set for $BG_1(G)$. Hence, $\gamma_i(BG_1(G)) \leq 1+\delta(G)$.

(2) **Let $\gamma(G) = 2$:**If there exists a dominating edge $e = uv \in E(G)$ and $\gamma_i(G) > 2$,then $\gamma(BG_1(G)) = 3$, where $D = \{u, v, e\}$ is a dominating set. D is also restrained. Hence, $\gamma_r(BG_1(G)) = 3 = ir(BG_1(G)) = 3$. If $\gamma(G) = 2 = \gamma_i(G)$, then $\gamma(BG_1(G)) = 2 = \gamma_i(BG_1(G)) = ir(BG_1(G)) = \gamma_r(BG_1(G))$. (The independent dominating set of G is again an independent dominating set for $BG_1(G)$).

(3) **Let $\gamma(G) = 3$:** In this case, $\gamma(BG_1(G)) = 3$. If G has at least three independent vertices, then $\gamma_i(BG_1(G)) = 3$, since they dominate $BG_1(G)$. In general, $\gamma_i(BG_1(G)) \leq 1+\delta(G)$ and $\gamma_r(BG_1(G)) = 3$, since $D = \{u, v, e\}$, where $e = uv \in E(G)$ is a restrained dominating set. D is also irredundant.

Thus, the following theorems are proved.

Theorem 2.1: If G has a pendant vertex, then $\gamma(BG_1(G)) = 2$.

Theorem 2.2:Let G be a graph without pendant and isolated vertices.

(1) If $diam(G) = 1$, then $\gamma(BG_1(G)) = 3 = \gamma_r = ir(BG_1(G))$ and $\gamma_i(BG_1(G)) \leq p$.

(2) If $r(G) = 1$ and $diam(G) = 2$, and if there exists non-adjacent vertices $u, v \in V(G)$ such that $\{u, v\}$ dominates G, then $\gamma(BG_1(G)) = \gamma_i(BG_1(G)) = \gamma_r(BG_1(G)) = ir(BG_1(G)) = 2$.

(3) If $\gamma(G) = 2 = \gamma_c(G)$ and $\gamma_i(G) > 2$, then $\gamma(BG_1(G)) = \gamma_r(BG_1(G)) = ir(BG_1(G)) = 3$ and $\gamma_i(BG_1(G)) \leq 1+\delta(G)$.

(4) If $\gamma(G) = 2 = \gamma_i(G)$, then $\gamma(BG_1(G)) = \gamma_i(BG_1(G)) = ir(BG_1(G)) = \gamma_r(BG_1(G)) = 2$.

(5) If $\gamma(G) \geq 3$, then $\gamma(BG_1(G)) = \gamma_r(BG_1(G)) = ir(BG_1(G)) = 3$ and $\gamma_i(BG_1(G)) \leq 1+\delta(G)$.

Remark 2.1:

1. If $u \in V(G)$ and D is the set of all edges incident with u in G, then $\{u\} \cup D$ is a dominating set of $BG_1(G)$, which is also independent. Hence, $\gamma_i(BG_1(G)) \leq 1+\delta(G)$.

2. $D = V(G)$ is a dominating set of $BG_1(G)$ if and only if $G \neq K_2$.

3. $D = E(G)$ is a dominating set of $BG_1(G)$ if and only if $G \neq K_{1,n}$.

4. If $D \subseteq V(G)$ is a dominating set of G, then D is a dominating set of $BG_1(G)$ if and only if $|D| \geq 3$ or $|D| = 2$ and is independent.

A D-partition of G is a partition of V(G) into dominating sets. The maximum order of a D-partition is called the domatic number of G. Now, we shall evaluate the domatic numbers of $BG_1(G)$, when $G = C_n, P_n, K_{1,n}, K_n, K_{n,n}$ and nK_2.

Proposition 2.1: Domatic number of $BG_1(C_n)$ is $\lfloor(n+2)/2\rfloor$.

Proof: Let $v_1, v_2, v_3, \ldots, v_n$ form a cycle $C_n = G$ and let $v_1v_2 = e_{12}, \ldots, v_{n-1}v_n = e_{(n-1)n}, v_nv_1 = e_{n1} \in E(G)$.

Case 1: n is even, n = 2k.

$D_1 = \{v_1, e_{12}, e_{n1}\}, D_2 = \{v_3, e_{23}, e_{34}\}, \ldots D_{n/2} = \{v_{2k-1}, e_{(2k-2)(2k-1)}, e_{(2k-1)2k}\}, D_{(n+2)/2} = \{v_{2k}, v_{2k-2}, \ldots, v_2\}$. $D_1, D_2, \ldots, D_{(n+2)/2}$ is a maximum domatic partition of $BG_1(G)$. Therefore, Domatic number of $BG_1(C_n) = (n+2)/2 = n/2+1$.

Case 2: n is odd = 2k+1.

Take $D_1 = \{v_1, e_{12}, e_{n1}\}, D_2 = \{v_3, e_{23}, e_{34}\}, \ldots, D_{2k/2} = \{v_{2k-1}, e_{(2k-1)(2k-2)}, e_{(2k-1)2k}\}$. $D_{(n+1)/2} = D_{2k/2+1} = \{v_{2k+1}, v_{2k}, v_{2k-2}, \ldots, v_2, e_{2k(2k+1)}\}$. $D_1, D_2, \ldots, D_{n+1/2}$ is a domatic partition of $BG_1(G)$.

Proposition 2.2: Domatic number of $BG_1(P_n)$ is $\lfloor n+2/2\rfloor$.

Proof: Let $v_1, v_2, \ldots, v_n$ represent $P_n = G$. $v_iv_{i+1} = e_{i(i+1)} \in E(G)$.

Case 1: n = 2k+1 (odd).

Consider $D_1 = \{e_{12}, v_1\}, D_2 = \{v_3, e_{23}, e_{34}\}, \ldots, D_k = \{v_{2k-1}, e_{(2k-2)(2k-1)}, e_{(2k-1)2k}\}, D_{k+1} = \{v_2, v_4, \ldots, v_{n-1}, v_n, e_{2k(2k+1)}\}$. $D_1, D_2, \ldots, D_{k+1}$ is a domatic partition of $BG_1(P_n)$. Hence, domatic number of $BG_1(P_n)$ is $k+1 = \lfloor(n+2)/2]\rfloor$.

Case 2: n = 2k, even.

Consider $D_1 = \{v_1, e_{12}\}, D_2 = \{v_3, e_{23}, e_{34}\}, \ldots, D_{n/2} = \{v_{n-1}, e_{(n-2)(n-1)}, e_{(n-1)n}\}, D_{n/2+1} = \{v_2, v_4, \ldots, v_n\}$. $D_1, D_2, \ldots, D_{n/2+1}$ is a domatic partition of $BG_1(P_n)$.

Hence, domatic number of $BG_1(P_n)$ is $(n/2)+1 = (n+2)/2$.

Proposition 2.3: Domatic number of $BG_1(K_{1,n}) = n$.

Proof: Let v be the central vertex of $K_{1,n}$ and let $v_1, v_2, \ldots, v_n$ be the other vertices.

$D_1 = \{v_1, e_1\}$, $D_2 = \{v_2, e_2\}$, …,$D_n = \{e_n, v_n\}$, $e_j = vv_j \in E(G)$ is a domatic partition. Hence, domatic number of $BG_1(K_{1,n})$ is n.

Remark 2.2: $\delta(BG_1(K_{1,n})) = n-1$. Hence, $d(BG_1(K_{1,n})) = \delta(BG_1(K_{1,n}))+1 = n$.

Hence, $BG_1(K_{1,n})$ is domatically full.

Proposition 2.4: Domatic number of $BG_1(K_n)$ is $\lfloor n/2 \rfloor$.

Proof: Let $v_1, v_2, \ldots, v_n$ be the vertices of K_n and $v_iv_j = e_{ij} \in E(G)$, i, j = 1, 2, …, n.

Case 1: n is even.

$D_1 = \{v_1, v_2, e_{12}\}$; $D_2 = \{v_3, v_4, e_{34}\}$; …; $D_{n/2} = \{v_{n+1}, v_2, e_{(n-1)n}$, all other line vertices} is a domatic partition of $BG_1(K_n)$. Hence, domatic number of $BG_1(K_n) = n/2$.

Case 2: n is odd. Let $n = 2k+1$.

$D_1 = \{v_1, v_2, e_{12}\}$; $D_2 = \{v_3, v_4, e_{34}\}$ …; $D_k = \{v_{2k-1}, v_{2k}, v_{2k+1}, e_{(2k-1)2k}\} \cup$ {other line vertices} is a domatic partition of $BG_1(K_n)$. Therefore, domatic number of $BG_1(K_n) = \lfloor n/2 \rfloor = k = (n-1)/2$.

Proposition 2.5: Domatic number of $BG_1(K_{n,n})$ is n.

Proof: Let u_i, v_i, where i = 1, 2, …, n be the vertices of $K_{n,n}$ and $e_{ij} = u_iv_j$, i, j = 1, 2, ..., n. $D_1 = \{u_1, v_1, e_{22}\}$, $D_2 = \{u_2, v_2, e_{33}\}$, …,$D_{n-1} = \{u_{n-1}, v_{n-1}, e_{nn}\}$, $D_n= \{u_n, v_n, e_{11}\}$and remaining line vertices} is a domatic partition of $R(K_{n,n})$. Hence, domatic number of $BG_1(K_{n,n})$ is n.

Proposition 2.6: Domatic number of $BG_1(nK_2) = n$.

Proof: Let u_i, v_i, i = 1, 2, …, n be the vertices of nK_2 and let $u_iv_i = e_i \in E(G)$.

$D_1 = \{u_1, v_1, e_1\}$; $D_2 = \{u_2, v_2, e_2\}$,…, $D_n = \{u_n, v_n, e_n\}$ is a domatic partition of $BG_1(nK_2)$. Hence, the domatic number of $BG_1(nK_2)$ is n.

Now, we shall find the domination parameters of $\overline{BG}_1(G)$ for some classes of graphs and give the bounds for $\gamma(\overline{BG}_1(G))$.

Proposition 2.7:$\gamma(\overline{BG}_1(G)) = 1$ if and only if $G = K_2$.

Proof: Clearly $\gamma(\overline{BG}_1(K_2)) = 1$.

On the other hand, assume $\gamma(\overline{BG}_1(G)) = 1$. Hence, $r(\overline{BG}_1(G)) = 1$. $\overline{BG}_1(G)$ is self-centered with diameter two except when $G = K_2$. This proves the result.

Proposition 2.8: If $G \neq K_2$ and has a pendant vertex, then $\gamma(\overline{BG}_1(G)) = 2 = \gamma_c(\overline{BG}_1(G))$.

Proof: Let x be a pendant vertex of G and let e = xy∈E(G), y ∈ V(G). Consider, D = {x, e} in $\overline{BG_1(G)}$. D is a connected minimal dominating set in $\overline{BG_1(G)}$, since e is adjacent to every other line vertices and x, y; x is adjacent to all other point vertices except y in $\overline{BG_1(G)}$. Therefore, $\gamma(\overline{BG_1(G)}) = 2 = \gamma_c(\overline{BG_1(G)})$.

Proposition 2.9: If G has an isolated vertex, then $\gamma(\overline{BG_1(G)}) = 2$.

Proof: Let u ∈V(G) be an isolated vertex of G. Consider D = {u, e} , where e ∈ E(G). D is an independent dominating set of $\overline{BG_1(G)}$, since u dominates all the point vertices and e dominates all the line vertices of $\overline{BG_1(G)}$. Hence, $\gamma(\overline{BG_1(G)}) = 2 = \gamma_i(\overline{BG_1(G)})$.

Proposition 2.10: If $G \neq K_2$ and $p \leq 4$, then $\gamma(\overline{BG_1(G)}) = 2$.

Proof: As $G \neq K_2$, clearly $\gamma(\overline{BG_1(G)}) > 1$. If G has a pendant vertex or isolated vertex, then $\gamma(\overline{BG_1(G)}) = 2$. If G has no isolated vertices, there exists $e_1, e_2 \in E(G)$ such that $\{e_1, e_2\}$ is a line cover for G. Hence, $D = \{e_1, e_2\}$ is a dominating set of $\overline{BG_1(G)}$. This proves the proposition.

Theorem 2.3:$\gamma(\overline{BG_1(G)}) = 2$ if and only if any one of the following is true.

(1) G has a pendant vertex ($G \neq K_2$). (2) $p \leq 4$. (3) G has an isolated vertex. (4) G contains a triangle with at least one vertex of degree two in G.

Proof: Assume $\gamma(\overline{BG_1(G)}) = 2$. Let D be a dominating set of $\overline{BG_1(G)}$ with cardinality two.

Case 1: D = {u, v} ⊆V(G)

D dominates $\overline{BG_1(G)}$. Hence, all other point vertices and line vertices in $\overline{BG_1(G)}$ are adjacent to u or v or both. This gives $\gamma((\overline{G})) \leq 2$.

Sub case 1.1: $\gamma(\overline{G}) = 1$

In this case, G has an isolated vertex u. But D = {u, v}. Hence, in $\overline{BG_1(G)}$, u is adjacent to all point vertices and v is adjacent to all line vertices. Hence, all edges are incident with v in G. Hence, $G = K_{1,n} \cup mK_1$.

Sub case 1. 2:$\gamma(\overline{G}) = 2$.

In this case, G has no vertex, which is adjacent to both u and v, and all edges are incident with u or v or both in G. That is, G is a double star or $G = K_{1,n} \cup K_{1,m}$. So, in this case 1, G has a pendant vertex.

Case 2: $D = \{e_1, e_2\} \subseteq E(G)$.

This gives e_1, e_2 are incident with all point vertices. Hence $p \leq 4$.

Case 3: $D = \{u, e\}$, where $u \in V(G)$ and $e \in E(G)$.

In this case, e is adjacent to all line vertices in $\overline{BG_1(G)}$ and is adjacent to exactly two point vertices v_1, v_2, where $e = v_1v_2 \in E(G)$. Therefore, remaining point vertices must be adjacent to u in $\overline{BG_1(G)}$. Hence, $\deg_G u$ is at most 2.

Now, $\deg_G u = 0$ implies that u is isolated in G; $\deg_G u = 1$ implies u is pendant in G; $\deg_G u = 2$ implies u is adjacent to v_1 and v_2 such that $v_1v_2 = e \in E(G)$, that is G contains a triangle with a vertex of degree two in G.

Converse follows from the previous propositions.

Remark 2.3: From the definition of $\overline{BG_1(G)}$, the following results follow:

(1) If D is dominating set of $\overline{G}$, then $D \cup \{e\}$ is a dominating set of $\overline{BG_1(G)}$. Hence, $\gamma(\overline{BG_1(G)}) \leq \gamma(\overline{G})+1$.

(2) If $D \subseteq E(G)$ is a line cover of G, then D dominates $\overline{BG_1(G)}$. Converse is also true. Hence, $\gamma(\overline{BG_1(G)}) \leq \alpha_1(G)$.

(3) If there exists a minimal point cover D of G such that $N(v) \cap D \neq D$ for $v \in V-D$, then D is a dominating set of $\overline{BG_1(G)}$. $\gamma(\overline{BG_1(G)}) \leq \alpha_o(G)$. Here, D is a global dominating set of G. The converse is also true.

(4) The set of all point vertices is a dominating set for $\overline{BG_1(G)}$.

(5) The set of all line vertices is a dominating set of $\overline{BG_1(G)}$ if and only if G has no isolated vertices.

(6) If $u \in V(G)$ and $D \subseteq E(G)$ contains all the edges incident with u in G, $D \cup \{u\}$ is a dominating set of $\overline{BG_1(G)}$. Hence, $\gamma(\overline{BG_1(G)}) \leq 1+\delta(G)$.

Generally, $\gamma(\overline{BG_1(G)}) \leq \min\{1+\gamma(\overline{G}), \alpha_1(G), 1+\delta(G)\}$.

(7) D is dominating set of G, D dominates $\overline{BG_1(G)}$ if and only if D is global dominating set of G and is also a point cover of G.

Now, assume that G is a graph without isolated vertices and pendant vertices. Also, assume that $p \geq 5$.

Theorem 2.4: Let G be a graph such that $diam(G) \geq 3$.

(1) If G contains a triangle with at least one vertex of degree 2 in G then $\gamma(\overline{BG_1(G)}) = 2 = \gamma_i(\overline{BG_1(G)})$.

(2) If G contains no triangle with at least one vertex of degree 2 in G, then $\gamma(\overline{BG_1(G)})=3$.

Proof: $Diam(G) \geq 3$.

Case 1: G contains a triangle with one vertex of deg 2 in G.

Let u, v, w form this triangle and let $deg_G u = 2$, Let $e = vw \in E(G)$. D = {u, e} dominates $\overline{BG_1}(G)$, since u dominates all point vertices except u and w and e dominates v, w and all line vertices. Hence, $\gamma(\overline{BG_1}(G)) = 2$.

Case 2: G has no triangle with a vertex of deg 2 in G.

Diam(G) $\geq$ 3. Therefore, there exists u, v $\in V(G)$ such that $d_G(u, v) \geq 3$. Then D = {u, v} dominates $\overline{G}$ and hence {u, v, e}, $e \in E(G)$ dominates $\overline{BG_1}(G)$. {u, v, e} is connected, if e is incident with u or v. Hence, $\gamma(\overline{BG_1}(G)) = 3 = \gamma_c(\overline{BG_1}(G))$.

This proves the theorem.

Theorem 2.5: Let G be a graph with diameter two. (1) If G contains a triangle with at least one vertex of degree 2 in G, then $\gamma(\overline{BG_1}(G)) = 2$.

(2) If (1) is not true and if G has an edge e not in any triangle, then $\gamma(\overline{BG_1}(G)) = 3$.

(3) If $\delta(G) = 2$ such that $deg_G u = 2$ and u is not in any triangle, then $\gamma(\overline{BG_1}(G)) = 3$.

(4) $\gamma(\overline{BG_1}(G)) = 1+\alpha_1(N(v))$, where $deg_G v = \delta(G)$.

Proof: Assume that diameter of G is 2.

Proof of (1): Similar to the proof of Theorem 2.4.

Proof of (2): Let $e = uv \in E(G)$ such that e is not in any triangle. D = {u, v, e} dominates $\overline{BG_1}(G)$. Hence, $\gamma(\overline{BG_1}(G)) \leq 3$. Therefore, $\gamma(\overline{BG_1}(G)) = 3$ by Theorem 2.4.

Proof of (3): Let $deg_G u = \delta(G) = 2$, u is not in any triangle. Let v, w be adjacent to u in G and v, w be not adjacent in G. Hence D = {u, e_1, e_2}, where $e_1 = uv$, $e_2 = uw \in E(G)$ is a minimal dominating set of $\overline{BG_1}(G)$. Hence, $\gamma(\overline{BG_1}(G)) = 3$.

Proof of (4): If the hypothesis of (1), (2) and (3) are not true, then let $\delta(G) = deg_G u$. Let D be the set of edges incident with u. Then D $\cup$ {u} is a dominating set of $\overline{BG_1}(G)$. Hence, $\gamma(\overline{BG_1}(G)) \leq 1+\delta(G)$. Since every edge of G is in a triangle,

<N(u)> has no isolated vertices. Consider a line cover of <N(u)> in G. Then {u} $\cup$ (Line cover of <N(u)) form a dominating set for $\overline{BG_1}(G)$.

Therefore, $\gamma(\overline{BG_1}(G)) \leq 1+ \alpha_1(<N(u)>)$.

Proposition 2.11: If G = K_n, then $\gamma(\overline{BG_1}(G)) = \begin{cases} n/2, \text{ n is even.} \\ (n+1)/2, \text{ n is odd.} \end{cases}$

Proof: In G, all vertices are adjacent to each other. Consider any minimal line cover for G. If G = K_n, $\alpha_1(G) = n/2$ or $(n+1)/2$ and this minimal line cover is the minimal dominating set for $\overline{BG_1}(G)$ with minimum cardinality. This proves the proposition.

Proposition 2.12: $\gamma(\overline{BG_1}(G)) \leq \delta(G)$, if $\delta(G) \geq 3$.

Proof: If there exists $e \in E(G)$ such that e is not in any triangle, then $D = \{u, v, e\}$, where $e = uv \in E(G)$ dominates $\overline{BG_1}(G)$. Hence, $\gamma(\overline{BG_1}(G)) \leq 3$. If there exists no such e in G, that is if every edge lies in some triangle, then let $S = \{e \in E(G) : e \text{ is incident with } u\}$, where $deg_G u = \delta(G)$. Consider N(u). N(u) has no isolated vertices and so $\alpha_1(< N(u) >) \leq \delta(G)-1$. Let D be a line cover of N(u) with cardinality $\alpha_1(< N(u) >)$. $\{u\} \cup D$ is a dominating set for $\overline{BG_1}(G)$. Hence, $\gamma(\overline{BG_1}(G)) \leq 1+\delta(G)-1 = \delta(G)$.

Remark 2.4: Let G be a graph without isolated vertices and pendant vertices, with $p \geq 5$. Then (1) If $deg_G u = 2$ and u lies on a triangle, then $\gamma(\overline{BG_1}(G)) = 2$. If $deg_G u = 2$ and u is not on any triangle, then $\gamma(\overline{BG_1}(G)) = 3$.
(2) If $\delta(G) \geq 3$, then $\gamma(\overline{BG_1}(G)) \leq \delta(G)$.

Theorem 2.6: If $G \neq K_2$, then $4 \leq \gamma(\overline{BG_1}(G))+\gamma(\overline{BG_1}(G)) \leq 4+k$ where $k = \min\{\delta(G), \gamma(\overline{G})\}$.

Proof: Since $G \neq K_2$, $\gamma(\overline{BG_1}(G)) \geq 2$, and $\gamma(BG_1(G)) = 2$ or 3.

Therefore, $4 \leq \gamma(BG_1(G))+\gamma(\overline{BG_1}(G))$ -------------------------------------- (1)

Also, $\gamma(BG_1(G)) \leq 3$ and $\gamma(\overline{BG_1}(G)) \leq \min\{1+\delta(G), \gamma(G)+1\}$

Therefore, $\gamma(BG_1(G)) + \gamma(\overline{BG_1}(G)) \leq 4+k$, where $k = \min\{\delta(G), \gamma(\overline{G})\}$ ------ (2)

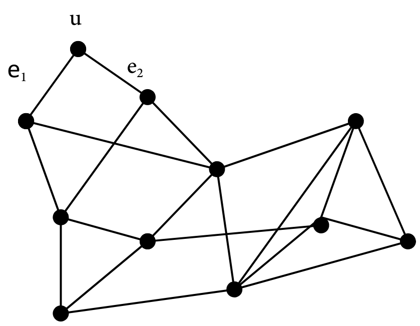
From (1) and (2), it is clear that, $4 \leq \gamma(BG_1(G))+\gamma(\overline{BG_1}(G)) \leq 4+k$.

Remark 2.5:(1) When $G = K_2$, $\gamma(BG_1(G)) = 2$ and $\gamma(\overline{BG_1}(G)) = 1$. Hence, $\gamma(BG_1(G))+\gamma(\overline{BG_1}(G)) = 3$. (2) If $\delta(G) > 2$, $4 \leq \gamma(BG_1(G))+\gamma(\overline{BG_1}(G)) \leq 3+k$.

Examples (1) If G has a pendant vertex then $\gamma(\overline{BG_1}(G)) = 2$ and $\gamma(BG_1(G)) = 2$.

Hence, the lower bound is sharp.

(2)



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Here, D = {u, e_1, e_2}. $\gamma(\overline{BG_1(G)}) = 3$, $\delta(G) = 2$ and $\gamma(\overline{G}) = 2$.

$\gamma(BG_1(G)) = 3$, since $\gamma(G) \geq 3$. Hence, $\gamma(BG_1(G)) + \gamma(\overline{BG_1(G)}) = 6 = 4+2 = 4+k$.
Hence, the upper bound in the inequality is also sharp.

3. Irredundant number of $BG_1(G)$

Next, properties related to irredundant sets of $BG_1(G)$ and $\overline{BG_1(G)}$ can be studied.

Proposition 3.1:(1) Set of all point vertices is an irredundant set of $BG_1(G)$ if and only if $G = K_3$.
(2) Set of all line vertices is an irredundant set of $BG_1(G)$ if and only if $G = 2K_2$, K_3 and $K_{1,2}$.

Proof of (1): Let V(G) be irredundant in $BG_1(G)$. Let $v \in V(G)$. v has a private neighbor in $BG_1(G)$ if there exists an edge e not incident with v but incident with all other vertices, which is true for every $v \in V(G)$. This gives p = 3 and $G = K_3$. Converse is obvious.

Proof of (2): Let D = E(G) be irredundant in $BG_1(G)$. Take $e \in D$. e has a private neighbor in $BG_1(G)$, if there exists $u \in V(G)$ such that e is not incident with u and all other edges are incident with u. This is true for all $e \in E(G)$. Hence, $G = 2K_2$, K_3 or $K_{1,2}$.

Theorem 3.1:$ir(\overline{BG_1(G)}) = 2$ if G satisfies any one of the following conditions.

(1) G is a graph with $p \leq 4$. (2) G has a pendant vertex. (3) G has an isolated vertex. (4) G has a triangle with a vertex of degree two.

Proof: Let G be a graph with $p \leq 4$. If G has no pendant or isolated vertices, let D = {e_1, e_2}. e_1, e_2 have private neighbors in $\overline{BG_1(G)}$, and hence D is irredundant. Also, $\gamma(\overline{BG_1(G)}) = 2$. Hence, $ir(\overline{BG_1(G)}) = 2$ [since $ir(\overline{BG_1(G)}) \leq \gamma(\overline{BG_1(G)})$].

If G has a pendant vertex u, then {u, e}, where e is incident with u, is an irredundant set (maximal). Hence, $ir(\overline{BG_1(G)}) = 2$.

If G has an isolated vertex u, D = {u, e}, where $e \in E(G)$ is a maximal irredundant set with minimum cardinality. Hence, $ir(\overline{BG_1(G)}) = 2$.

If G has a triangle with a vertex u of degree 2 in G, then in $\overline{BG_1(G)}$, {u, e}, where $e = u_1, u_2 \in E(G)$ and $N(u) = \{u_1, u_2\}$ is irredundant. Hence the theorem is proved.

Theorem 3.2: Let G be a graph without isolated vertices. Let $diam(G) \geq 3$.

(1) If G has a pendant vertex, then $ir(\overline{BG_1(G)}) = 2$.

(2) If G has no pendent vertices, then $ir(\overline{BG_1(G)}) = 3$.

Proof: Proof of (1) follows from Theorem 3.1. If v, u $\in V(G)$ are not pendant in G, then D = {u, v, e}, where $d_G(u, v) \geq 3$ and $e \in E(G)$ is a maximal irredundant set with minimum cardinality. Hence, ir($\overline{BG_1}(G)$) = 3.

Following results are stated without proof, since they are easy to follow.

Theorem 3.3: Let diam(G) = 2. (1) If G contains a triangle with a vertex u of degree two and e an edge not incident with u, then D = {u, e} is a maximal irredundant set for $\overline{BG_1}(G)$.

(2) If G has an edge e = uv$\in$V(G), which is not in any triangle, then D = {u, v, e} is maximal irredundant.

(3) Let u $\in V(G)$, with $e(u) \neq 1$ and D = {e $\in$ E(G) : e is incident with u}. Then D is maximal irredundant in $\overline{BG_1}(G)$.

(4) If every edge of G is in a triangle for u $\in V(G)$ with $e(u) \neq 1$. D = {u} $\cup$ D^1, where D^1 is a minimal line cover of N(u) in G is maximal irredundant in $\overline{BG_1}(G)$.

Proposition 3.2: Let r(G) = 1. Let u $\in V(G)$ such that e(u) = 1. Then (1) N(u) is a maximal irredundant set of $\overline{BG_1}(G)$.

(2) Let D = {e $\in$E(G):e is incident with u}. Then D is also a maximal irredundant set of $\overline{BG_1}(G)$.

Proposition 3.3:If G = K_n, any set containing (n−1) point vertices is an irredundant subset of $\overline{BG_1}(G)$.

Proposition 3.4: Let G be a graph without isolated vertices and let D be a minimal line cover for G. Then D is an irredundant set of $\overline{BG_1}(G)$

4. Independent domination of $\overline{BG_1}(G)$

We have already found out the independent domination number of $BG_1(G)$. Now, independent domination number of $\overline{BG_1}(G)$ can be studied.

Theorem 4.1: γ_i($\overline{BG_1}(G)$) = 2 if and only if any one of the following is true. (1) G = $K_2 \cup mK_1$, m > 1. (2) G has an isolated vertex. (3) G has a pendant vertex u such that uv$\in$E(G) and $deg_G v \geq 2$. (4) G has a vertex u lying on a triangle and $deg_G u = 2$.

Proof: Assume γ_i($\overline{BG_1}(G)$) = 2, G $\neq K_2$. Since γ($\overline{BG_1}(G)$) $\neq$ 1, γ($\overline{BG_1}(G)$) = 2. Let D be an independent dominating set of $\overline{BG_1}(G)$.

Case 1: D = {u, v} $\subseteq$V(G).

Since D is independent, u and v are adjacent in G and u, v are not in any triangle in G. Also u, v dominates only one line vertex. Hence, G must be $K_2 \cup mK_1$, m > 1. (Because for $G = K_2$, $\gamma(\overline{BG_1(G)}) = 1 = \gamma_i(\overline{BG_1(G)})$).

Case 2: $D = \{e_1, e_2\} \subseteq E(G)$.

This is not possible, since line vertices form a complete graph in $\overline{BG_1(G)}$.

Case 3: D = {u, e}, where $u \in V(G)$ and $e \in E(G)$.

Since D is independent in G, e is not incident with u.

Sub case 3.1: u is isolated.

In this case, e may be any line vertex.

Sub case 3.2: $deg_G u = 1$.

If $deg_G u = 1$. u is pendant in G. Let $e_1 = uv \in E(G)$. In $\overline{BG_1(G)}$, u cannot dominate v. Therefore, D is an independent dominating set implies that e must be incident with v. Hence, $deg_G v \geq 2$.

Sub case 3.3:$deg_G u = 2$.

Let N(u) = {v, w} in G. In $\overline{BG_1(G)}$, u dominates all the point vertices except v and w. Hence, e must be incident with both v and w.

Sub case 3.4:$deg_G u \geq 3$.

In this case, D cannot be a dominating set.

This proves (1), (2), (3) and (4). Converse is obvious.

Proposition 4.1:$\gamma_i(\overline{BG_1(G)}) \leq \gamma_i(\overline{G})$ or $\gamma_i(\overline{G})+1$.

Proof: Let D be an independent dominating set with cardinality $\gamma_i(\overline{G})$ for $\overline{G}$. If D is a point cover for G, then D is an independent dominating set of $\overline{BG_1(G)}$, otherwise $D \cup \{e\}$, where $e \in E(G)$ is not incident with any element of D, is an independent dominating set for $\overline{BG_1(G)}$. Hence, $\gamma_i(\overline{BG_1(G)}) \leq \gamma_i(\overline{G})$ or $\gamma_i(\overline{G})+1$.

Remark 4.1: If G is a graph with $p \leq 4$, then $\gamma_i(\overline{BG_1(G)}) = 2$ or 3.

Theorem 4.2: Let G be a graph not satisfying the conditions of the Theorem 4.1. (1) If G has an edge not lying in any triangle, then $\gamma_i(\overline{BG_1(G)}) = 3$.

(2) If each edge of G is lying on a triangle, then $\gamma_i(\overline{BG_1(G)}) \leq \delta(G)+1$.

Proof: Let G be a graph not satisfying any conditions of Theorem 4.1. Then $\gamma_i(\overline{BG_1(G)}) > 2$.

Case 1: G has an edge not lying in any triangle.

Let $e = uv \in E(G)$, such that e is not lying in any triangle. u and v dominates all point vertices in $\overline{BG_1(G)}$. Take $D = \{u, v, e_1\}$, where e_1 is not incident with u and v in G. D is an independent dominating set for $\overline{BG_1(G)}$. Hence, $\gamma_i(\overline{BG_1(G)}) = 3$.

Case 2: Every edge of G is lying in some triangle and $G \neq K_1$.

Let $deg_G u = \delta(G)$. In $\overline{BG_1(G)}$, u dominates all point vertices in $V(G)-N(u)$. Since, every edge is lying on a triangle, $<N(u)>$ has no isolated vertices. Consider a minimal independent dominating set for $<N(u)>$. Let it be D_1. If $G \neq K_n$, then $D_1 \cup \{u\}$ is an independent dominating set of $\overline{G}$. $D = D_1 \cup \{u\} \cup \{e\}$, where e is not incident with elements of D_1, is an independent dominating set of $\overline{BG_1(G)}$ and $|D| \leq |D_1| +1+1 \leq (\delta(G)-1)+1+1 = \delta(G)+1$.

Case 3: $G = K_n$

Let $e = v_1v_2 \in E(G)$ and $v_1, v_2, \ldots, v_n \in V(G)$. $D = \{e, v_3, v_4, \ldots, v_n\}$ is an independent dominating set for $\overline{BG_1(G)}$. Hence, $\gamma_i(\overline{BG_1(G)}) \leq n-1 \leq \delta(G)+1$.

This proves the theorem.

5. Connected, total and cycle domination of $BG_1(G)$ and $\overline{BG_1(G)}$

In this section, connected, total and cycle domination of $BG_1(G)$ and its complement are studied.

Observations: (1) If G has a pendant vertex with $p > 2$ and $q \geq 2$, then $\gamma_c(BG_1(G)) = 3$. If u is pendant, v is adjacent to u in G, then $D = \{u, v\} \cup \{e\}$, where $e \neq uv \in E(G)$ is a dominating set.

(2) Let G be a connected graph with $p \geq 3$ and $\gamma(G) = 2$. Then $\gamma_c(BG_1(G)) = 3$. [$D = \{u_1, u_2, u_3\}$, where $\{u_1, u_2\}$ is a connected dominating set of G and u_3 is adjacent to u_1 or u_2 or $D = \{u_1, u_2, e\}$, where e is not incident with u_1, u_2 if $\{u_1, u_2\}$ is not a connected dominating set of G].

(3) If $\gamma_c(G) > 3$ and G has no pendant vertex, then $\gamma_c(BG_1(G)) \leq 4$. [The connected dominating set D is given by $D = \{u, v, w, e\}$, $e = uv \in E(G)$, where $\{u, v, w\}$ is a connected set of G]. $\gamma_c(BG_1(G))$ is at least 3. (4) $D \subseteq V(G)$ such that $|D| = 2$ cannot be a connected dominating set of $BG_1(G)$.

Theorem 5.1: If $G \neq K_2, 2K_2$, then $\gamma_t(BG_1(G)) \leq 4$.

Proof: Let $e = uv \in E(G)$ and let w be adjacent to u or v in G. Then $< D > = < \{u, v, e, w\} >$ is connected. Hence, D is a connected dominating set of $BG_1(G)$ and hence $\gamma_c(BG_1(G))$

≤ 4 and $\gamma_t(BG_1(G)) \leq 4$. If $G = 2K_2$, then $BG_1(G)$ is disconnected and $\gamma_t(BG_1(G)) = 4$. If $G = nK_2$ for $n > 2$, $\gamma_c(BG_1(G)) = 4 = \gamma_t(BG_1(G))$. This proves the theorem.

Theorem 5.2:$\gamma_o(BG_1(G)) = 3$ if and only if $\gamma_o(G) = 3$.

Proof: If $\gamma_o(G) = 3$, then G has a dominating set D = {u, v, w}, where u, v, w form a C_3 in G. This D is also a dominating set for $BG_1(G)$. Hence, $\gamma_o(BG_1(G)) = 3$.

(Here, < D > is an induced cycle). Conversely, assume that $\gamma_o(BG_1(G)) = 3$. Then there exists $D = \{x, y, z\} \subseteq V(BG_1(G))$ such that D is a cycle dominating set of $BG_1(G)$.

Case1: x, y, z $\in$ V(G).

Since, G is an induced subgraph of $BG_1(G)$, $\gamma_o(G) = 3$.

Case2: x, y $\in$ V(G) and z $\in$ E(G).

x, y and z form a C_3 in $BG_1(G)$. Thus, in G, z is not incident with x and y. Also in G, x and y are adjacent. Take $e = xy \in E(G)$. In $BG_1(G)$, e is not dominated by D. Hence, this case is not possible. Similarly, other cases are also not possible.

This proves the theorem.

Theorem 5.3: Let G be a graph with $p > 4$, $\gamma_o(BG_1(G)) = 4$ if and only if $\gamma_o(G) \neq 3$ and any one of the following is true. (1) G has a vertex of degree at least 3.

(2) There exists two non-adjacent vertices u and v in G and two edges e_1 and e_2 not incident with both u and v such that either they are not adjacent or they are incident at w, where w is adjacent to u or v.

Proof: Assume $\gamma_o(G) \neq 3$ and (1) or (2) is true. Since $\gamma_o(G) \neq 3$, $\gamma_o(BG_1(G)) \neq 3$. Now, assume that G has a vertex v_o of degree at least 3. Let $N(v_o) = \{v_1, v_2, v_3\}$ and $e_i = v_o v_i \in E(G)$. In $BG_1(G)$, $D = \{v_0, e_1, v_2, v_3\}$ is a cycle dominating set. Hence $\gamma_o(BG_1(G)) = 4$.

If (2) is true, $D = \{u, v, e_1, e_2\}$ is a cycle dominating set in $BG_1(G)$. Therefore, $\gamma_o(BG_1(G)) = 4$.

Conversely, assume that $\gamma_o(BG_1(G)) = 4$. $\gamma_o(BG_1(G)) = 4$ implies that $\gamma_o(BG_1(G)) \neq 3$ and hence $\gamma_o(G) \neq 3$ by Theorem 5.2.

Case 1: All vertices in the cycle dominating set D are point vertices.

Let $D = \{u, v, w, z\} \subseteq V(G)$. D is also a cycle dominating set of G. Since $p \geq 5$, there exists another vertex x in G, which is adjacent to any one of this four vertices (say u). [x cannot be in another component, since D is a dominating set of $BG_1(G)$]. Thus, $\deg_G u \geq 3$.

Case 2: D contains one line vertex and three point vertices

Let D = {u, v, w, e}, $e \in E(G)$. Let u v w e u be an induced C_4. In this case, {u, v, w} is not a cycle dominating set of G; u, w are not adjacent; e is not incident with u and w; and e must be incident with v, (otherwise, e and v are adjacent in $BG_1(G)$). Thus, $deg_G v \geq 3$.

Case 3: D contains two point vertices and 2 line vertices.

Let D = {u, v, e_1, e_2}. Here u e_1 v e_2 u is a C_4,where e_1, and e_2 are not incident with u and v in G. Also, as D is a dominating set, either e_1, e_2 are not adjacent in G or they are incident at a vertex, which is adjacent to u or v. This proves the theorem. (Other cases are not possible)

Remark 5.1: (1) For all connected graphs, which are not a path or cycle, $\gamma_o(BG_1(G)) \leq 4$.

(2) If $G = P_n \cup mK_1$, or $C_n \cup mK_1$, or $P_n \cup C_n \cup mK_1$, for $n > 3$ and $m \geq 2$, then $\gamma_o(BG_1(G)) = 4$.

(3) If $G = C_3 \cup mK_1$ for $m \geq 2$, then $\gamma_o(BG_1(G)) = 5$ and if $G = P_3 \cup mK_1$ for $m \geq 2$, then $\gamma_o(BG_1(G)) = 6$.

(4) If $G = P_4$ or P_5 or C_5, then $\gamma_o(BG_1(G)) = 5$.

(5) If $G = P_n$ or C_n for $n \geq 5$, then $\gamma_o(BG_1(G)) = 4$.

(6) If $G = P_4 \cup K_1$ or $C_4 \cup K_1$, then $\gamma_o(BG_1(G)) = 5$.

(7) If $G = P_5 \cup K_1$ or $C_5 \cup K_1$, then $\gamma_o(BG_1(G)) = 4$.

Next, we shall find out the connected, total and cycle domination numbers of $\overline{BG_1}(G)$.

Theorem 5.4: If $G \neq K_2$ and G has a pendant vertex, or $p \leq 4$, then $\gamma_c(\overline{BG_1}(G)) = 2 = \gamma_t(\overline{BG_1}(G))$ and $\gamma_o(\overline{BG_1}(G)) = 3$.

Proof: Case 1: Let $p \leq 4$ and let G has no pendant vertex.

Then D = {e_1, e_2} is a connected dominating set for $\overline{BG_1}(G)$. D = {e_1, e_2, e_3} or {u, e_1, e_2} is a cycle dominating set. Hence, $\gamma_o(\overline{BG_1}(G)) = 3$.

Case 2: G has a pendant vertex u.

Let $e = uv \in E(G)$ be incident with u in G. Then D = {u, e} is a connected dominating set for $\overline{BG_1}(G)$. D = {u, v, e} is a cycle dominating set for $\overline{BG_1}(G)$. Hence, $\gamma_c(\overline{BG_1}(G)) = 2 = \gamma_t(\overline{BG_1}(G))$ and $\gamma_o(\overline{BG_1}(G)) = 3$.

Theorem 5.5: If G is a graph with ($p > 4$), no pendant vertex and has an isolated vertex, then $\gamma_c(\overline{BG_1}(G)) = \gamma_t(\overline{BG_1}(G)) = 3$ and $\gamma_o(\overline{BG_1}(G)) = 4$.

Proof: Let u be an isolated vertex in G and v not isolated. Let $e = vw \in E(G)$. D = {u, v, e} is connected dominating set and there exists no connected dominating set with cardinality two. {u, v, e, w} is a cycle dominating set. Hence, $\gamma_c(\overline{BG_1(G)}) = \gamma_t(\overline{BG_1(G)}) = 3$ and $\gamma_o(\overline{BG_1(G)}) = 4$.

Now, let us assume that G has no isolated vertices and has no pendant vertices with $p \geq 5$.

Theorem 5.6: If $diam(G) \geq 3$, then (1) $\gamma_c(\overline{BG_1(G)}) = \gamma_t(\overline{BG_1(G)}) = 3$. (2) $\gamma_o(\overline{BG_1(G)}) = 4$.

Proof: Let u, v $\in V(G)$ such that of $d_G(u, v) \geq 3$. Then $\overline{G}$ has a dominating edge uv. Consider D = {u, v, e}, where e is incident with u or v in G. D is a connected dominating set for $\overline{BG_1(G)}$ and there exists no connected dominating set with cardinality two. Hence, $\gamma_c(\overline{BG_1(G)}) = \gamma_t(\overline{BG_1(G)}) = 3$. Let $e_1 \in E(G)$, $e_2 \in E(G)$ such that e_1 is incident with u and e_2 is incident with v in G. Then D = {u, v, e_1, e_2} is a cycle dominating set for $\overline{BG_1(G)}$. Therefore, $\gamma_o(\overline{BG_1(G)}) = 4$.

Theorem 5.8: (1) If diam(G) = 2 and G contains a triangle with at least one vertex of degree 2 in G and every edge of G lies in a triangle, then $\gamma_c(\overline{BG_1(G)}) = \gamma_t(\overline{BG_1(G)}) = \gamma_o(\overline{BG_1(G)}) = 3$.

(2) If diam(G) = 2 and G has an edge e not in any triangle, then $\gamma_c(\overline{BG_1(G)}) = \gamma_t(\overline{BG_1(G)}) = \gamma_o(\overline{BG_1(G)}) = 3$.

(3) If diam(G) = 2 and G has no triangle with a vertex of degree 2 and every edge of G lies in a triangle, then $\gamma_c(\overline{BG_1(G)}) = \gamma_t(\overline{BG_1(G)}) = \gamma_o(\overline{BG_1(G)}) \leq 2+\alpha_1(\langle N(v)\rangle)$.

Proof of (1): If G contains a triangle with one vertex u of degree 2 in G. Let N(u) = {v, w} and e = v w $\in$ E(G), e_1 = uv, e_2 = uw$\in$ E(G). Consider D = {u, e}. D dominates $\overline{BG_1(G)}$. {u, e, e_1} dominates $\overline{BG_1(G)}$ and is connected. {u, e_1, e_2} is a cycle dominating set.

Proof of (2): If G has an edge e = uv not in any triangle, D = {u, v, e} dominates $\overline{BG_1(G)}$, where D forms a cycle.

Proof of (3): Let v $\in V(G)$ such that $deg_G v = \delta(G)$. Since every edge of G lies on a triangle, $\langle N(v)\rangle$ has no isolated vertex. Let D be a line cover of $\langle N(v)\rangle$ in G. $D_1 = \{v\} \cup D$ is a dominating set for $\overline{BG_1(G)}$ and $D_2 = D_1 \cup \{e\}$, where e is incident with v, is a connected dominating set for $\overline{BG_1(G)}$ and $D_3 = \{e, e_1\} \cup D_1$, where e, e_1 are incident with v is a cycle dominating set for $\overline{BG_1(G)}$ (not induced). This proves the theorem.

Theorem 5.9: If G = K_n, then $\gamma(\overline{BG_1(G)}) = \gamma_t(\overline{BG_1(G)}) = \gamma_c(\overline{BG_1(G)}) = n/2$ or $(n+1)/2$.

6. Global domination number of $BG_1(G)$

Following theorems and propositions deal with the global domination number of $BG_1(G)$. Some bounds for global domination number are also found out.

Theorem 6.1: Let G be a graph without isolated vertices. Then $\gamma_g(BG_1(G)) \leq 1+\delta(G)$.

Proof: Let $u \in V(G)$ such that $deg_G u = \delta(G)$. Let $D = \{e \in E(G) : e$ is incident with u in G$\}$. Then $D \cup \{u\}$ is a dominating set for $BG_1(G)$ and $\overline{BG_1}(G)$. Hence, $\gamma_g(BG_1(G)) \leq 1+\delta(G)$.

Remark 6.1:If G has a pendant vertex, then $\gamma_g(BG_1(G)) = 2$.

Theorem 6.2: If G has an isolated vertex, then $\gamma_g(BG_1(G)) \leq 4$.

Proof: Let $u \in V(G)$ be an isolated vertex of G. $\{u, e\}$, $e \in E(G)$ dominates $\overline{BG_1}(G)$. If $e = vw \in E(G)$, $\{v, w, e\}$ dominates $BG_1(G)$. Therefore, $\{u, v, w, e\}$ is a global dominating set for $BG_1(G)$. Hence, $\gamma_g(BG_1(G)) \leq 4$.

Remark 6.2: If G has a pendant vertex and has some isolated vertices, then $\gamma_g(BG_1(G)) = 3$ or 4.

Proposition 6.1: For $p \geq 5$, if G has no pendant vertex and if G has a vertex of degree 2 lying on a triangle; or if G has an edge e not lying on a triangle, then $\gamma_g(BG_1(G)) = 3$.

Proof: Let v be the vertex of degree two lying in a triangle. $N(v) = \{v_1, v_2\}$, $e = v_1v_2 \in E(G)$. $\{v, e\}$ dominates $\overline{BG_1}(G)$. Let $u \in V(G)$ be not adjacent to v. Then $\{u, v, e\}$ dominates $BG_1(G)$. Hence, $\gamma_g(BG_1(G)) = 3$. Let $uv = e \in E(G)$ such that e is not lying on any triangle. Then $D = \{u, v, e\}$ dominates $BG_1(G)$ and $\overline{BG_1}(G)$. Hence, $\gamma_g(BG_1(G)) = 3$.

Proposition 6.2: If $G = K_n$, then $\gamma_g(BG_1(G)) = n/2 +2$ or $(n+1)/2 +2$.

Proof: Follows from Proposition 2.11 and Theorem 2.2.

Proposition 6.3:If $diam(G) \geq 3$, then $\gamma_g(BG_1(G)) \leq 4$.

Proof: Let $u, v \in V(G)$ such that $d_G(u, v) \geq 3$. Let w be adjacent to u and let $e = uw \in E(G)$. Then $\{u, v, w, e\}$ is a global dominating set. Hence, $\gamma_g(BG_1(G)) \leq 4$.

Proposition 6.4: Let D be a minimal dominating set for G. Then (1) D is a dominating set for $BG_1(G)$ if and only if $|D| \geq 3$ or $|D| = 2$ and D is independent. (2) D dominates $\overline{BG_1}(G)$ if and only if D is a global dominating set for G and D is a line cover for G.

Proof of (1): As $\gamma(BG_1(G)) > 1$, $|D| > 1$. Also, if $|D| = 2$ and $< D >$ is connected, then the edge in D can not be dominated by D in $BG_1(G)$. This proves (1).
Proof of (2): Proof is obvious.

Proposition 6.5: Let D be a global dominating set of G. Then D is a global dominating set of $BG_1(G)$ if and only if (1) $|D| \geq 3$ or $|D| = 2$ and D is independent. (2) D is a point cover for G.
Proof: Follows from Proposition.6.4.

Remark 6.2: In Proposition 6.5, $|D| = 2$ and D is independent is true only when G is disconnected, otherwise D cannot be a dominating set of $BG_1(G)$ or $\overline{BG_1}(G)$.

Theorem 6.3: $\gamma_g(BG_1(G)) \leq \gamma_g(G)+1$.
Proof: Let D be a global dominating set of G. Then it is clear that $|D| \geq 2$. If $|D| = 2$, then take $D_1 = \{u, v, e\}$, for $u, v \in D$ and e edge in $< D >$. Clearly, D_1 dominates $BG_1(G)$ and $\overline{BG_1}(G)$. If $|D| \geq 3$, then $D_1 = D \cup \{e\}$ for $e \in E(G)$ dominates $BG_1(G)$ and $\overline{BG_1}(G)$. This proves the theorem.

7. Total global domination of $BG_1(G)$

A total dominating set D of a graph is a total global dominating set, if D is also a total dominating set of $\overline{G}$. In this section, bounds for total domination number of $BG_1(G)$ are found out.

Theorem 7.1: Let G be a graph without isolated vertices and $diam(G) > 1$. Then $\gamma_{tg}(BG_1(G)) \leq \delta(G)+3$.
Proof: Let $u \in V(G)$ such that $deg_G u = \delta(G)$. Let $D = \{e \in E(G) : e$ is incident with $u\}$. Let $e = uv \in E(G)$ and $w \in V(G)$ is such that it is adjacent to v, not to u. Let $D_1 = D \cup \{u, v, w\}$. Then D_1 is a global dominating set of $BG_1(G)$ and $\overline{BG_1}(G)$ and is also a total dominating set in $BG_1(G)$ and $\overline{BG_1}(G)$. Hence, $\gamma_{tg}(BG_1(G)) \leq \delta(G)+3$.

Remark 7.1:If $G \neq K_2$, and connected with $\delta(G) = 1$, then $\gamma_{tg}(BG_1(G)) \leq 4$.

Proposition 7.1: If G is a connected graph with $diam(G) \geq 3$, then $\gamma_{tg}(BG_1(G)) \leq 5$.

Proof: If diam(G) $\geq$ 3, then there exists a path u v w z in G. Let e = uv, e_1 = vw. Consider D = {u, v, z, e, e_1} $\subseteq V(BG_1(G))$. D is a total dominating set for $BG_1(G)$ and $\overline{BG_1(G)}$. Therefore, $\gamma_{tg}(BG_1(G)) \leq 5$.

Proposition 7.2: Let G $\neq K_2$. If there exists e $\in$E(G) such that e is not lying in a triangle in G, then $\gamma_{tg}(BG_1(G)) \leq 4$.

Proof: Let u, v $\in$V(G) such that e = uv$\in$ E(G) and G $\neq K_2$ and e is not lying on a triangle. D = {u, v, e} dominates $BG_1(G)$ and $\overline{BG_1(G)}$. D_1 = {u, v, e, w}, w not incident with e, is a total global dominating set of $BG_1(G)$. Therefore, $\gamma_{tg}(BG_1(G)) \leq 4$.

Proposition 7.3: If G has a vertex of degree two lying on a triangle, then $\gamma_{tg}(BG_1(G)) \leq 4$.

Proof: Let v $\in$V(G) such that $deg_G v = 2$ and v lies in a triangle formed by v, v_1, v_2. D = {v, e}, where e = $v_1v_2 \in$E(G) dominates $\overline{BG_1(G)}$. Now, let u be any other vertex, which is not adjacent to v in G. Then D_1 = {u, v, e} dominates $BG_1(G)$ and is a total dominating set. Therefore, D_2 = {u, v, e, e_1}, where $e_1 = vv_1 \in$E(G) is a total dominating set for $BG_1(G)$ and $\overline{BG_1(G)}$. Therefore, $\gamma_{tg}(BG_1(G)) \leq 4$.

Proposition 7.4: If diam(G) = 1, $\gamma_{tg}(BG_1(G))$ = (p/2)+3 or ((p+1)/2)+3.

Proof: Let diam(G) = 1. Hence, G = K_n. Let e = uv$\in$E(G). D = {u, v, e} dominates $BG_1(G)$. D_1 = {u, v, e} $\cup$ D_2, where $D_2 \subseteq$ E(G) is a line cover of G. D_1 dominates $\overline{BG_1(G)}$ and D_1 is total. Therefore, S = {u, v, e} $\cup$ $D_2 \cup$ {w}, where e $\in$ D_2 is a total dominating set for $BG_1(G)$ and $\overline{BG_1(G)}$. Therefore, $\gamma_{tg}(BG_1(G)) \leq$ (p/2)+3 or ((p+1)/2)+3.

Proposition 7.5: Let G be a graph with no pendant vertices and diam(G) = 2. Then $\gamma_{tg}(BG_1(G)) \leq (\delta(G)+5)/2$.

Proof: Case 1: e $\in$E(G) is not lying on a triangle.

D = {u, v, e}, e = uv$\in$E(G) dominates $BG_1(G)$ and $\overline{BG_1(G)}$. w$\in$ V(G) is not adjacent to u or v, D_1 = {u, v, e, w} is a total dominating set. Therefore, $\gamma_{tg}(BG_1(G)) \leq 4$.

Case 2: Every edge of G is lying on a triangle.

Let D be a line cover of <N(v) >, where $deg_G v = \delta(G)$. Take D_1 = D $\cup$ {v} $\cup$ {u}. This is a total dominating set of $BG_1(G)$ and $\overline{BG_1(G)}$. Therefore, $\gamma_{tg}(BG_1(G)) \leq (\delta(G)/2)+2$ or $((\delta(G)+1)/2)+2 \leq (\delta(G)+5)/2$.

Theorem 7.2: (1) If diam(G) = 1, then $\gamma_{tg}(BG_1(G)) \leq$ (p/2)+3 or ((p+1)/2)+3.

(2) If diam(G) = 2, then $\gamma_{tg}(BG_1(G)) \leq$ min {4, (δ(G)+5)/2}.

(3) If diam(G) $\geq$ 3, then $\gamma_{tg}(BG_1(G)) \leq 5$.

Proof: Follows from Propositions 7.1, 7.2, 7.3, 7.4 and 7.5.

8. Efficient domination of $BG_1(G)$ and $\overline{BG_1}(G)$

If G $\neq$ K_2, $\overline{BG_1}(G)$ is self-centered with diameter two. Therefore, $\overline{BG_1}(G)$ has no efficient domination. Also, if p > 4 and G has no pendant vertices, then $BG_1(G)$ is self-centered with diameter two. So, if p > 4 and G has no pendant vertices, then $BG_1(G)$ has no efficient domination.

Proposition 8.1: If G has a pendant vertex, then $\gamma_e(BG_1(G)) = 2$.

Proof: Let u $\in$ V(G) be pendant in G and e $\in$ E(G) be incident with u in G. D = {u, e} is an efficient dominating set of $BG_1(G)$, since D is a dominating set for $BG_1(G)$ and d(u, e) = 3 in $BG_1(G)$. Hence, $\gamma_e(BG_1(G)) = 2$.

Proposition 8.2: If p = 4 and G has no pendant vertices, then $BG_1(G)$ has no efficient domination.

Proof: Since p = 4 and G has no pendant vertices, G is any one of $K_3 \cup K_1$, K_4, C_4 or $K_4 - e$. In all these cases, $BG_1(G)$ has no efficient domination.

Proposition 8.3: If p = 3 and G has no pendant vertices, then $\gamma_e(BG_1(G)) = 3$.

Proof: In this case, G = K_3 and the set of all point vertices is a dominating set and distance between any two line vertices is three. Therefore, $\gamma_e(BG_1(G)) = 3$.

Theorem 8.1: (1) $\gamma_e(BG_1(G)) = 2$ if and only if G has a pendant vertex.

(2) $\gamma_e(BG_1(G)) = 3$ if and only if G = K_3.

(3) If G $\neq$ K_3 and has no pendant vertices, then $BG_1(G)$ has no efficient domination.

Proof of (1): Let $\gamma_e(BG_1(G)) = 2$. Let D be an efficient dominating set of $BG_1(G)$.

Case1: D = {u, v} $\subseteq$V(G).

D is a dominating set of $BG_1(G)$. Hence, $d_G(u, v) \geq 2$. This implies d(u, v) $\geq$ 3 in $BG_1(G)$ and u and v have no common non-incident edge in G. Hence, G is of the form $K_{1,m} \cup K_{1,n}$ or $K_{1,m} \cup nK_1$. Thus, G has pendant vertices.

Case 2: D = {u, e}.

D is a dominating set implies u is pendant and e is incident with it in G.

Case 3: D = {e_1, e_2} $\subseteq$ E(G).

D dominates $BG_1(G)$ and $d(e_1, e_2) \geq 3$ in $BG_1(G)$. Therefore, q = 2 and e_1, e_2 has no common non-incident vertex in G. Therefore, $G = 2K_2$ or $K_{1,2}$. If $G = K_{1,2}$, then D is not a dominating set. Hence, $G = 2K_2$. Therefore, G has a pendant vertex. Converse follows from Proposition 8.1.

Proof of (2): Assume $\gamma_e(BG_1(G)) = 3$. Let D be a minimal efficient dominating set with cardinality 3 for $BG_1(G)$.

Case 1: $D = \{u, v, w\} \subseteq V(G)$.
D is efficient implies distance between any two elements of D in G is at least 3 and there is no edge not incident with any two elements of D in G. This is not possible.

Case 2: $D = \{u, v, e\}$ $u, v \in V(G)$, $e \in E(G)$.
D is efficient implies u and v are at distance at least 3 in G and e is incident with u and v, which is not possible.
Case 3: $D = \{u, e_1, e_2\}$.
D is efficient implies, e_1, e_2, are incident with u in G. But, $d(e_1, u) = d(e_2, u) = 2$ in $BG_1(G)$. Therefore, this is also not possible.
Case 4: $D = \{e_1, e_2, e_3\}$.
D is a dominating set implies q = 3 in G. D is efficient implies any two elements of D cannot have a common non-incident point vertex in G. Hence, $G = K_3$ only.
Converse follows from Proposition 8.3.
Proof of (3): Already proved.

9. Restrained domination of $BG_1(G)$ and $\overline{BG_1}(G)$

Following results deal with the **restrained domination** of $\overline{BG_1}(G)$.

Proposition 9.1: If $G \neq K_2$ and $p \leq 4$, then $\gamma_r(\overline{BG_1}(G)) = 2$.
Proof: Case 1: G has a pendant vertex u.

$D = \{u, e\}$, $e = uv \in E(G)$ is a restrained dominating set of $\overline{BG_1}(G)$.
Case 2: G has no pendant vertex and no isolated vertex.

If $G = C_3$, $D = \{u, e\}$, where $u \in V(G)$ and e not incident with u in G and in other cases, $\{e_1, e_2\}$, where e_1, e_2 are independent edges of G is a restrained dominating set.
Case 3: G has an isolated vertex u.

$D = \{u, e\}$ is a restrained dominating set. Hence, in all cases, $\gamma_r(\overline{BG_1}(G)) = 2$.

Proposition 9.2: Let G be a graph with at least four vertices. If G has a pendant vertex or isolated vertex, then $\gamma_r(\overline{BG_1}(G)) = 2$.

Proof: If u is pendant in G, then {u, e}, where $e = uv \in E(G)$ is a restrained dominating set of $\overline{BG_1(G)}$. If v is an isolated vertex, then {v, e}, where $e \in E(G)$ is a restrained dominating set of $\overline{BG_1(G)}$. Hence, $\gamma_r(\overline{BG_1(G)}) = 2$.

Now, assume that G is a graph without isolated vertices and pendant vertices and $p \geq 5$. Following theorems give the restrained domination of $\overline{BG_1(G)}$. Since they are easy to follow, statements are given without proof.

Theorem 9.1:(1) Let $diam(G) \geq 3$. If G contains a triangle with at least one vertex of degree two in G, then $\gamma_r(\overline{BG_1(G)}) = 2$; otherwise, $\gamma_r(\overline{BG_1(G)}) = 3$.

(2) Let $diam(G) \leq 2$. If G contains a triangle with at least one vertex of degree two in G, then $\gamma_r(\overline{BG_1(G)}) = 2$; If G has no such triangle and has an edge e, which is not in any triangle then $\gamma_r(\overline{BG_1(G)}) = 3$; otherwise, $\gamma_r(\overline{BG_1(G)}) = 1+\alpha_1(<N(v)>)$, where $deg_G v = \delta(G)$.

(3) If $G = K_n$, then $\gamma_r(\overline{BG_1(G)}) = n/2$ or $(n+1)/2$.

Remark 9.1: (1) $\gamma_r(\overline{BG_1(G)}) = \gamma(\overline{BG_1(G)})$.

(2) If $q \geq 2$ and D is restrained dominating set of $\overline{G}$, then $D \cup \{e\}$ is a restrained dominating set of $\overline{BG_1(G)}$.

(3) If $q \geq 2$, the set of all point vertices is a restrained dominating set of $\overline{BG_1(G)}$.

(4) Set of all line vertices is a restrained dominating set if and only if radius of G is greater than one.

(5) Let D be a restrained dominating set of $\overline{G}$. D is a restrained dominating set of $\overline{BG_1(G)}$ if and only if D is a point cover for G.

10. Neighborhood Number of $BG_1(G)$ and $\overline{BG_1(G)}$

Neighborhood number of $BG_1(G)$ and $\overline{BG_1(G)}$ have been studied here. Bounds for $n_o(BG_1(G))$ and $n_o(\overline{BG_1(G)})$ are found out.

Proposition 10.1: (1) If $G \neq K_2$, then set of all point vertices is a neighborhood set for $BG_1(G)$.

(2) If $G \neq K_2 \cup mK_1$ and $q > 1$, then set of all line vertices is a neighborhood set of $BG_1(G)$.

Proof of (1): Since set of all line vertices is independent in $BG_1(G)$, set of all point vertices covers all the edges of $BG_1(G)$. Hence, it is a neighborhood set of $BG_1(G)$.

Proof of (2): Since $G \neq K_{1,n} \cup mK_1$ and $q > 1$, for any two adjacent vertices u, v in G, there exists an edge not incident with u and v. Hence, in $BG_1(G)$, all the edges of $BG_1(G)$ is covered by the neighborhood of line vertices. Therefore, $D = E(G)$ is a neighborhood set of $BG_1(G)$.

Theorem 10.1:$n_o(BG_1(G)) \leq \min\{p, q\}$, if $G \neq K_{1,n} \cup mK_1$.
Proof: Proof follows from Proposition 10.1.

Theorem 10.2: Let G be a graph without isolated vertices.
(1) If $G = K_n$, then $n_o(\overline{BG_1(G)}) = \alpha_1(G)$.
(2) If $G \neq K_n$, $\alpha_1(G) \leq n_o(\overline{BG_1(G)}) \leq p-1$.
Proof of (1): Let $G = K_n$. Consider a line cover D of G. $\bigcup_{x\in D} < N[x] >$covers all the edges in K_q and edges joining point vertices to line vertices.

Hence, $\overline{BG_1(G)} = \cup < N[x] >$. Hence, $n_o(\overline{BG_1(G)}) \leq \alpha_1(G)$.

Proof of (2): Let $G \neq K_n$. Consider $e = uv \in E(G)$; $u, v \in V(G)$. Let $D = V(G) - \{u, v\}$ and $S = \{e\} \cup D$. $< N[e] >$ covers all the edges in $\overline{BG_1(G)}$ joining line vertices and edges joining e to u and v and edges joining elements of D to other line vertices and $< N[x] >$, where $x \in D$ covers all the edges of $\overline{G}$. Hence, $\overline{BG_1(G)} = \cup<N[x]>$. Therefore, $n_o(\overline{BG_1(G)}) \leq p-1$. Also, $n_o(\overline{BG_1(G)}) \geq \alpha_1(G)$ for G.
Hence, $\alpha_1(G) \leq n_o(\overline{BG_1(G)}) \leq p-1$.

Conclusion: In this paper, we have studied connected, efficient, independent, restrained, total and cycle dominations of $BG_1(G)$ and its Complement. Irredundance and neighborhood numbers are also studied. Other domination parameters and properties are also studied and submitted.

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