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Number of Minimum Eccentric Dominating sets in Paths

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Abstract: *In a graph G = (V, E), a set D ⊆ V(G) is an eccentric dominating set if D is a dominating set of G and for every v ∈ V−D, there exists at least one eccentric point of v in D. The minimum cardinality of eccentric dominating set is called the eccentric domination number and is denoted by $\gamma_{ed}(G)$. In this paper, we determine the number of minimum eccentric dominating sets in paths.*

Keywords: *Eccentric dominating set, minimum eccentric dominating set, paths*

1. Introduction

Let G be a finite, simple, connected, undirected graph on n vertices with vertex set V(G) and edge set E(G). For graph theoretic terminology refer to Harary [4], Buckley and Harary [2].

The concept of distance in graph plays a dominant role in the study of structural properties of graphs in various angles using related concept of eccentricity of vertices in graphs. The length of any shortest path between any two vertices u and v of a connected graph G is called the *distance between u and v* and it is denoted by $d_G(u, v)$. For a connected graph G, the *eccentricity* e(v) of v is the distance to a vertex farthest from v. Thus, e(v) = max{d(u, v) : u ∈ V}. The **radius** r(G) is the minimum eccentricity of the vertices, whereas the **diameter** diam(G) = d(G) is the maximum eccentricity. For a vertex v, each vertex at a distance e(v) from v is an *eccentric vertex of v***.**

The concept of domination in graphs was introduced by Ore in his famous book 'Theory of Graphs' [6] in 1962. In 1977, Cockayne and Hedetniemi unfolded its diverse aspects, by surveying all the available results, bringing to light new ideas and citing its application potential in a variety of scientific areas, in their paper 'Towards a theory of domination in graphs' [3]. A set D ⊆ V is said to be a *dominating set* of G, if every vertex in V−D is adjacent to some vertex in D. The cardinality of minimum dominating set is called the *domination number* and is denoted by γ(G). A dominating set with cardinality γ(G) is known as *minimum dominating set or a* γ-set. The number of minimum dominating sets in a graph G is denoted by $\gamma_{ED}(G)$.

Janakiraman, Bhanumathi and Muthammai [5] introduced and studied the concept of eccentric dominating sets in 2010. In [1], they have studied the Eccentric domination in

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Trees. A set $D \subseteq V(G)$ is an *eccentric dominating set* if D is a dominating set of G and for every $v \in V-D$, there exists at least one eccentric vertex of v in D. The minimum cardinality of an eccentric dominating set is called the *eccentric domination number* and is denoted by $\gamma_{ed}(G)$. An eccentric dominating set with cardinality $\gamma_{ed}(G)$ is known as *minimum eccentric dominating set or a* γ_{ed}-set. The *number of minimum eccentric dominating sets* in a graph G is denoted by $\gamma_{ED}(G)$

In 2007, Walikar et.al [7] determined the number of minimum dominating sets in paths and cycles. Motivated by this, in this paper we study the number of minimum eccentric dominating sets in Paths.

We need the following results to study the *number of minimum eccentric dominating sets* in a path.

Theorem 1.1: [3] For any integer $n \geq 1$, $\gamma(P_n) = \left\lceil \frac{n}{3} \right\rceil$, where $\lceil x \rceil$ denote the smallest integer not smaller than x.

Theorem 1.2: [5] $\gamma_{ed}(P_n) = \begin{cases} \left\lceil \frac{n}{3} \right\rceil + 1 = k+1, & \text{if } n = 3k \\ \left\lceil \frac{n}{3} \right\rceil = k+1, & \text{if } n = 3k+1 \\ \left\lceil \frac{n}{3} \right\rceil + 1 = k+2, & \text{if } n = 3k+2. \end{cases}$

Theorem 1.3: [6] There are exactly k-minimum dominating sets containing v_1 and v_{3k} in P_{3k+1} with labellings $v_1, v_2, v_3, \ldots, v_{3k-1}, v_{3k}, v_{3k+1}$.

2. Minimum Eccentric dominating sets in Paths

Let us denote the total number of minimum eccentric dominating sets in a graph G by $\gamma_{ED}(G)$. In this paper, $\gamma_{ED}(P_n)$, for n = 3k, 3k+1, 3k+2 for any integer $k \geq 1$ is found out.

Observation 2.1: In P_n with labellings $v_1, v_2, v_3, \ldots, v_{n-2}, v_{n-1}, v_n$, the pendent vertices $v_1, v_n \in D$, where D is any arbitrary minimum eccentric dominating set of P_n.

Theorem 2.1: $\gamma_{ED}(P_{3k}) = k$.

Proof: Let $v_1 v_2 v_3 \ldots v_{3k} v_{3k+1}$ represents the path P_{3k+1}. Number of minimum dominating sets in P_{3k+1} containing v_1 and v_{3k} is the number of minimum eccentric dominating sets of P_{3k}.

By Theorem 1.1, $\gamma(P_n) = \left\lceil \frac{n}{3} \right\rceil$, $\gamma(P_{3k}) = k$, $\gamma(P_{3k+1}) = k+1$. By Theorem 1.2, we have $\gamma_{ed}(P_{3k}) = k+1$ and $\gamma_{ed}(P_{3k+1}) = k+1$. Hence $\gamma_{ed}(P_{3k}) = \gamma_{ed}(P_{3k+1})$. By Theorem 1.3, there are exactly k minimum dominating sets containing v_1 and v_{3k} in P_{3k+1} and these are the minimum eccentric dominating sets of P_{3k}. Hence $\gamma_{ED}(P_{3k}) = k$.

Theorem 2.2: $\gamma_{ED}(P_{3k+1}) = 1$.

Proof: Let $v_1\ v_2\ v_3 \ldots v_{n-2}\ v_{n-1}\ v_n$ represents the path P_n on n vertices. When n = 3k+1, each eccentric dominating set of P_n contains the end vertices and there is exactly only one eccentric dominating set containing the end vertices of P_{3k+1}. Hence $\gamma_{ED}(P_{3k+1}) = 1$.

Lemma 2.1: There is only one minimum eccentric dominating set containing v_1, v_{3k+1} and v_{3k+2} in P_{3k+2} with labeling $v_1, v_2, v_3, \ldots, v_{3k}, v_{3k+1}, v_{3k+2}$, where v_1 and v_{3k+2} are the pendent vertices.

Proof: Let $v_1\ v_2\ v_3 \ldots v_{n-2}\ v_{n-1}\ v_n$ represents the path P_n on n vertices, with n = 3k+2. By Theorem 1.2, $\gamma_{ed}(P_{3k+1}) = k+1$, $\gamma_{ed}(P_{3k+2}) = k+2$. By Theorem 2.2, there is only one minimum eccentric dominating set D containing v_1 and v_{3k+1} in P_{3k+1}. $D\cup\{v_{3k+2}\}$ is a minimum eccentric dominating set of P_{3k+2} containing v_1, v_{3k+1} and v_{3k+2}, since $|D\cup\{v_{3k+2}\}| = k+1+1 = k+2 = \gamma_{ed}(P_{3k+2})$.

Lemma 2.2: There are exactly k-minimum eccentric dominating sets containing v_1, v_{3k} and v_{3k+2} in P_{3k+2} with labeling $v_1, v_2, v_3, \ldots, v_{3k}, v_{3k+1}, v_{3k+2}$.

Proof: Let $v_1\ v_2\ v_3 \ldots v_{n-2}\ v_{n-1}\ v_n$ represents the path P_n on n vertices, with n = 3k+2. By Theorem 1.2, $\gamma_{ed}(P_{3k}) = k+1$, $\gamma_{ed}(P_{3k+2}) = k+2$. By Theorem 2.1, there are exactly k-minimum eccentric dominating sets D_i, i=1, 2, 3, …, k, containing v_1 and v_{3k} in P_{3k}. Now, $D_i\cup\{v_{3k+2}\}$, i=1, 2, 3, …, k is a minimum eccentric dominating set of P_{3k+2} containing v_1, v_{3k} and v_{3k+2}, since $|D_i\cup\{v_{3k+2}\}| = k+1+1 = k+2 = \gamma_{ed}(P_{3k+2})$. Hence, there are exactly k-minimum eccentric dominating sets containing v_1, v_{3k} and v_{3k+2} in P_{3k+2}.

Lemma 2.3: Number of minimum eccentric dominating sets of P_{3k+2} containing v_1, v_{3k-1} and v_{3k+2} is equal to the number of minimum eccentric dominating sets of P_{3k-1}.

Proof: Let $v_1\ v_2\ v_3 \ldots v_{n-2}\ v_{n-1}\ v_n$ represents the path P_n on n vertices, with n = 3k+2. By Theorem 1.2, $\gamma_{ed}(P_{3k-1}) = \gamma_{ed}(P_{3(k-1)+2}) = (k-1)+2 = k+1$, $\gamma_{ed}(P_{3k+2}) = k+2$. Also the distance between v_{3k-1} and v_{3k+2} is 3. If D is a minimum eccentric dominating set containing v_1 and v_{3k-1} in P_{3k-1}, then $D\cup\{v_{3k+2}\}$ is a minimum eccentric dominating set of P_{3k+2} containing v_1, v_{3k-1} and v_{3k+2}. Hence the lemma is proved.

From Lemmas 2.1, 2.2 and 2.3, we see that $\gamma_{ED}(P_{3k+2}) = \gamma_{ED}(P_{3k-1}) + k+1 = \gamma_{ED}(P_{3(k-1)+2}) + k+1$.

That is, $\gamma_{ED}(P_{3k+2}) = \gamma_{ED}(P_{3(k-1)+2}) + k+1$.

From the recurrence relation established above, we find the value of $\gamma_{ED}(P_{3k+2})$ as a function of k as follows:

$\gamma_{ED}(P_{3k+2}) - \gamma_{ED}(P_{3(k-1)+2}) = k+1.$

$\gamma_{ED}(P_{3(k-1)+2}) - \gamma_{ED}(P_{3(k-2)+2}) = (k-1)+1.$

$\gamma_{ED}(P_{3(k-2)+2}) - \gamma_{ED}(P_{3(k-3)+2}) = (k-2)+1.$

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$\gamma_{ED}(P_{11}) - \gamma_{ED}(P_8) = 3+1.$

$\gamma_{ED}(P_8) - \gamma_{ED}(P_5) = 2+1.$

Summing up all the above equations, we get

$$\gamma_{ED}(P_{3k+2}) - \gamma_{ED}(P_5) = k+(k-1)+(k-2)+\ldots+3+2+(1+1+1+\ldots+1)$$

$$= \frac{k(k+1)}{2} - 1 + (k-1) = \frac{k^2+3k-4}{2}$$

$$\gamma_{ED}(P_{3k+2}) - \gamma_{ED}(P_5) = \frac{k^2+3k-4}{2}$$

$$\gamma_{ED}(P_{3k+2}) = \frac{k^2+3k-4}{2} + \gamma_{ED}(P_5) = \frac{k^2+3k-4}{2} + 3$$

$$\gamma_{ED}(P_{3k+2}) = \frac{k^2+3k+2}{2}$$

Hence, we have the following theorem.

Theorem 2.3: $\gamma_{ED}(P_n) = \begin{cases} k, & \text{when } n = 3k \\ 1, & \text{when } n = 3k+1 \\ \frac{k^2+3k+2}{2}, & \text{when } n = 3k+2 \end{cases}$

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