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One Modulo Three Mean Labeling of Some Family of Trees

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Abstract: *The concept of mean labeling was introduced by Somasundaram and Ponraj [26]. Different kinds of mean labeling are further studied by Gayathri and Gopi [9-17]. Swaminathan and Sekar [28] introduced the concept of modulo three graceful labeling. As an analogue Jayanthi and Maheswari [22] introduced one modulo three mean labeling and proved that some standard graphs are one modulo three mean graphs. In [20], we have obtained some necessary conditions and properties for the one modulo three mean labeling. In this paper, we obtain the one modulo three mean labeling of some family of trees.*

Key words: *Mean labeling, Mean graphs, One modulo three mean labeling, one modulo three mean graphs.*
AMS (MOS) Subject Classification: 05C78

1. Introduction

This paper deals with graph labeling. All graphs considered here are simple, finite and undirected. The terms not defined here are used in the sense of Harary [21].

The **graph labeling** is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a **vertex labeling (or an edge labeling)**. By a (p,q) graph G, we mean a graph $G=(V,E)$ with $|V|=p$ and $|E|=q$.

Graph labeling was first introduced in the late 1960's. Many studies in graph labeling refer to Rosa's research in 1967 [25]. Rosa introduced a function f from a set of vertices in a graph G to the set of integers {0, 1, 2, 3...}, so that each edge xy is assigned the label $|f(x)-f(y)|$, with all labels distinct, Rosa called this labeling **β-valuation**. Independently, Golomb studied the same type of labeling and called this labeling as **graceful labeling**.

Labeled graphs serve as useful models for a broad range of applications such as x-ray crystallography, radar, coding theory, astronomy, circuit design and communication network addressing. Particularly, interesting applications of graph labeling can be found in [2-5].

Somasundaram and Ponraj [26-27] have introduced the notion of mean labeling of a (p,q) graph. A graph G is said to have a **mean labeling** if there is an injective function

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f from the vertices of G to {0, 1, 2, 3.., q} such that when each edge uv is labeled with $\frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $\frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$ is odd then the resulting edge labels are distinct. A graph that admits a mean labeling is called a **mean graph**.

Vaidya [31-34] and et al. have investigated several families of mean graphs. Nagarajan [30] and et al. have found some new results on mean graphs. Ponraj, Jayanthi and Ramya extended the notion of mean labeling to super mean labeling in [24]. Gayathri and Tamilselvi [18-19, 29] extended super mean labeling to k–super mean, (k,d)–super mean, k–super edge mean and (k,d)–super edge mean labeling. Manikam and Marudai [23] introduced the concept of odd mean graph. Gayathri and Amuthavalli [1,6-8] extended this concept to k–odd mean and (k,d)–odd mean graphs.

Different kinds of mean labeling are studied by Gayathri and Gopi in [9-17]. Swaminathan and Sekar [28] introduced the concept of modulo three graceful labeling. As an analogue, Jayanthi and Maheswari [22] introduced one modulo three mean labeling and proved that some standard graphs are one modulo three mean graphs. In [20], we have obtained some necessary conditions and properties for the one modulo three mean labeling. In this paper, we obtain the one modulo three mean labeling of some family of trees.

2. Prior Results

Observation 2.1 [20]:

(i) $3q - 2 \equiv 1 \pmod 3$ for all q

(ii) $3q - 2$ is odd if q is odd and
$3q - 2$ is even if q is even

(iii) $$3q-2 \equiv \begin{cases} 1 \pmod 6 & \text{if } q \text{ is odd} \\ 4 \pmod 6 & \text{if } q \text{ is even} \end{cases}$$

(iv) $3q - 3$ is even if q is odd and
$3q - 3$ is odd if q is even

(v) $$3q-3 \equiv \begin{cases} 0 \pmod 6 & \text{if } q \text{ is odd} \\ 3 \pmod 6 & \text{if } q \text{ is even} \end{cases}$$

Property 2.2 [21]:

Let $G=(p,q)$ be a one modulo three mean graph with one modulo three mean labeling f. Let t be the number of edges whose one vertex label is even and the other is odd. Then $\sum_{v\in V(G)} d(v)f(v)+t=q(3q-1)$ where $d(v)$ denotes the degree of vertex v.

In [20], we have obtained the results listed below.

Property 2.3:

If a graph G is a one modulo three mean graph then 0 and 1 are vertex labels.

Property 2.4:

If a graph $G=(p,q)$ is a one-modulo three mean graph then $3q-3$ and $3q-2$ are ought to be the vertex labels.

Corollary 2.5:

If $G=(p,q)$ is a one-modulo three mean graph with one modulo three mean labeling *f*, then $\sum_{v\in V(G)} d(v)f(v)\geq q^2$.

Property 2.6:

Let $G=(p,q)$ be a l regular one modulo three mean graph with I even. Let t be the number of edges whose one vertex label is even and other is odd then t is even.

Property 2.7:

Let $G=(p,q)$ be a one modulo three mean graph.

(i) If q is odd then $0,1, 3q-2$ cannot be the vertex labels of the cycle C_3 contained in G.

(ii) If q is even then $0,1, 3q-3$ cannot be the vertex labels of the cycle C_3 contained in G.

(iii) If q is odd then $1, 3q-3, 3q-2$ cannot be the vertex labels of the cycle C_3 contained in G.

(iv) If q is even then $0, 3q-3, 3q-2$ cannot be the vertex labels of the cycle C_3 contained in G.

Theorem 2.8:

Let G be a connected one modulo three mean graph. Let u be a vertex with label $f(u)$. Let v be any vertex adjacent to the vertex u with label $f(v)$

a) If $f(u)\equiv 0 \pmod 6$ then $f(v)\equiv 1 \pmod 6$

b) If $f(u)\equiv 1 \pmod 6$ then $f(v)\equiv 0$ (or) $1 \pmod 6$

c) If $f(u)\equiv 3 \pmod 6$ then $f(v)\equiv 4 \pmod 6$

d) If $f(u)\equiv 4 \pmod 6$ then $f(v)\equiv 3$ (or) $4 \pmod 6$

Theorem 2.9:

If G is a connected one modulo three mean graph then all its vertices receive labels as either 0 (or) 1 (mod 6).

Theorem 2.10:

If $G=(p,q)$ is a connected one modulo three mean graph then q is odd.

Corollary 2.11:

If G is a connected graph with q even then it is not a one modulo three mean graph.

Theorem 2.12:

If $G=(p,q)$ is a tree of odd order then it is not a one modulo three mean tree.

Theorem 2.13:

If G is a unicyclic connected graph with q even then it is not a one modulo three mean graph.

Theorem 2.14:

If G is a connected one modulo three mean graph of odd size and then $\Delta \le \frac{q+1}{2}$. Where Δ is the maximum degree of a vertex in G.

3. Main Results

Definition 3.1:

A graph $G=(p,q)$ is said to be **one modulo three mean graph (OMTMG)** if there is an injective function f from the vertex set of G to the set $\{0, 1, 3, 4, \ldots, 3q-5, 3q-3, 3q-2\}$ with f is one-one and f induces a bijection f^* from the edge set of G to the set $\{1, 4, 7, 10, \ldots, 3q-5, 3q-2\}$ where $f^*(uv)=\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ and the function f is called as **one modulo three mean labeling** (OMTML) of G. Here, $f^*(uv) \equiv 1 \pmod 3$ for every edge uv in G.

Definition 3.2:

A ***Y*-tree** is a graph obtained from a path by appending an edge to a vertex of a path adjacent to an end point and it is denoted by Y_n where n is the number of vertices in the tree.

Theorem 3.3:

The Y-tree $Y_n\,(n \geq 6)$ is a one modulo three mean graph if and only if n is even.

Proof:

Assume n is even.

To prove that $Y_n\,(n \geq 6)$ is a one modulo three mean graph.

Let $\{v_1, v_2, v_3, \ldots, v_n\}$ be the vertices of Y_n and $\{e_1, e_2, e_3, \ldots, e_{n-1}\}$ be the edges of Y_n which are denoted as in Figure 3.1.

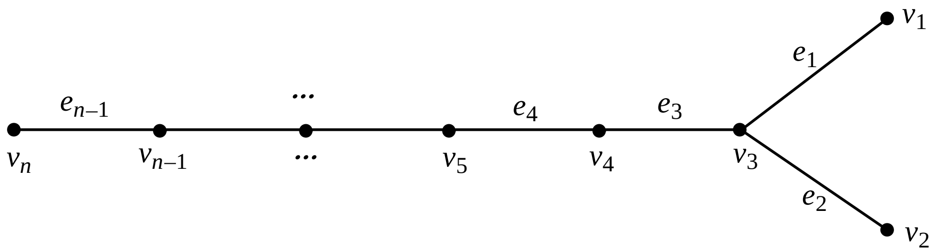

Figure 3.1: Ordinary labeling of Y_n

First we label the vertices as follows:

Define $f : V \rightarrow \{0,\ 1,\ 3,\ 4,\ 6,\ 7, \ldots, 3q-5,\ 3q-3,\ 3q-2\}$ by

$$f(v_1) = 0$$

$$f(v_{n-1}) = 3(n-2) + 1$$

$$f(v_n) = 3(n-2) - 5$$

$$f(v_{2i}) = 6i, \qquad \text{for } 1 \leq i \leq \frac{n-2}{2}$$

$$f(v_{2i+1}) = 6(i-1) + 1, \qquad \text{for } 1 \leq i \leq \frac{n-4}{2}$$

Then the induced edge labels are:

$$f^*(e_i) = 3i - 2, \qquad \text{for } 1 \leq i \leq n-3$$

$$f^*(e_{n-2}) = 3(\mathrm{n}-2) + 1$$

$$f^*(e_{n-1}) = 3(n-2) - 2$$

The above defined function f provides one modulo three mean labeling of the graph $Y_n(n \geq 6)$. Hence, the Y-tree $Y_n\,(n \geq 6)$ is a one modulo three mean graph when n is even.

Conversely, let us assume that Y_n is a one modulo three mean graph. To prove that n is even.

Suppose not, let n be odd, that is p is odd. Then by Theorem 2.12, Y_n is not a one modulo three mean graph, which is a contradiction to our assumption that Y_n is a one modulo three mean graph.

Hence n is even.

One modulo three mean labeling of the graph Y_8 and Y_{12} are given in Figure 3.2 and Figure 3.3 respectively.

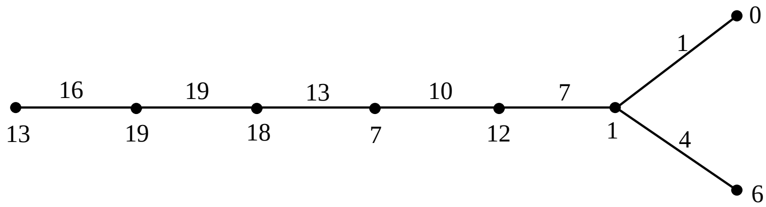

Figure 3.2: OMTML of Y_8

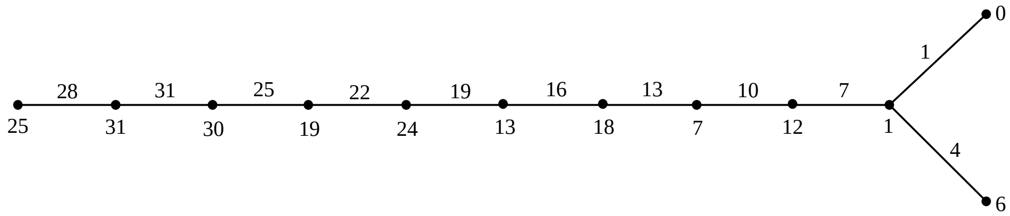

Figure 3.3: OMTML of Y_{12}

Definition 3.4:

A **twig** is a tree obtained from a path by attaching exactly two pendant edges to each internal vertex of the path.

Theorem 3.5:

The twig $TW(n)$ $(n \geq 4)$ is a one modulo three mean graph if and only if n is even.

Proof:

Assume n is even.

To prove that $TW(n)$ $(n \geq 4)$ is a one modulo three mean graph.

Let $\{v_i,\ 1 \leq i \leq n,\ u_i,\ w_i,\ 1 \leq i \leq n-2\}$ be the vertices and $\{e_i\ 1 \leq i \leq 3n-5\}$ be the edges of $TW(n)$ which are denoted as in Figure 3.4.

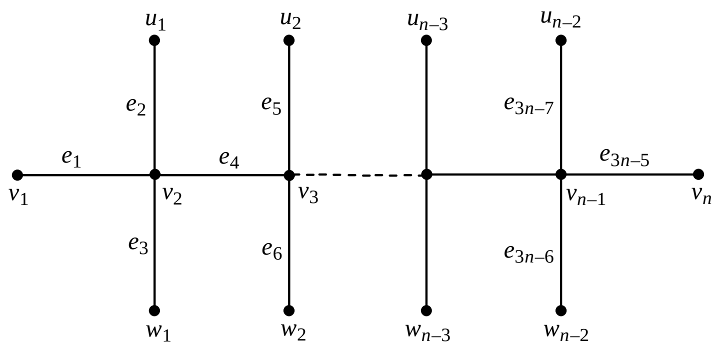

Figure 3.4: Ordinary labeling of $TW(n)$

First we label the vertices as follows:

Define $f : V\left(TW\left(n\right)\right) \to \{0,\ 1,\ 3,\ 4,\ldots,3q-3,\ 3q-2\}$

For $1 \leq i \leq n - 2$,

$$f\left(u_i\right) = \begin{cases} 9i-3, & \text{if } i \text{ is odd} \\ 9\left(i-1\right)-2, & \text{if } i \text{ is even} \end{cases}$$

$$f\left(w_i\right) = \begin{cases} 9i+3, & \text{if } i \text{ is odd} \\ 9i-5, & \text{if } i \text{ is even} \end{cases}$$

For $1 \leq i \leq n$,

$$f\left(v_i\right) = \begin{cases} 9\left(i-1\right), & \text{if } i \text{ is odd} \\ 9\left(i-2\right)+1, & \text{if } i \text{ is even} \end{cases}$$

Then the induced edge labels are

$$f^*\left(e_i\right) = 3i - 2, \qquad \text{for } 1 \leq i \leq 3n - 5$$

The above defined function f provides one modulo three mean labeling of the graph $TW(n)$ $(n \geq 4)$. Hence, the twig $TW(n)$ $(n \geq 4)$ is a one modulo three mean graph when n is even.

Conversely, let us assume that $TW(n)$ $(n \geq 4)$ is a one modulo three mean graph. To prove that n is even.

Suppose not, let n be odd, that is p is odd. Then by Theorem 2.12, $TW(n)$ $(n \geq 4)$ is not a one modulo three mean graph, which is a contradiction to our assumption that $TW(n \geq 4)$ is a one modulo three mean graph.

Hence n is even.

One modulo three mean labeling of the graph $TW(6)$ and $TW(10)$ are given in Figure 3.5 and Figure 3.6 respectively.

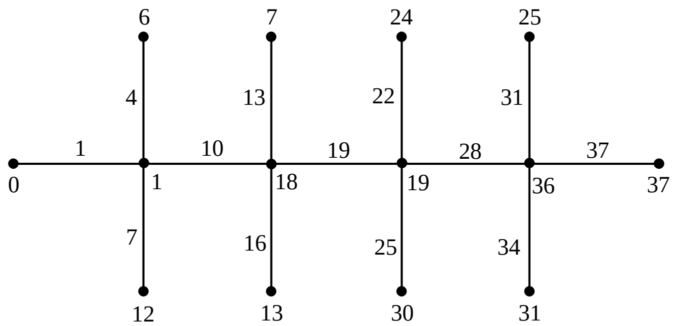

Figure 3.5: OMTML of $TW(6)$

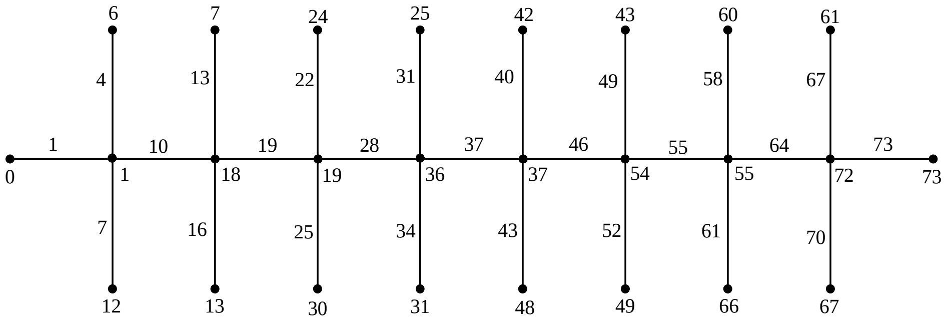

Figure 3.6: OMTML of $TW(10)$

Definition 3.6:

The **Bistar $B_{m,n}$** is the graph obtained from K_2 by joining m pendant edges to one end of K_2 and n pendant edges to the other end of K_2. The edge of K_2 is called the central edge of $B_{m,n}$.

Theorem 3.7:

The graph $S(B_{n,n+1})$ obtained by the subdivision of the central edge of the bistar $B_{n,n+1}$ is a one modulo three mean graph.

Proof:

Let $\{u,\ v,\ w,\ u_i,\ 1 \le i \le n,\ v_i,\ 1 \le i \le n+1\}$ be the verttices and $\{a,\ b,\ a_i,\ 1 \le i \le n,\ b_i,\ 1 \le i \le n+1\}$ be the edges which are denoted as in Figure 3.7.

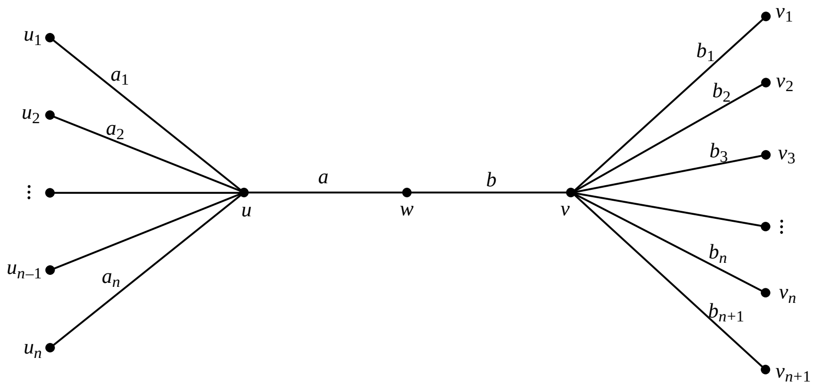

Figure 3.7: Ordinary labeling of $S(B_{n,n+1})$

First we label the vertices as follows:

Define $f : V \to \{0,\ 1,\ 3,\ 4,\ 6,\ 7, \ldots, 3q-3,\ 3q-2\}$

$f(u) = 1$; $f(w) = 0$; $f(v) = 6(n+1)+1$

$f(u_i) = 6i + 1$, for $1 \le i \le n$

$f(v_i) = 6i$, for $1 \le i \le n + 1$.

Then the induced edge labels are

$f^*(a) = 1;$ $\quad$ $f^*(b) = 3(n + 2) - 2$

$f^*(a_i) = 3i + 1,$ $\quad$ for $1 \leq i \leq n$

$f^*(b_i) = 3(i - 1) + 3(n + 2) + 1,$ $\quad$ for $1 \leq i \leq n + 1.$

The above defined function f provides one modulo three labeling of $S\left(B_{n,n+1}\right)$. Hence, the graph $S\left(B_{n,n+1}\right)$ is a one modulo three mean graph.

One modulo three mean labeling of the graph $S\left(B_{4,5}\right)$ and $S\left(B_{5,6}\right)$ are given in Figure 3.8 and Figure 3.9 respectively.

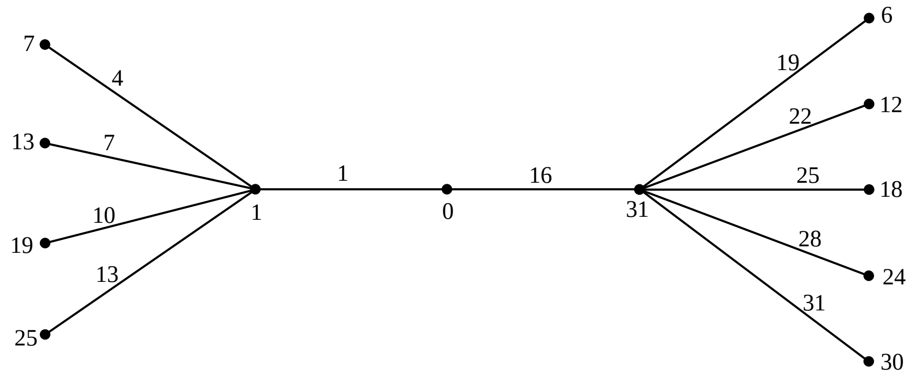

Figure 3.8: OMTML of $S\left(B_{4,5}\right)$

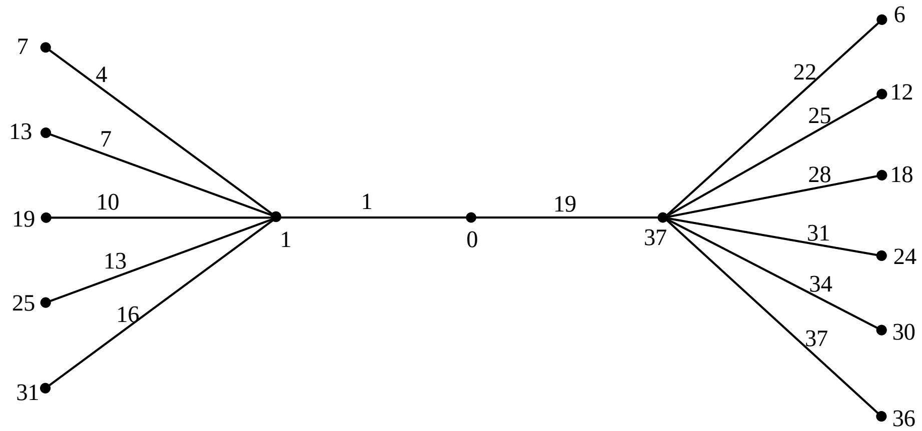

Figure 3.9: OMTML of $S\left(B_{5,6}\right)$

Theorem 3.8:

If $m = n$, then $S\left(B_{m,n}\right)$ is not a one modulo three mean graph.

Proof:

Suppose $S(B_{m,n})$ is a one modulo three mean graph. The number of vertices in $p = m + n + 3$ is odd, that is $q = m + n + 2$ is even, a contradiction to Theorem 2.11.

Hence the Theorem.

Theorem 3.9:

The graph $S(B_{m,n})$, $m \geq n+2$ is not a one modulo three mean graph.

Proof:

Suppose $S(B_{m,n})$, $m \geq n+2$ is a one modulo three mean graph.

Case 1: *m* even, *n* even (or) *m* odd, *n* odd

In this case, the number of vertices in $p = m + n + 3$ is odd, that is $q = m + n + 2$ is even.

a contradiction to Theorem 2.11.

Case 2: *m* even, *n* odd (or) *m* odd, *n* even

In this case, the number of vertices $p = m + n + 3$ is even, that is $q = m + n + 2$ is odd. Therefore by Theorem 2.14,

$$\Delta(G) \leq \frac{q+1}{2} \qquad \text{... (1)}$$

Here $G = S(B_{m,n})$, $\Delta = m + 1$, $q = m + n + 2$

From (1), we have

$$m + 1 \leq \frac{m+n+2+1}{2}$$

$$2m + 2 \leq m + n + 3$$

$$m \leq n + 1$$

a contradiction to $m \geq n+2$.

Definition 3.10:

The graph $G = (m : \langle K_{1,n} : K_{1,n} \rangle)$ is obtained by joining the centres of m copies of $\langle K_{1,n} : K_{1,n} \rangle$ in parallel.

Theorem 3.11:

The graph $(m : \langle K_{1,n} : K_{1,n} \rangle)$ is a one modulo three mean graph if and only if m is even.

Proof:

Assume m is even.

To prove that $\left(m:\left\langle K_{1,n}:K_{1,n}\right\rangle\right)$ is a one modulo three mean graph.

Let $V\left[\left(m:\left\langle K_{1,n}:K_{1,n}\right\rangle\right)\right]=\left\{u_i,v_i,w_i,1\le i\le m,u_{ij},v_{ij},1\le i\le m,1\le j\le n\right\}$

be the set of vertices and

$E\left[\left(m:\left\langle K_{1,n}:K_{1,n}\right\rangle\right)\right]=\left\{e_i,f_i,1\le i\le m,g_i,1\le i\le m-1,e_{ij},f_{ij},1\le i\le m,1\le j\le n\right\}$

be the edges which are denoted as in Figure 3.10.

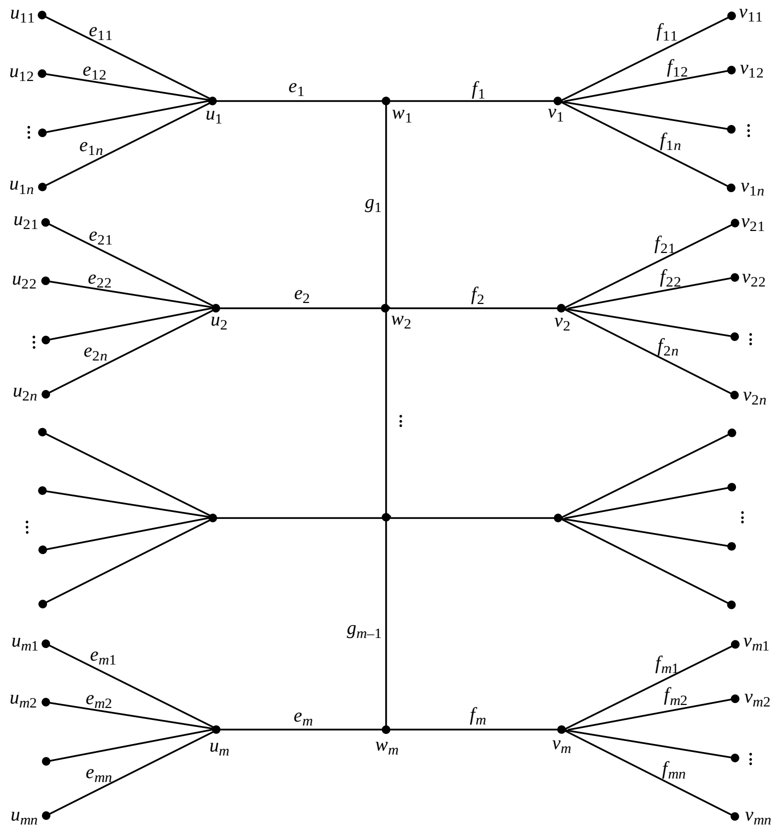

Figure 3.10: Ordinary labeling of $\left(m:\left\langle K_{1,n}:K_{1,n}\right\rangle\right)$

First we label the vertices as follows:

Define $f:V\rightarrow\{0,\ 1,\ 3,\ 4,\ldots,3q-3,\ 3q-2\}$

$$f\left(u_{ij}\right)=\begin{cases}(6n+9)(i-1)+6(j-1), & \text{for } 1\le i\le m,\ i \text{ is odd and } 1\le j\le n\\ (6n+9)(i-2)+6(j-1)+6(n+1)+1, & \text{for } 2\le i\le m,\ i \text{ is even and } j=1\\ (6n+9)(i-2)+6(j-2)+6(n+3)+1, & \text{for } 2\le i\le m,\ i \text{ is even and } 2\le j\le n\end{cases}$$

$$f\left(v_{ij}\right)=\begin{cases}(6n+9)(i-1)+6j+1, & \text{for } 1\le i\le m,\ i \text{ is odd and } 1\le j\le n-1\\ (6n+9)(i-1)+6(j+1), & \text{for } 1\le i\le m,\ i \text{ is odd and } j=n\\ (6n+9)(i-2)+6(j-1)+6(n+3), & \text{for } 1\le i\le m,\ i \text{ is even and } 1\le j\le n\end{cases}$$

$$f\left(u_{i}\right)=\begin{cases}(6n+9)(i-1)+1, & \text{for } 1\le i\le m,\ i \text{ is odd}\\ (6n+9)(i-2)+6(n+2), & \text{for } 2\le i\le m,\ i \text{ is even}\end{cases}$$

$$f\left(w_{i}\right)=\begin{cases}(6n+9)(i-1)+6n, & \text{for } 1\le i\le m,\ i \text{ is odd}\\ (6n+9)(i-2)+6(n+2)+1, & \text{for } 2\le i\le m,\ i \text{ is even}\end{cases}$$

$$f\left(v_{i}\right)=\begin{cases}(6n+9)(i-1)+6n+1, & \text{for } 1\le i\le m,\ i \text{ is odd}\\ (6n+9)(i-2)+12(n+1)+1, & \text{for } 2\le i\le m,\ i \text{ is even}\end{cases}$$

Then the induced edge labels are

$$f^{*}\left(e_{ij}\right)=\begin{cases}(6n+9)(i-1)+3(j-1)+1, & \text{for } 1\le i\le m,\ i \text{ is odd and } 1\le j\le n\\ (6n+9)(i-2)+3(j-2)+6(n+2)+4, & \text{for } 2\le i\le m,\ i \text{ is even and } 2\le j\le n\\ (6n+9)(i-2)+3(j-1)+6(n+1)+4, & \text{for } 2\le i\le m,\ i \text{ is even and } j=1\end{cases}$$

$$f^{*}\left(f_{ij}\right)=\begin{cases}(6n+9)(i-1)+3(j-1)+3(n+1)+1, & \text{for } 1\le i\le m,\ i \text{ is odd and } 1\le j\le n-1\\ (6n+9)(i-1)+6j+4, & \text{for } 1\le i\le m,\ i \text{ is odd and } j=n\\ (6n+9)(i-1)+3(j-1)+6(n+2)+3(n+1)+1, & \text{for } 1\le i\le m,\ i \text{ is even and } 1\le j\le n\end{cases}$$

$$f^{*}\left(e_{i}\right)=\begin{cases}(6n+9)(i-1)+3n+1, & \text{for } 1\le i\le m,\ i \text{ is odd}\\ (6n+9)(i-2)+6(n+2)+1, & \text{for } 1\le i\le m,\ i \text{ is even}\end{cases}$$

$$f^{*}\left(f_{i}\right)=\begin{cases}(6n+9)(i-1)+6n+1, & \text{for } 1\le i\le m,\ i \text{ is odd}\\ (6n+9)(i-2)+6(n+1)+3(n+1)+4, & \text{for } 1\le i\le m,\ i \text{ is even}\end{cases}$$

$f^*(g_i) = (6n + 9)(i - 1) + 6(n + 1) + 1,$ for $1 \le i \le m - 1$

The above defined function f provides one modulo three mean labeling of the graph $\left(m : \left\langle K_{1,n} : K_{1,n} \right\rangle\right)$. Hence, the graph $\left(m : \left\langle K_{1,n} : K_{1,n} \right\rangle\right)$ is a one modulo three mean graph when m is even.

Conversely, let us assume that $\left(m : \left\langle K_{1,n} : K_{1,n} \right\rangle\right)$ is a one modulo three mean graph. To prove that m is even.

Suppose not, let m be odd. Then the number of vertices in $p = m(2n + 3)$ is odd. Now, by Theorem 2.12, $\left(m : \left\langle K_{1,n} : K_{1,n} \right\rangle\right)$ is not a one modulo three mean graph, a contradiction to our assumption that $\left(m : \left\langle K_{1,n} : K_{1,n} \right\rangle\right)$ is a one modulo three mean graph.

Hence m is even.

One modulo three mean labeling of the graph $\left(6\left\langle K_{1,2} : K_{1,2} \right\rangle\right)$ and $\left(4\left\langle K_{1,5} : K_{1,5} \right\rangle\right)$ are given in Figure 3.11 and Figure 3.12 respectively.

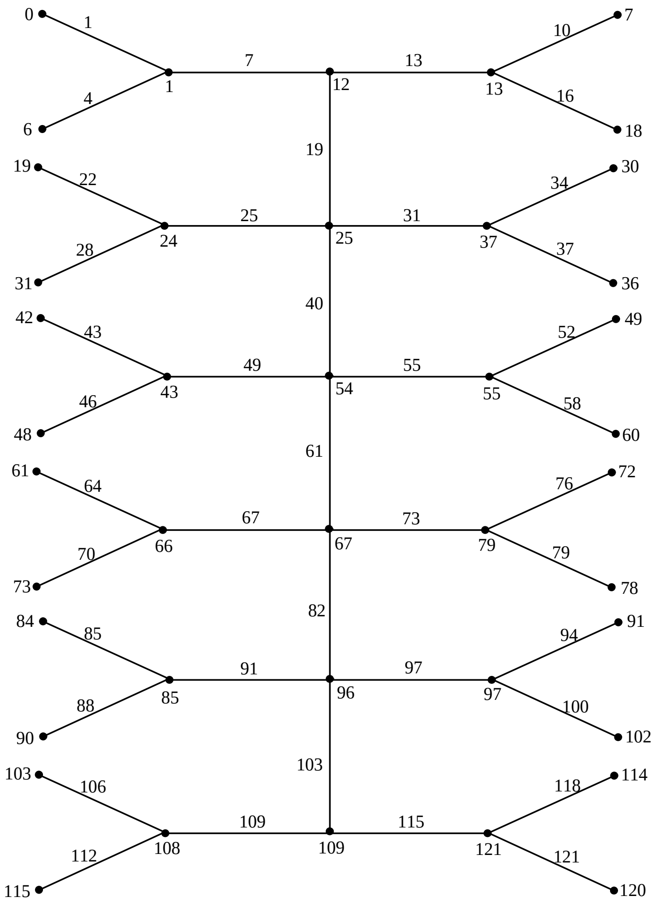

Figure 3.11: OMTML of $\left(6\left\langle K_{1,2} : K_{1,2} \right\rangle\right)$

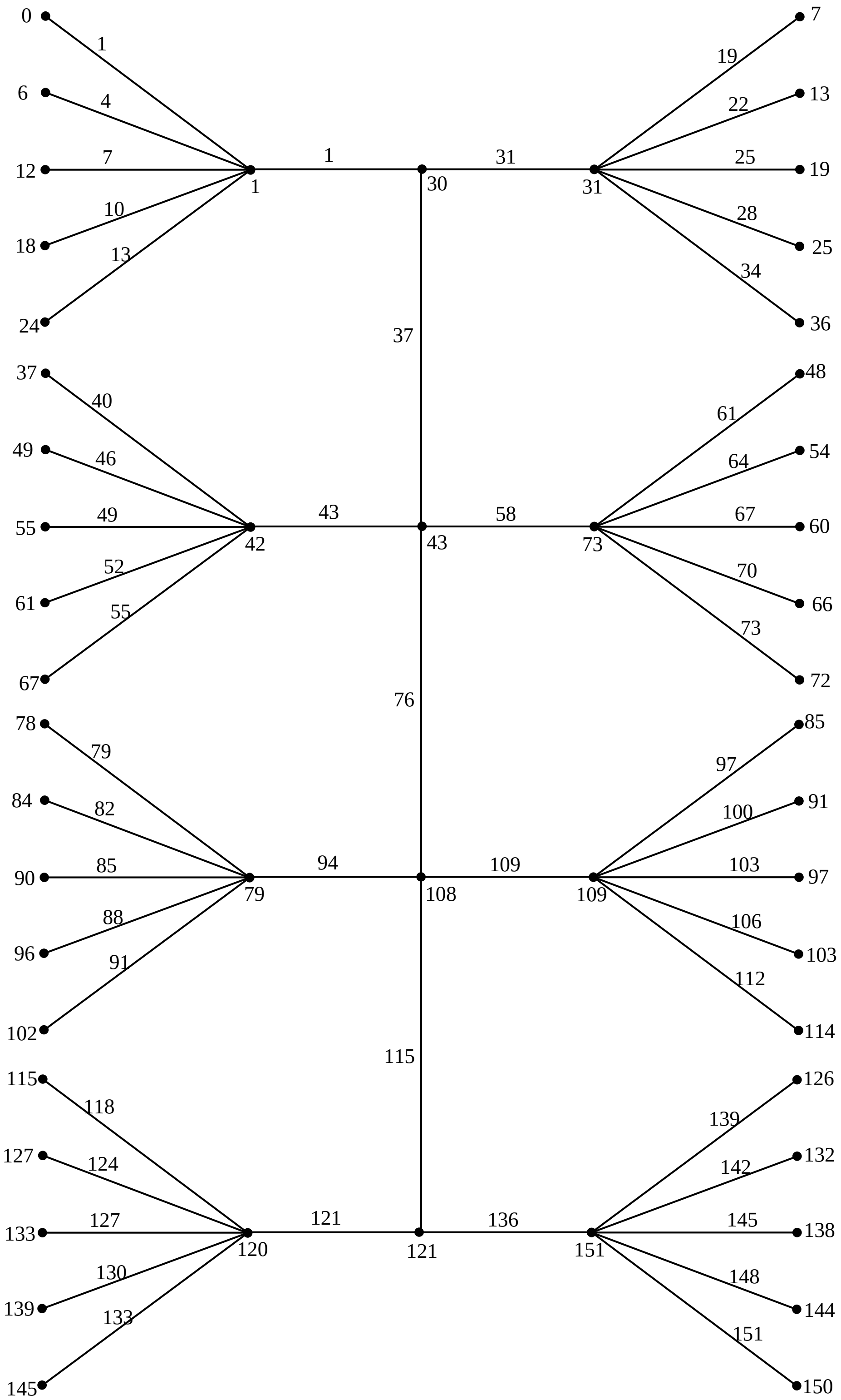

Figure 3.12: OMTML of $\left(4\left\langle K_{1,5} : K_{1,5}\right\rangle\right)$

Definition 3.12:

$T_{t,n,m}$ is a graph obtained by joining the centers of $K_{1,n}$ and $K_{1,m}$ by a path P_t. It consists of $t + n + m$ vertices and $t + n + m - 1$ edges.

Theorem 3.13:

The graph $T_{t,n,m}$ is a one modulo three mean graph for $t(t \geq 2)$ is even and $n = m$.

Proof:

Let $V\left[t_{t,n,m}\right] = \{w_1, w_2, \ldots, w_t,\ u_1, u_2, \ldots, u_n,\ v_1, v_2, \ldots, v_m\}$ be the set of vertices and $E\left[T_{t,n,m}\right] = \{e_1, e_2, \ldots, e_n,\ e_{n+1}, e_{n+2}, \ldots, e_{n+t-1},\ e_{n+t}, e_{n+t+1}, e_{n+t+2}, \ldots, e_{n+t+m-1}\}$ be the edges which are denoted as in the Figure 3.13.

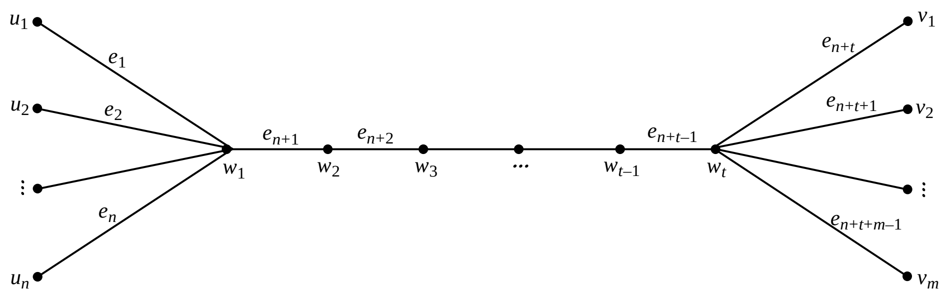

Figure 3.13: Ordinary labeling of $T_{t,n,m}$

First we label the vertices as follows:

Define $f : V\left(T_{t,n,m}\right) \rightarrow \{0,\ 1,\ 3,\ 4, \ldots, 3q-3,\ 3q-2\}$ where $q = n + t + m - 1$.

$f(u_i) = 6(i - 1)$, for $1 \leq i \leq n$

$f(v_i) = 3t + 6(i - 1) + 1$, for $1 \leq i \leq m$

For $1 \leq i \leq t$,

$$f(w_i) = \begin{cases} 3i-2, & \text{if } i \text{ is odd} \\ 3(i-2)+6n, & \text{if } i \text{ is even} \end{cases}$$

Then the induced edge labels are

$f^*(e_i) = 3i - 2$, for $1 \leq i \leq t + n + m - 1$

The above define function f provides one modulo three mean labeling of the graph $T_{t,n,m}$ for $t(t \geq 2)$ is even and $n = m$.

One modulo three mean labeling of the graph $T_{4,5,5}$ and $T_{6,6,6}$ are given in Figure 3.14 and Figure 3.15 respectively.

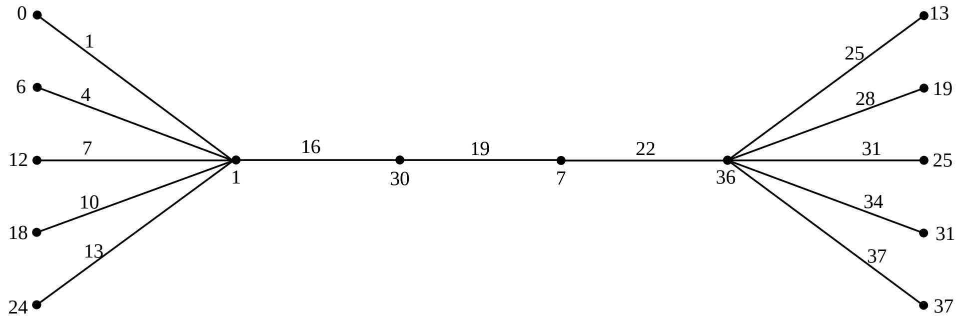

Figure 3.14: OMTML of $T_{4,5,5}$

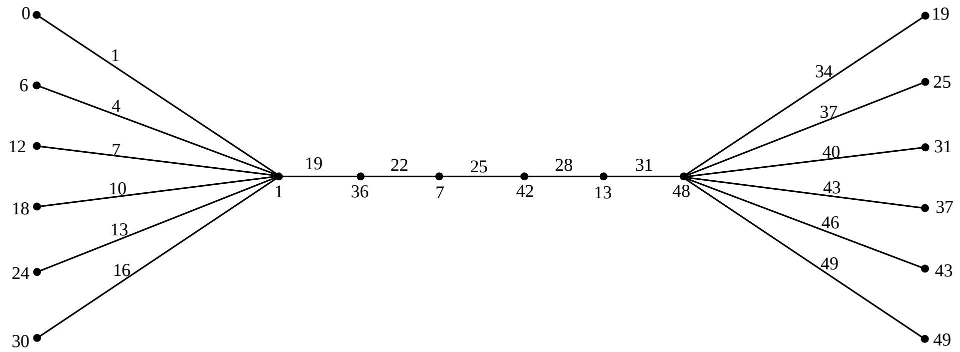

Figure 3.15: OMTML of $T_{6,6,6}$

Theorem 3.14:

The graph $T_{t,n+1,n}$ is a one modulo three mean graph for $t(t \geq 3)$ is odd.

Proof:

Let $V\left[T_{t,n+1,n}\right]=\left\{w_1,w_2,...,w_t,\ u_1,u_2,...,u_n,u_{n+1},\ v_1,v_2,...,v_n\right\}$ be the set of vertices and $E\left[T_{t,n+1,n}\right]=\left\{e_1,e_2,...,e_{t-2},e_{t-1},\ a_1,a_2,...,a_n,a_{n+1},\ b_1,b_2,...,b_n\right\}$ be the edges which are denoted as in the Figure 3.16.

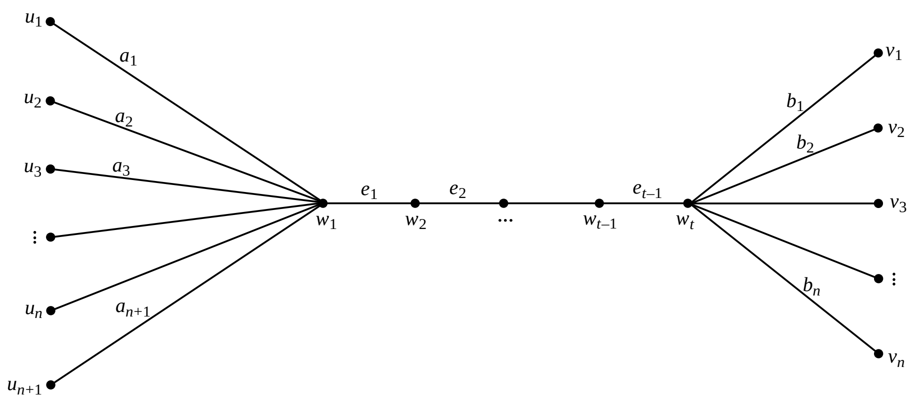

Figure 3.16: Ordinary labeling of $T_{t,n+1,n}$

First we label the vertices as follows:

Define $f : V\left(T_{t,n+1,n}\right) \rightarrow \{0,\ 1,\ 3,\ 4,\dots,3q-3,\ 3q-2\}$ where $q = 2n + t$

$$f\left(u_i\right) = 6(i-1), \qquad \text{for } 1 \leq i \leq n+1$$

$$f\left(v_i\right) = 3t + 6(i-1) - 2, \qquad \text{for } 1 \leq i \leq n$$

For $1 \leq i \leq t-1$,

$$f\left(w_i\right) = \begin{cases} 3i-2, & \text{if } i \text{ is odd} \\ 3(i-2)+6(n+1), & \text{if } i \text{ is even} \end{cases}$$

$$f\left(w_t\right) = 3(2n+t) - 2$$

Then the induced edge labels are

$$f^*\left(a_i\right) = 3i - 2, \qquad \text{for } 1 \leq i \leq n+1$$

$$f^*\left(b_i\right) = 3(i-1) + 3(t+n-1) + 1, \qquad \text{for } 1 \leq i \leq n$$

$$f^*\left(e_i\right) = 3(i-1) + 3(n+1) + 1, \qquad \text{for } 1 \leq i \leq t-2$$

$$f^*\left(e_{t-1}\right) = 3(2n+t) - 2$$

Then above defined function f provides one modulo three mean labeling of the graph $T_{t,n+1,n}$ for $t\left(t \geq 3\right)$ is odd.

One modulo three mean labeling of the graph $T_{5,4,3}$ and $T_{7,6,5}$ are given in Figure 3.17 and Figure 3.18 respectively.

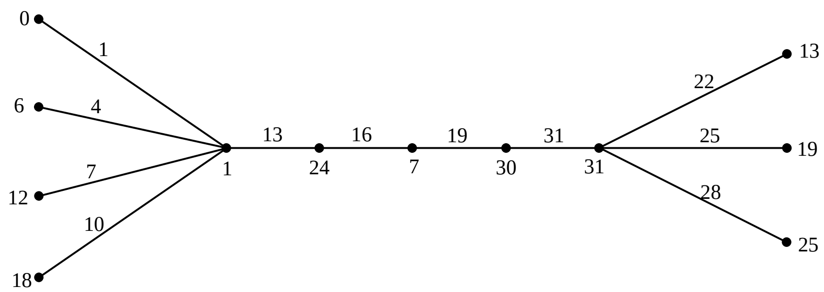

Figure 3.17: OMTML of $T_{5,4,3}$

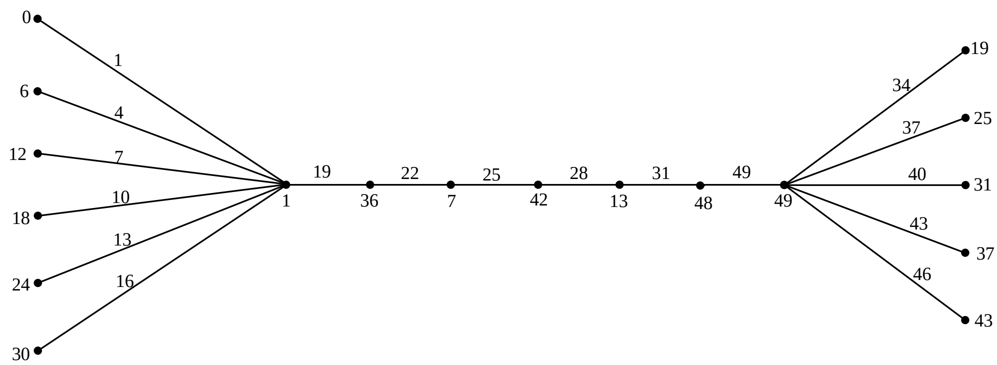

Figure 3.18: OMTML of $T_{7,6,5}$

Definition 3.15:

The ***H*-graph** of path P_n is the graph obtained from two copies of P_n with vertices $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ by joining the vertices $u_{\frac{n+1}{2}}$ and $v_{\frac{n+1}{2}}$ by an edge if n is odd and the vertices $u_{\frac{n}{2}+1}$ and $v_{\frac{n}{2}}$ if n is even.

Theorem 3.16:

The H-graph $H(P_n)$ of a path P_n $(n \geq 3)$ is a modulo three mean graph for any n.

Proof

Let $\{u_i, v_i, 1 \leq i \leq n\}$ be the vertices and $\{e_i, 1 \leq i \leq 2n-1\}$ be the edges which are denoted as in Figure 3.19 and Figure 3.20 respectively.

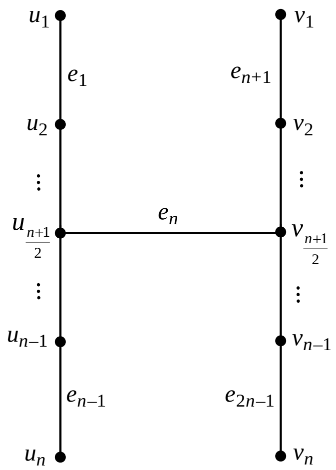

Figure 3.19: Ordinary labeling of $H(P_n)$, n is odd

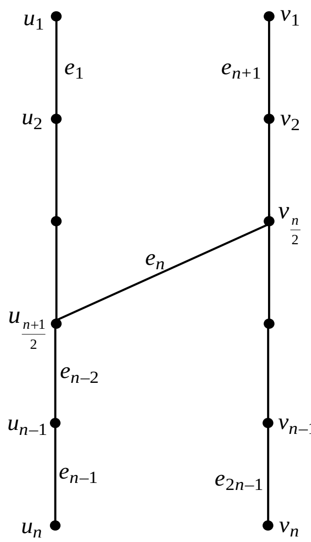

Figure 3.20: Ordinary labeling of $H(P_n)$, n is even

First we label the vertices as follows:

Define $f: V \rightarrow \{0, 1, 3, 4, \ldots, 3q-3, 3q-2\}$

Case 1: *n* is odd

For $1 \leq i \leq n$,

$$f(u_i) = \begin{cases} 3(i-1), & i \text{ is odd} \\ 3i-5, & i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} 3(i-1)+3(n-1)+1, & i \text{ is odd} \\ 3(i-2)+3(n+1), & i \text{ is even} \end{cases}$$

Case 2: n is even

For $1 \leq i \leq n$,

$$f(u_i) = \begin{cases} 3(i-1), & i \text{ is odd} \\ 3i-5, & i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} 3(i-1)+3n, & i \text{ is odd} \\ 3(i-2)+3n+1, & i \text{ is even} \end{cases}$$

Then the induced edge labels are:

$$f^*(e_i) = 3i - 2, \text{for } 1 \leq i \leq 2n - 1,$$

The above defined function f provides one modulo three mean labeling of the $H(P_n)$.

One modulo three mean labeling of the $H(P_7)$ and $H(P_8)$ as given in Figure 3.21 and Figure 3.22 respectively.

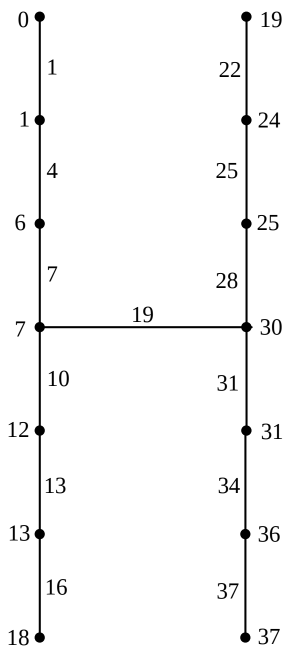

Figure 3.21: OMTML of $H(P_7)$

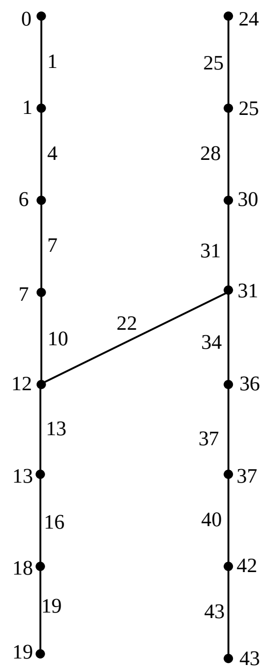

Figure 3.22: OMTML of $H(P_8)$

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