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# Domination Numbers on the Boolean Function Graph $B(K_p, INC, \overline{K}_q)$ of a Graph

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**Abstract:** *For any graph* G, *let* V(G) *and* E(G) *denote the vertex set and edge set of* G *respectively. The Boolean function graph* $B(K_p, INC, \overline{K}_q)$ *of G is a graph with vertex set* $V(G) \cup E(G)$ *and two vertices in* $B(K_p, INC, \overline{K}_q)$ *are adjacent if and only if they correspond to two adjacent vertices of* G, *two nonadjacent vertices of G or to a vertex and an edge incident to it in* G, *For brevity, this graph is denoted by* $B_4(G)$ *In this paper, various domination numbers of* $B_4(G)$ *are determined.*

**Key Word:** *Boolean Function Graph, domination number*

## 1. Introduction

Graphs discussed in this paper are undirected and simple graphs. For a graph G, let V(G) and E(G) denote its vertex set and edge set respectively. A vertex and an edge are said to cover each other if they are incident. A set of vertices which covers all the edges of a graph is called a point cover for G, while a set of edges which covers all the vertices is a line cover. The smallest number of vertices in any point cover for G is called a point covering number and is denoted by $\alpha_o(G)$ or $\alpha_o$. Similarly, $\alpha_1(G)$ or $\alpha_1$ is the smallest number of edges in any line cover of G and is called its line covering number. A set of vertices in G is independent if no two of them are adjacent. The largest number of vertices in such a set is called the point independence number of G and is denoted by $\beta_o(G)$ or $\beta_o$. Analogously, an independent set of edges of G has no two of its edges adjacent and the maximum cardinality of such a set is the line independence number $\beta_1(G)$ or $\beta_1$. A set of independent edges covering all the vertices of a graph G is called *perfect matching*. An edge e = (u, v) is a *dominating edge* in a graph G, if every vertex of G is adjacent to at least one of u and v, where u, v$\in$V(G).

The concept of domination in graphs was introduced by Ore [25]. A set $D \subseteq V(G)$ is said to be a *dominating set* of G, if every vertex in V(G)$-$D is adjacent to some vertex in D. D is said to be a minimal dominating set if D$-${u} is not a dominating set, for any $u \in D$. The *domination number* $\gamma(G)$ of G is the minimum cardinality of a

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dominating set. We call a set of vertices a $\gamma$ - set, if it is a dominating set with cardinality $\gamma(G)$. Different types of dominating sets have been studied by imposing conditions on dominating sets. A dominating set D is called a *connected* (*independent*) *dominating* set, if the induced subgraph <D> is connected [29] (independent) . D is called a total dominating set, if every vertex in V(G) is adjacent to some vertex in D [2]. By $\gamma_c$, $\gamma_i$ and $\gamma_t$, we mean the minimum cardinality of a connected dominating set, independent dominating set and total dominating set respectively.

Sampathkumar and Pushpalatha [28] introduced the concept of point-set domination number of a graph. A set $D \subseteq V(G)$ is called a *point-set dominating set* (psd-set), if for every set $T \subseteq V(G) - D$, there exists a vertex $v \in D$ such that the subgraph $\langle T \cup \{v\} \rangle$ induced by $T \cup \{v\}$ is connected. The point-set domination number $\gamma_{ps}(G)$ is the minimum cardinality of a psd-set of G. Kulli and Janakiram introduced the concept of split [23] and nonsplit [24] domination in graphs. A dominating set D of a connected graph G is a *split* (*non-split*) *dominating set*, if the induced subgraph <V(G)−D> is disconnected (connected). The split (non-split) domination number $\gamma_s(G)$ ($\gamma_{ns}(G)$) of G is the minimum cardinality of a split (non-split) dominating set. Sampathkumar[26] introduced the concept of global domination in graphs. Kulli and Janakiram [22] introduced the concept of total global domination in graphs. Pushpalatha [26] introduced the concept of global point-set domination in graphs.

A dominating set of G is a *global dominating set* [27], if it is a dominating set of both G and its complement $\overline{G}$. For a co-connected graph G = (V, E), a set $D \subseteq V(G)$ is said to be a *global point set dominating set* [26], if it is a psd-set of both G and $\overline{G}$. The *global domination number* $\gamma_g(G)$ of G *is* defined as the minimum cardinality of a global dominating set. The *total global dominating number* $\gamma_{tg}(G)$ of G and *global point set domination number* $\gamma_{pg}(G)$ of G is defined similarly.

Using L(G), the line graph of G, G, incident and non-incident, complementary operations, complete and totally disconnected structures, thirty-two graph operations can be obtained. As already total graphs, semi-total edge graphs, semi-total vertex graphs and quasi-total graphs and their complements (8 graphs) are defined and studied, Janakiraman, Muthammai and Bhanumathi [7 – 21] studied all other similar remaining graph operations and called as Boolean Function and Boolean Graphs.

The *Boolean Function graph* $B(K_p, INC, \overline{K_q})$ of G is a graph with vertex set V(G) ∪ E(G) and two vertices in $B(K_p, INC, \overline{K_q})$ are adjacent if and only if they correspond to two adjacent vertices of G, two nonadjacent vertices of G or to a vertex and an edge incident to it in G. For brevity, this graph is denoted by $B_4(G)$. In this paper, various domination numbers for the graph $B_4(G)$ are determined. For graph theoretic terminology, Harary [4] is referred.

## 2. Prior Results

In this section, we list some results with indicated references, which will be used in the subsequent main results. Let G be any (p, q) graph.

**Theorem 2.1.[28]**

Let G = (V, E) be a graph. A set $S \subseteq V$ is a point-set dominating set of G if and only if for every independent set W in V−S, there exists a vertex u in S such that $W \subseteq N_G(u) \cap (V - S)$.

**Theorem 2.2. [22]**

A total dominating set T of G is a total global dominating set if and only if for each vertex $v \in V$, there exists a vertex $u \in T$ such that v is not adjacent to u.

**Theorem 2.3.[26]**

For a graph G, a set $S \subseteq V(G)$ is a global point-set dominating set if and only if the following conditions are satisfied.

(i). For every independent set W in V−S, there exists u in S such that $W \subseteq N_G(u) \cap (V - S)$ in G; and

(ii). For every set $D \subseteq V - S$ such that <D> is complete in G, there exists v in S such that $D \cap N(v) = \phi$ in G.

**Observation 2.4.[21]**

1. $K_p$ is an induced subgraph of $B_4(G)$ and the subgraph of $B_4(G)$ induced by q vertices is totally disconnected.
2. Number of vertices in $B_4(G)$ is p + q, since $B_4(G)$ contains vertices of both G and the line graph L(G) of G.
3. Number of edges in $B_4(G)$ is $\left(\frac{p(p-1)}{2}\right) + 2q$
4. For every vertex $v \in V(G)$, $d_{B_4(G)}(v) = p - 1 + d_G(v)$
   (a). If G is complete, then $d_{B_4(G)}(v) = 2(p - 1)$.
   (b). If G is totally disconnected, then $d_{B_4(G)}(v) = p - 1$.
   (c). If G has atleast one edge, then $2 \leq d_{B_4(G)}(v) \leq 2(p - 1)$
   and $d_{B_4(G)}(v) = 1$ if and only if $G \cong 2K_1$.

5. For an edge $e \in E(G)$, $d_{B_4(G)}(e) = 2$. $B_4(G)$ is always connected.

## 3. Main results

## Domination, Connected and Total domination numbers in $B_4(G)$.

In the following, the graphs G for which the domination number $\gamma(B_4(G))$ is 1 or 2 are found.

It is to be noted that $V(B_4(G)) = V(G) \cup V(L(G))$

**Observation 3.1.**

$\gamma(B_4(G)) = 1$ if and only if $G \cong nK_1, K_{1,m}$ or $nK_1 \cup K_{1,m}$, $n, m \geq 1$.

**Theorem 3.1.**

For any graph G, $\gamma(B_4(G)) = 2$ if and only if there exists a point cover of G containing two vertices.

**Proof:**

Let D be a dominating set of $B_4(G)$ such that $|D| = \gamma(B_4(G))$. Let D = {u, v}, where $u, v \in V(B_4(G))$)

**Case (1):** $u, v \in V(G)$

Then $D \subseteq V(B_4(G)) \cap V(G)$ and $V(B_4(G)) - D$ contains vertices in $V(G) - D$ and the vertices corresponding to the edges of G.

Let $x \in V(B_4(G))$ be a vertex corresponding to an edge, say e in G. Then e must be incident with a vertex in D. That is, each edge in G is incident with a vertex in D. Therefore, D is a point cover of G.

**Case (2):** $u \in V(G)$ and $v \in V(L(G))$

Then all the vertices in $V(B_4(G)) - D$ are adjacent to atleast one of u and v. Since, vertices of L(G) in $V(B_4(G))$ are independent, a vertex of L(G) in $V(B_4(G)) - D$ is adjacent to u only. That is, each edge in G is incident with a vertex in D. Therefore, D is a point cover of G.

**Case (3):** $u, v \in V(L(G))$

In this case, $G \cong 2K_2$.

From Case (1) to Case (3), D is a point cover of G.

Conversely, let there exist a point cover D of G such that $|D| = 2$. Then D is a dominating set of $B_4(G)$. Since, $\gamma(B_4(G)) \neq 1$, $\gamma(B_4(G)) = 2$.

In the following, relationships between $\gamma(B_4(G))$ and point covering number of G is found.

**Theorem 3.2.**

For any graph G, $\gamma(B_4(G)) = \alpha_0(G)$.

**Proof:**

Let D be a minimum point cover of G. Then D covers all the edges of G. Therefore, $D \subseteq V(B_4(G))$ dominates all the vertices in $B_4(G)$ corresponding to the edges in G. Also, since any two vertices of G in $B_4(G)$ are adjacent in $B_4(G)$, D dominates the vertices in $V(G) \cap V(B_4(G))$. Hence, D is a dominating set of $B_4(G)$ and $\gamma(B_4(G)) \leq \alpha_0(G)$

Let D be a minimum dominating set of $B_4(G)$. To prove D is a point cover of $B_4(G)$.

**Case (1):** $D \subseteq V(G)$

Then D dominates all the vertices corresponding to the edges in G. That is, each edge in G is incident with atleast one vertex in D. Therefore, D is a point cover of G.

**Case (2):** $D \subseteq V(L(G))$

Let $D_1$ = {v ∈ V(G): v is incident an edge corresponding to a vertex in L(G)}. Then $D_1$ is a point cover of G.

**Case (3):** $D \subseteq V(G) \cup V(L(G))$

Let $D_1$ be the set defined as in Case 2 and $D_2$ be the of vertices in $D \cap V(G)$. Then $D_1 \cup D_2$ is a point cover of G.

From Case (1) to Case (3), $\alpha_0(G) \leq \gamma(B_4(G))$.

Hence, $\gamma(B_4(G)) = \alpha_0(G)$.

**Theorem 3.3.**

For any graph G, $\gamma(B_4(G)) \leq \alpha_1(G) + \alpha_0(< E(G) - S >)$, where S is a minimum line cover of G.

**Proof:**

Let S be a line cover of G such that $|S| = \alpha_1(G)$. Then $S \subseteq V(G)$ Let S′ be the vertices in $B_4(G)$ corresponding to the edges in S. Then S′ dominates all the vertices in $V(G) \cap V(B_4(G))$. Let D be a minimum point cover of <E(G) – S>. Then $D \subseteq V(B_4(G))$ and $S' \cup D$ is dominating set of $V(B_4(G))$.

Hence, $\gamma(B_4(G)) \leq \alpha_1(G) + \alpha_0(< E(G) - S >)$.

**Remark 3.1.**

1. If G contains no isolated vertices, then V(L(G)) is a dominating set of $B_4(G)$.
2. Any proper subset S of L(G) is not a dominating set $B_4(G)$, since V(L(G)) is totally disconnected in $B_4(G)$.

**Theorem 3.4.**

Let G be a (p, q) graph having no isolated vertices. Then $\gamma(B_4(G)) = \beta_0(B_4(G))$ if and only if $G \cong nK_2$, $n \geq 1$.

**Proof:**

Assume $\gamma(B_4(G)) = \beta_0(B_4(G))$. Since G has no isolated vertices, $\beta_0(B_4(G)) = q$. Therefore, $\gamma(B_4(G)) = q$. Since any two vertices of G in $B_4(G)$ are adjacent, each vertex in $V(G) \cap V(B_4(G))$ is adjacent to exactly one vertex in $V(L(G)) \cap V(B_4(G))$. Therefore, $\deg_G(v) = 1$ for all v in G. Since G has no isolated vertices, $G \cong nK_2$, $n \geq 1$.
Conversely, if $G \cong nK_2$, $n \geq 1$, , then $\gamma(B_4(G)) = \beta_0(B_4(G))$.

In the following, independent domination number $\gamma_i(B_4(G))$ of $B_4(G)$ is found.

**Theorem 3.5.**

Let G be any graph such that $G \neq K_{1,n}$. Then $S = \{v, e\}$, where $e \in E(G)$ is not incident with $v \in V(G)$, is an independent dominating set of $B_4(G)$ if and only if G is one of the following graphs., $K_3, K_3 \cup nK_1, P_4, P_4 \cup nK_1, 2K_2, 2K_2 \cup nK_1,$ $K_{1,n} + e, (K_{1,n} + e) \cup mK_1, K_{1,n} \cup K_2, (K_{1,n} \cup K_2) \cup mK_1,$ where $m, n \geq 1$.

**Proof:**

Let G be any graph such that $G \neq K_{1,n}$.

Let $S = \{v, e\}$ be an independent dominating set of $B_4(G)$, where $e \in E(G)$ is not incident with $v \in V(G)$. Since S is a dominating set of $B_4(G)$, vertex in V($B_4(G)$)–S is adjacent to atleast one vertex in S. Since any two vertices in $B_4(G)$ corresponding to the edges of G are nonadjacent, any vertex in $B_4(G)$ corresponding to the edge of G must be adjacent to $v \in S$. That is, each edge (except e) in G is incident with $v \in V(G)$. Therefore, G is one of the graphs given in the Theorem.
Conversely, let G be one of the graphs given in the Theorem. Then $S = \{v, e\}$, where $e \in E(G)$ is not incident with $v \in V(G)$ is an independent dominating set of $B_4(G)$.

**Theorem 3.6.**

For any (p, q) graph G, independent domination number $\gamma_i(B_4(G)) \leq q - \Delta(G) + 1$

**Proof:**

Let v be a vertex of maximum degree in G. That is, $\deg_G(v) = \Delta(G)$. Then $v \in V(B_4(G))$ is adjacent to $\Delta(G)$ vertices in $B_4(G)$. Therefore, v dominates all the vertices of G in $B_4(G)$ and $\Delta(G)$ vertices in $B_4(G)$ corresponding to $\Delta(G)$ edges in G. Since any two vertices of V(L(G)) are adjacent in $B_4(G)$, $q - \Delta(G)$ vertices of L(G) together with v form an independent dominating set of $B_4(G)$ . Hence, $\gamma_i(B_4(G)) \leq q - \Delta(G) + 1$.

The equality is obtained, when $G \cong K_{1,n} + e$, where e is an edge joining any two pendant vertices of $K_{1,n}, n \geq 2$.

In the following, global domination number of $B_4(G)$ is found. $\overline{B}_4(G)$ denotes the complement of $B_4(G)$.

**Theorem 3.7.**

Let G be a graph having no isolated vertices. Then V(G) is a global dominating set of $B_4(G)$ if and only if V(G) has atleast three vertices.

**Proof:**

Assume V(G) has atleast three vertices. Then V(G) is a dominating set of $B_4(G)$. Since V(G) has atleast three vertices, each vertex in $V(B_4(G))$ – V(G) is adjacent to atleast one vertex in V(G). Therefore, V(G) is a dominating set of $\overline{B}_4(G)$. Conversely, let V(G) be a global dominating set of $B_4(G)$. If V(G) has atmost two vertices, then $G \cong K_2$. Then D is a dominating set of $B_4(G)$, but not a dominating set of $\overline{B}_4(G)$. Therefore, D has atleast three vertices.

**Theorem 3.8.**

Let G be a graph without isolated vertices. Then V(L(G)) is a global dominating set of $B_4(G)$ if and only if G is not a star.

**Proof:**

Let D = V(L(G)). Assume G is a global dominating set of $B_4(G)$ and G is a star. Let v be the center vertex of the star. Then v is in

$V(\overline{B}_4(G))$– D and v is not adjacent to any of the vertices in D, which is a contradiction. Therefore, G is not a star.

Conversely, let G be not a star. $V(\overline{B}_4(G))$ – D = V(G). Since G is not a star, for each vertex in G, there is an edge not incident with it. That is, for each vertex in $V(\overline{B}_4(G))$– D, there is atleast one vertex in D adjacent to it. Therefore, D is a dominating set of $\overline{B}_4(G)$. Since G contains no isolated vertices, D is a dominating set of $B_4(G)$ and hence D is a global dominating set of $B_4(G)$.

**Theorem 3.9.**

Let G be not totally disconnected and $(u,v)\in E(G)$ . Then $D=\{u,v,e\}\subseteq V(B_4(G))$ is a global dominating set of $B_4(G)$ if and only if each edge in G is incident with u or v. That is, eccentricity of e in L(G) is 1.

**Proof:**

Let D = {u, v, e} be a global dominating set of $B_4(G)$. Since, D is a dominating set of $B_4(G)$, each vertex of $V(L(G))\cap V(B_4(G))$ is adjacent to atleast one of u and v. That is, each edge in G is incident with u or v.

Conversely, assume each edge in G is incident with u or v. That is, eccentricity of e in L(G) is 1.Then $D=\{u,v,e\}\subseteq V(B_4(G))$ is a dominating set of $\overline{B}_4(G)$. It is enough to prove D is dominating set of $B_4(G)$. Since, eccentricity of e in L(G) is 1, each edge in G is adjacent to e. Therefore, vertices in V($B_4(G)$) - D corresponding to edges in G is adjacent to e . Also, vertices of G in V($B_4(G)$) – D is adjacent to u or v. Hence, D is a dominating set of $B_4(G)$ and D is a global dominating set of $B_4(G)$.

**Theorem 3.10.**

Let $e_1$ and $e_2$ be any two adjacent edges in a graph G with atleast three vertices and let u be the vertex in G common to both $e_1$ and $e_2$. Then D = { $e_1$, $e_2$, u} is a global dominating set of $B_4(G)$ if and only if G is a star on atleast three vertices.

**Proof:**

Let D = { $e_1$, $e_2$, u} be a global dominating set of $B_4(G)$. Then D is a dominating set of $B_4(G)$.

Therefore, each vertex of L(G) in V($B_4(G)$) – D is adjacent to u. That is, each edge in G is incident with u. Hence, G is a star on atleast three vertices.

Conversely, let G be a star on atleast three vertices. Let $e_1$ and $e_2$ be any two adjacent edges in G and let u be the vertex in G common to both $e_1$ and $e_2$. Then D = {u, v, e}is a global dominating set of $B_4(G)$.

In the following, split domination number $\gamma_s(B_4(G))$ of $B_4(G)$ is found.

**Theorem 3.11.**

For any graph G, $\gamma_s(B_4(G))=\alpha_0(G)$ if and only if there exists a point cover D of G with $|D|=\alpha_0(G)$ such that the subgraph <D> of G induced by D is not totally disconnected.

**Proof:**

Let D be a point cover of G with $|D|=\alpha_0(G)$ and $e=(u,v)\in E(<D>)$.

Then the vertex e in $B_4(G)$ is isolated in <V($B_4(G)$) – D>.

Therefore, D is a split dominating set of $B_4(G)$ and is minimum and hence $\gamma_s(B_4(G)) = \alpha_0(G)$.

Conversely, assume $\gamma_s(B_4(G)) = \alpha_0(G)$ and D is a point cover of G with $|D| = \alpha_0(G)$. Therefore, D is a split dominating set of $B_4(G)$. Assume <D> is totally disconnected. Then each vertex in $B_4(G)$ corresponding to the edge in G is adjacent to atleast to one vertex of G in $B_4(G)$. Also, the subgraph of $B_4(G)$ induced by the vertices of G in V($B_4(G)$) – D is complete. Therefore, <V($B_4(G)$) – D> is not disconnected, which is a contradiction. Therefore, <D> is totally disconnected in G.

**Theorem 3.12.**

For any graph G, if D is an independent point cover of G such that $|D| = \alpha_0(G)$, then $\gamma_s(B_4(G)) = \alpha_0(G) + 1$.

**Proof:**

Let D be an independent point cover such that $|D| = \alpha_0(G)$. Then D is a dominating set of $B_4(G)$. Let $u \in V(G) - D$. Then u is adjacent to atleast one vertex, say v in D. Let $e = (u, v) \in E(G)$. Then $D' = D \cup \{u\}$ is a minimum split dominating set of $B_4(G)$, since the vertex in corresponding to the edge e is isolated in $< V(B_4(G) - D' >$. Therefore, $\gamma_s(B_4(G)) = \alpha_0(G) + 1$.

In the following point set domination number of $B_4(G)$ is found.

**Theorem 3.13.**

Let D be any subset of vertex set V(G) of a graph G. Then D is a point set dominating set of $B_4(G)$ if and only if

(a) D is a point cover for G

(b) There exists atleast one vertex $v \in D$ such that all the edges of G are incident with v.

**Proof:**

Let $D \subseteq V(G)$ be a point set dominating set of $B_4(G)$. Then D is a dominating set of $B_4(G)$ and hence D is point cover of G.

Let $W = V(L(G)) \subseteq V(B_4(G))$. Then W is an independent set in V($B_4(G)$) – D. Since D is a point set dominating set of $B_4(G)$, there exists atleast one vertex $v \in D$ such that all the edges of G are incident with v.

Conversely, (a) and (b) imply that, $D \subseteq V(G)$ is a point set dominating set of $B_4(G)$.

**Theorem 3.14.**

For any graph G, $\gamma_{ps}(B_4(G)) \le q - \Delta(G) + 1$.

**Proof:**

Let v be a vertex in G of maximum degree and let $\{e_1, e_2, ..., e_{\Delta(G)}\} \subseteq V(B_4(G))$. Let $D'$ be the set of vertices in $B_4(G)$ corresponding to the edges of G which are not incident with v. Then $|D'| = q - \Delta(G)$ and $D' \cup \{v\}$ is a point set dominating set of $B_4(G)$. Hence, $\gamma_{ps}(B_4(G)) \le q - \Delta(G) + 1$.

**Remark 3.2.**

a. $D' \cup \{v\}$ is also an independent dominating set of $B_4(G)$.

b. $\gamma_{ps}(B_4(G)) = q - \Delta(G) + 1$, if $G \cong K_{1,n}, K_{1,n} \cup nK_1, n \ge 2$, $K_{1,n} + x, (K_{1,n} + x) \cup nK_1, n \ge 2$, where $x \in E(G)$.

c. $\gamma_{ps}(B_4(G)) = \alpha_0(G)$ if $G \cong K_{1,n}, n \ge 2$.

In the following, global point set domination number of $B_4(G)$ is found.

**Theorem 3.15.**

For any graph G, if radius of L(G) is atleast 2, then global point set domination number $\gamma_{pg}(B_4(G)) \le q - \Delta(G) + 3$

**Proof:**

Let v be a vertex of maximum degree in G and let e = (u, v) be an edge in G incident with v. Let $D'$ be the set of vertices in $B_4(G)$ corresponding to the edges which are incident with v in G and let e* be the vertex in $B_4(G)$ corresponding to e in G. Then D = {u,v,e*}$\cup D'$

is a point set dominating set of $B_4(G)$.

Let $S \subseteq V(B_4(G)) - D$ be such that The subgraph <S> induced by S is complete in V( $B_4(G)$ ) - D .

(a) Let S contain vertices of G.

Then e* $\in$ D is not adjacent to any of the vertices in S.

(b) Let S contain a vertex of G and a vertex of L(G).

Since radius of L(G) is atmost 2, there exists a vertex in D corresponding to an edge in G, not adjacent to any of the vertices in S.

These are the only possibilities that <S> to be complete in

V( $B_4(G)$ ) - D . Therefore, D is a global dominating set of $B_4(G)$. Hence, $\gamma_{pg}(B_4(G)) \leq q - \Delta(G) + 3$.

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