International Journal of Engineering Science, Advanced Computing and Bio-Technology
Vol. 6, No. 1, January – March 2015, pp. 17 - 27

Co-Isolated Locating Domination Number for Cartesian Product of Two Graphs

S. Muthammai[1] and N. Meenal[2]

[1]Government Arts College for Women (Autonomous), Pudukkottai – 622 001

E-mail:muthammai.sivakami@gmail.com

[2]J.J. College of Arts and Science, Pudukkottai – 622 422.

E-mail:meenal_thillai@rediffmail.com

Abstract: *Let G (V, E) be a simple, finite, undirected connected graph. A non – empty set $S \subseteq V$ of a graph G is a dominating set, if every vertex in V – S is adjacent to at least one vertex in S. A dominating set $S \subseteq V$ is called a locating dominating set, if for any two vertices v, w ∈ V–S, $N(v) \cap S \neq N(w) \cap S$. A locating dominating set $S \subseteq V$ is called a co-isolated locating dominating set, if there exists at least one isolated vertex in <V – S >. The co-isolated locating domination number γ_{cild} is the minimum cardinality of a co-isolated locating dominating set.*

In this paper, upper bounds of co-isolated locating domination number for the Cartesian product of two graphs namely, $P_2 \times P_n$ (n ≥ 3), $P_3 \times P_n$ (n ≥ 2), $P_4 \times P_n$ (n ≥ 2), $P_2 \times C_n$ (n ≥ 3), $P_3 \times C_n$ (n≥3), $P_4 \times C_n$ (n≥3), $P_2 \times K_n$ (n ≥ 2), $P_3 \times K_n$ (n ≥ 2), $P_4 \times K_n$ (n ≥ 3) and $K_n \times K_m$ (m, n ≥ 3), are established.

Keywords: *Dominating set, locating dominating set, co-isolated locating dominating set, Cartesian product of two graphs.*

1. Introduction

Let G = (V, E) be a simple connected graph with vertex set V and edge set E. The (open) neighbourhood N(v) of a vertex v consists of the vertices adjacent to v. The closed neighbourhood of v is defined by $N[v] = N(v) \cup \{v\}$ and $d_G(v) = |N(v)|$ is the degree of v. The minimum degree of a graph G is denoted by $\delta(G)$.The concept of domination in graphs was introduced by Ore [11]. A non – empty set $S \subseteq V(G)$ of a graph G is a dominating set, if every vertex in V(G) – S is adjacent to some vertex in S. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set in G.A special case of a dominating set is a locating dominating set. It was introduced by Slater [12, 13]. On recent studies on locating domination [2, 3] and [4] are referred. A dominating set S in a graph G is called a locating dominating set in G, if for any two vertices v, w ∈V(G)–S, $N_G(v) \cap S$, $N_G(w) \cap S$ are distinct . The locating domination number of G is defined as the minimum cardinality of a locating dominating set in G. A locating dominating set $S \subseteq V(G)$ is called a co-isolated locating dominating set, if<V – S> contains atleast one isolated vertex. The co-isolated locating domination number $\gamma_{cild}(G)$ is the minimum cardinality of a co-isolated locating

Received: 28December, 2014; Revised: 26February, 2015; Accepted: 20March, 2015

dominating set. A locating dominating set of minimum cardinality is called a $\gamma_L(G)$ – set γ_{cild} – set is defined likewise.

The Cartesian product of two graphs G and H is the graph denoted by G × H with V(G × H)=V(G)×V(H) (where × denotes the cartesian product of sets) and ((u,u'),(v,v')) ∈ E(G × H) if and only if u = v and (u',v') ∈ E(H) or u' = v' and (u,v) ∈ E(G). If each of G and H is a path P_m and P_n respectively, then the graph $P_m \times P_n$ is called a grid graph. The domination number of k × n grid graphs $P_k \times P_n$, for $1 \le k \le 10$, $n \ge 1$ have been previously established by Jacobson and Kinch.

In this paper, upper bounds of co-isolated locating domination number for the grid graphs $P_2 \times P_n$($n \ge 3$), $P_3 \times P_n$ ($n \ge 2$), $P_4 \times P_n$($n \ge 2$), and the graphs $P_2 \times C_n$($n \ge 3$), $P_3 \times C_n$($n \ge 3$), $P_4 \times C_n$($n \ge 3$), $P_2 \times K_n$ ($n \ge 2$), $P_3 \times K_n$ ($n \ge 2$), $P_4 \times K_n$ ($n \ge 3$) and $K_n \times K_m$ (m, $n \ge 2$) are found. For graph theoretic notations, [5] is referred.

2. Prior Results

In the following, some results involving co-isolated locating domination, already proved in [8] and [9] are given

Theorem2.1 [8]: For every non–trivial simple connected graph G with p vertices, $1 \le \gamma_{cild}(G) \le p - 1$.

Theorem 2.2 [8]: $\gamma_{cild}(G) = 1$ if and only if $G \cong K_2$.

Theorem 2.3 [8]: $\gamma_{cild}(K_n) = n - 1$, where K_n is the complete graph on n vertices.

Theorem 2.4 [8]: $\gamma_{cild}(K_n - e) = n - 1$, where $e \in E(K_n)$

Theorem 2.5 [9]: For the path P_n on n vertices,

$$\gamma_{cild}(P_n) = \left\lfloor \frac{2n+4}{5} \right\rfloor, n \ge 3$$

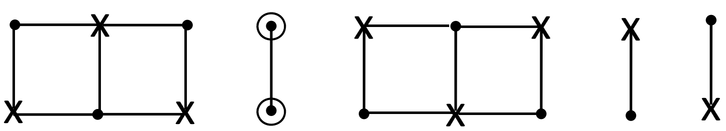
Observation 2.6 [9]: If S is a co-isolated locating dominating set of G(V, E) with |S| = k, then V(G) – S contains atmost $nC_1 + nC_2 + \ldots + nC_k$ vertices.

3. Main Results

In the following, $\gamma_{cild}(P_m \times P_n)$, for m = 2, 3 and 4 are found.

Theorem 3.1: For $n \ge 3$, $\gamma_{cild}(P_2 \times P_n) \le \left\lceil \frac{3n+3}{4} \right\rceil$

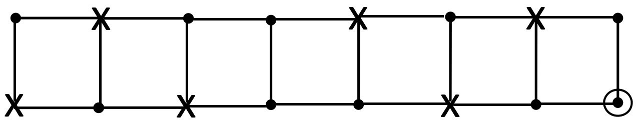
Proof: Let $G \cong P_2 \times P_n$. The blocks A_1, B_1, A_2, B_2 and B_3 are constructed as given below.



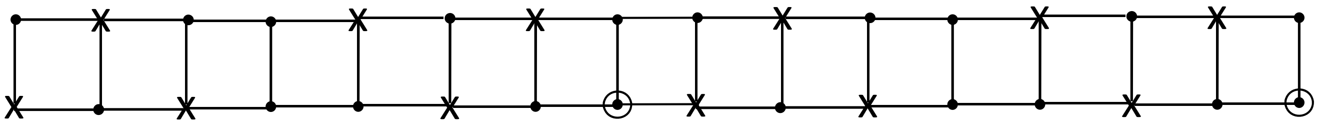
A_1 B_1 A_2 B_2 B_3

Let S be a co-isolated locating dominating set of G. The vertices with the symbol '✕' in each of the blocks represent the vertices that are to be included in S. The vertices with the symbol ○ in the blocks indicate that those vertices are not in the locating dominating set S constructed upto this stage and to be considered while concatenation. In a sequence of concatenations, if B_i, $1 \le i \le 3$ is the last block, then |S| is increased by 1, otherwise S will not be a locating dominating set of G.

A new block C obtained by concatenating the blocks A_1, B_1, A_2 and B_1 and the block C^2 are given below.



$C \cong A_1B_1A_2B$



C^2

Also, let $a_i = |A_i \cap S| = 3$, $i = 1,2$;

$b_1 = |B_1 \cap S| = 0$

$b_2 = |B_2 \cap S| = 1$, $b_3 = |B_3 \cap S| = 1$.

Then, $a_i = 3$, $b_1 = 0$, $b_2 = 1$, $b_3 = 1$.

$|C \cap S| = a_1 + b_1 + a_2 + b_1 = 6$

If n = 3, then $G \cong A_1$ and $|S| = a_1 = 3$.

If n = 4, then $G \cong A_1B_1$ and $|S| = a_1 + b_1 + 1 = 4$, since G ends with the block B_1.

If n = 5, then $G \cong A_1B_1B_2$ and $|S| = a_1 + b_1 + b_2 + 1 = 5$, since G ends with the block B_2.

If n = 6, then $G \cong A_1B_1B_2B_3$ and $|S| = a_1 + b_1 + b_2 + b_3 + 1 = 6$, since the graph ends with the block B_3.

If n = 7, then $G \cong A_1B_1A_2$ and $|S| = a_1 + b_1 + a_2 = 6$.

If n = 8, then $G \cong C$ ($\cong A_1B_1A_2B_1$) and it ends with the block B_1.

Therefore, $|S| = a_1 + b_1 + a_2 + b_1 + 1 = 7$.

Let n > 8 and n = 8q + r; $0 \le r \le 7$. Then, $q = \dfrac{n - r}{8}$

Then, the following cases arise.

Case (i): $n \equiv 0 \pmod 8$. Then $G \cong C^q$ and $|S| \le 6q + 1 = \dfrac{3n + 4}{4} = \left\lceil \dfrac{3n + 3}{4} \right\rceil$

Case (ii): $n \equiv 1 \pmod 8$. Then $G \cong C^qB_3$ and $|S| \le 6q + 2 = \dfrac{3n + 5}{4} = \left\lceil \dfrac{3n + 3}{4} \right\rceil$

Case (iii): $n \equiv 2 \pmod 8$. Then $G \cong C^q B_3 B_2$ and $|S| \le 6q + 3 = \frac{3n+6}{4} = \left\lceil \frac{3n+3}{4} \right\rceil$

Case (iv): $n \equiv 3 \pmod 8$. Then $G \cong C^q A_1$ and $|S| \le 6q + 3 = \frac{3n+3}{4}$

Case (v): $n \equiv 4 \pmod 8$. Then $G \cong C^q A_1 B_1$ and $|S| \le 6q + 4 = \frac{3n+4}{4} = \left\lceil \frac{3n+3}{4} \right\rceil$

Case (vi): $n \equiv 5 \pmod 8$. Then $G \cong C^q A_1 B_1 B_2$ and $|S| \le 6q+5 = \frac{3n+5}{4} = \left\lceil \frac{3n+3}{4} \right\rceil$

Case (vii): $n \equiv 6 \pmod 8$. Then $G \cong C^q A_1 B_1 B_2 B_3$ and $|S| \le 6q+6 = \frac{3n+6}{4} = \left\lceil \frac{3n+3}{4} \right\rceil$

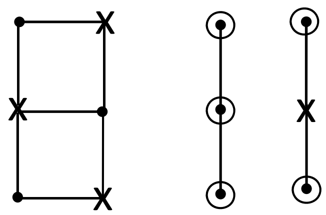
Case (viii): $n \equiv 7 \pmod 8$. Then $G \cong C^q A_1 B_1 A_2$ and $|S| \le 6q + 6 = \frac{3n+3}{4}$

In all the above cases, $\gamma_{cild}(P_2 \times P_n) \le |S| = \left\lceil \frac{3n+3}{4} \right\rceil$

Theorem 3.2: For $n \ge 2$,

$$\gamma_{cild}(P_3 \times P_n) \le \begin{cases} \left\lfloor \frac{6n+3}{5} \right\rfloor, & \text{if } n \equiv 0, 1, 2, 3 \pmod 5 \\ \left\lceil \frac{6n+3}{5} \right\rceil, & \text{if } n \equiv 4 \pmod 5 \end{cases}$$

Proof: Let $G \cong P_3 \times P_n$, $n \ge 2$. The blocks A_1, B_1, B_2 are constructed as below



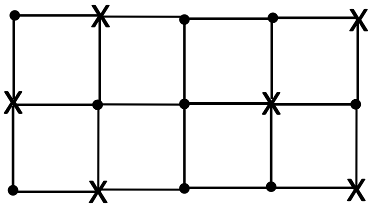
A_1 B_1 B_2

Let S be a co-isolated locating dominating set. The vertices with the symbol ✕ in each of the blocks represent that the vertices are to be included in S and the vertices with the symbol ◯ in the blocks indicate that those vertices are not in the locating dominating set S constructed upto this stage and to be considered while concatenation. For S to be a locating dominating set of G, in a sequence of concatenations, if B_1 is the last bock, then $|S|$ is increased by 1. For B_2, it is increased by 2. Then, $a_1 = |A_1 \cap S| = 3$, $b_1 = |B_1 \cap S| = 0$ and $b_2 = |B_2 \cap S| = 1$,

If $n = 2$, then $G \cong A_1$ and $|S| = 3$.

If n = 3, then $G \cong A_1 B_1$. Since B_1 is the last block, |S| is increased by 1 and hence, |S| = 4. If n = 4, then $G \cong A_1 B_1 B_2$ and |S| = 6.

Let C be the new block obtained by concatenating the blocks A_1, B_1 and A_1 and is given below.



$C \cong A_1B_1A_1$

Then, $|C \cap S| = 6$. If n = 5, then $G \cong C$ and |S| = 6.Let $n \geq 6$and $n = 5q + r; 0 \leq r \leq 4$

Case (i): $n \equiv 0 \pmod 5$. Then $G \cong C^q$ and $|S| = 6q = \frac{6n}{5} = \left\lfloor \frac{6n+3}{5} \right\rfloor$

Case (ii): $n \equiv 1 \pmod 5$. Then $G \cong C^qB_1$ and $|S| = 6q + 1 = \left\lfloor \frac{6n+3}{5} \right\rfloor$

Case (iii): $n \equiv 2 \pmod 5$. Then $G \cong C^q A_1$ and $|S| = 6q + 3 = \left\lfloor \frac{6n+3}{5} \right\rfloor$

Case (iv): $n \equiv 3 \pmod 5$. Then $G \cong C^qA_1B_1$ and $|S| = 6q + 4 = \left\lfloor \frac{6n+3}{5} \right\rfloor$

Case (v): $n \equiv 4 \pmod 5$.Then $G \cong C^q A_1B_1B_2$ and $|S| = 6q + 6 = \left\lceil \frac{6n+3}{5} \right\rceil$

From the above cases,

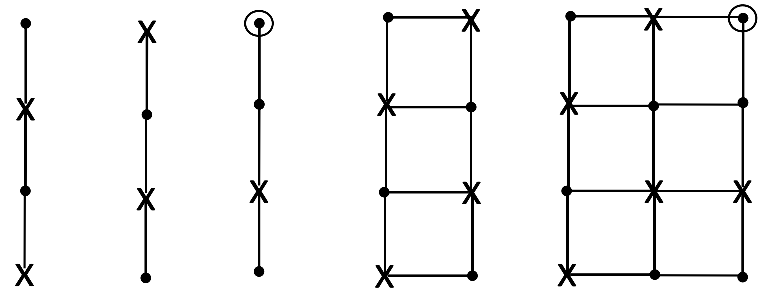
$$\gamma_{cild}(P_3 \times P_n) \leq |S| = \begin{cases} \left\lfloor \frac{6n+3}{5} \right\rfloor, & \text{if } n \equiv 0, 1, 2, 3 \pmod 5 \\ \left\lceil \frac{6n+3}{5} \right\rceil, & \text{if } n \equiv 4 \pmod 5 \end{cases}$$

This completes the proof of the theorem.

Theorem 3.3: For $n \geq 2$, $\gamma_{cild}(P_4 \times P_n) \leq \left\lceil \frac{5n+1}{3} \right\rceil$

Proof:

Let $G \cong P_4 \times P_n$, $n \geq 2$. The blocks A_1, A_2, A_3, A_1A_2, $A_1A_2A_3$ are constructed as before and are given below.



A_1 A_2 A_3 A_1A_2 $C \cong A_1\, A_2A_3$

Let S be a co-isolated locating dominating set of G. While concatenation, if C is the last block, then |S| is increased by 1, for S to be a locating dominating set of G.

Then $a_1 = |A_1 \cap S| = 2$; $a_2 = |A_2 \cap S| = 2$; $a_3 = |A_1 \cap S| = 1$ and $c = |C \cap S| = 5$.

If n = 2, then $G \cong A_1A_2$ and $|S| = a_1 + a_2 = 4$.

If n = 3, then $G \cong C$ and $|S| = c + 1 = 6$.

Let $n = 3q + r; 0 \leq r \leq 2$

Case (i): $n \equiv 0 \pmod 3$. Then $G \cong C^q$ and $|S| = cq + 1 = 5q + 1 = \left\lceil \frac{5n+1}{3} \right\rceil$

Case (ii): $n \equiv 1 \pmod 3$. Then $G \cong C^qA_1$; hence, $|S| = cq + 2 = 5q + 2 = \left\lceil \frac{5n+1}{3} \right\rceil$

Case (iii): $n \equiv 2 \pmod 3$. Then $G \cong C^qA_1A_2$; and $|S| = 5q + 2 + 2 = 5q + 4 = \left\lceil \frac{5n+1}{3} \right\rceil$

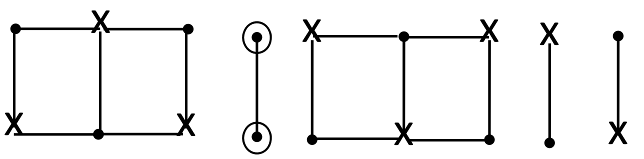
From the above cases, $\gamma_{cild}(P_4 \times P_n) \leq |S| = \left\lceil \frac{5n+1}{3} \right\rceil$.

In the following, $\gamma_{cild}(P_m \times C_n)$, for m= 2 and 3 and $n \geq 3$ are found.

Theorem 3.4: For $n \geq 3$,

$$\gamma_{cild}(P_2 \times C_n) \leq \begin{cases} \left\lceil \frac{3n}{4} \right\rceil, & \text{if } n \equiv 0, 2, 3, 5, 7 \pmod 8 \\ \left\lceil \frac{3n+4}{4} \right\rceil, & \text{if } n \equiv 1, 4, 6 \pmod 8 \end{cases}$$

Proof: Let $G \cong P_2 \times C_n$, $n \geq 3$. The blocks A_1, A_2, B_1, B_2, B_3 are constructed as before,

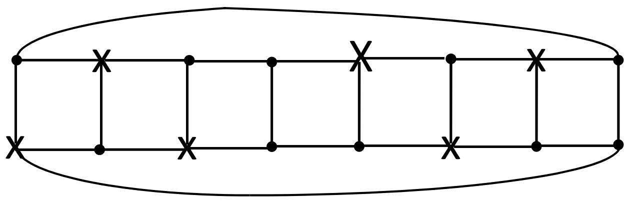


A_1 B_1 A_2 B_2 B_3

Let S be a co-isolated locating dominating set of G. The vertices with the symbol 'X' in each of the blocks represent the vertices that are to be included in S. The vertices with the symbol ◯ in the blocks indicate that those vertices are not in the locating dominating set S constructed upto this stage and to be considered while concatenation. For S to be locating dominating set of G, in a sequence of concatenations, if A_1B_1 or B_3 is the last block, then $|S|$ is increased by 1.

If the graph G' is obtained by a sequence of concatenations of two (or) more blocks mentioned above, then the graph G is obtained by joining the vertices in the first column of G' to the corresponding vertices in the last column of G'.

If $C' \cong A_1B_1A_2B_2$, then the corresponding **C** is as below.



Let $a_i = |A_i \cap S| = 3$; $i = 1,2$.

$b_1 = |B_1 \cap S| = 0$, $b_2 = |B_2 \cap S| = 1$, $b_3 = |B_3 \cap S| = 1$

$|C \cap S| = a_1 + b_1 + a_2 + b_1 = 6$.

If $n = 3$; then $G' \cong A_1$, and $|S| = a_1 = 3$.

If $n = 4$; then $G' \cong A_1B_1$, and $|S| = a_1 + b_1 + 1 = 4$.

If $n = 5$; then $G' \cong A_1B_1B_2$, and $|S| = a_1 + b_1 + b_2 + 1 = 4$.

If $n = 6$; then $G' \cong A_1B_1B_2\,B_3$ and $|S| = a_1 + b_1 + b_2 + b_3 + 1 = 6$.

If $n = 7$; then $G' \cong A_1B_1A_2$ and $|S| = a_1 + b_1 + a_2 = 6$.

If $n = 8$, then $G' \cong C'$ and $|S| = 6$.

Let $n > 8$ and $n = 8q + r$; $0 \leq r \leq 7$.

Then the following cases arise,

Case (i):$n \equiv 0 \pmod 8$. Then $G' \cong (C')^q$ and $|S| = 6q = \left\lceil \frac{3n}{4} \right\rceil$

Case (ii):$n \equiv 1 \pmod 8$. Then $G' \cong (C')^q B_3$ and $|S| = 6q + 2 = \left\lceil \frac{3n+4}{4} \right\rceil$

Case (iii):$n \equiv 2 \pmod 8$. Then $G' \cong (C')^q B_3B_2$ and $|S| = 6q + 2 = \left\lceil \frac{3n}{4} \right\rceil$

Case (iv):$n \equiv 3 \pmod 8$. Then $G' \cong (C')^q A_1$ and $|S| = 6q + 3 = \left\lceil \frac{3n}{4} \right\rceil$

Case (v):$n \equiv 4 \pmod 8$. Then $G' \cong (C')^q A_1B_1$ and $|S| = 6q + 4 = \left\lceil \frac{3n+4}{4} \right\rceil$

Case (vi):$n \equiv 5 \pmod 8$. Then $G' \cong (C')^q A_1 B_1 B_2$ and $|S| = 6q + 4 = \left\lceil \frac{3n}{4} \right\rceil$

Case (vii):$n \equiv 6 \pmod 8$. Then $G' \cong (C')^q A_1 B_1 B_2 B_3$ and $|S| = 6q + 6 = \left\lceil \frac{3n+4}{4} \right\rceil$

Case (vii):$n \equiv 7 \pmod 8$. Then $G' \cong (C')^q A_1 B_1 A_2$ and $|S| = 6q + 6 = \left\lceil \frac{3n}{4} \right\rceil$

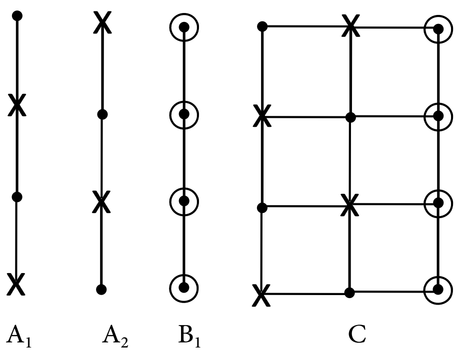
From the above cases,

$$\gamma_{cild}(P_2 \times C_n) \leq |S| = \begin{cases} \left\lceil \frac{3n}{4} \right\rceil, & \text{if } n \equiv 0, 2, 3, 5, 7 \pmod 8 \\ \left\lceil \frac{3n+4}{4} \right\rceil, & \text{if } n \equiv 1, 4, 6 \pmod 8 \end{cases}$$

Note 3.4: For $n \geq 3$, $\gamma_{cild}(P_3 \times C_n) = \gamma_{cild}(P_3 \times P_n)$.

Theorem 3.5: For $n \geq 3$, $\gamma_{cild}(P_4 \times C_n) \leq \left\lceil \frac{4n+2}{3} \right\rceil$

Proof: Let $G \cong P_4 \times C_n$, $n \geq 3$ .As before, the blocks A_1, A_2 and $B_1$are constructed. A new block C by concatenating the blocks A_1, A_2 and B_1 is also obtained below. Let S be a co-isolated locating dominating set of G. The vertices with the symbol '×' in each of the blocks represent the vertices that are to be included in S. The vertices with the symbol '◯'in the blocks indicate that those vertices are not in the locating dominating set S constructed upto this stage and to be considered while concatenation.



If the graph G' is obtained by a sequence of concatenations of two (or) more blocks mentioned above, then the graph G is obtained by joining the vertices in the first column of G' to the corresponding vertices in the last column of G' as mentioned in Theorem 3.3. Let $a_1 = |A_1 \cap S| = 2$, $a_2 = |A_2 \cap S| = 2$, $b_1 = |B_1 \cap S| = 0$ and $c = |C' \cap S| = 4$. If C' is the last block in the graph G' obtained by a sequence of concatenations, then $|S|$ increases by 1.

Let $n = 3q + r; \quad 0 \le r \le 2$

Case (i): $n \equiv 0 \pmod 3$. Then $G' \cong (C')^q$ and $|S| = cq + 1 = 4q + 1 = \left\lceil \frac{4n+2}{3} \right\rceil$

Case (ii): $n \equiv 1 \pmod 3$. Then $G' \cong (C')^q A_1$ and $|S| = cq + 2 = 4q + 2 = \left\lceil \frac{4n+2}{3} \right\rceil$

Case (iii): $n \equiv 2 \pmod 3$. Then $G' \cong (C')^q A_1 A_2$ and $|S| = 4q + 4 = \left\lceil \frac{4n+2}{3} \right\rceil$

From the above cases, $\gamma_{cild}(P_4 \times C_n) \le \left\lceil \frac{4n+2}{3} \right\rceil$. This completes the proof of the theorem.

In the following, $\gamma_{cild}(P_m \times K_n)$, for m = 2, 3 and 4 are found.

Theorem 3. 6:

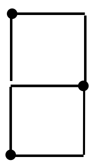
For $n \ge 2$; $\gamma_{cild}(P_2 \times K_n) = \begin{cases} 3; & \text{if } n = 2 \\ n; & \text{if } n \ge 3 \end{cases}$

Proof: If n = 2, then $P_2 \times K_n \cong C_4$ for which $\gamma_{cild}(C_4) = 3$.Let S be a co–isolated locating dominating set of G. It is to be observed that two copies of K_n are subgraphs of $P_2 \times K_n$. Let $V_1(K_n) = \{v_{11}, v_{12}, \ldots, v_{1n}\}$ and $V_2(K_n) = \{u_{21} u_{22}, \ldots, u_{2n}\}$ in $P_2 \times K_n$. For a co-isolated locating dominating set, there should exist atleast one isolated vertex in V – S. Let v_{11} be isolated in V – S. Then, $N(v_1) \subseteq S$. That is, $v_{12}, \ldots, v_{1n}, u_{21} \in S$ which implies that $|S| \le n$. Also, $N(u_2) \cap S = \{u_{21}, v_{12}\}$, $N(u_3) \cap S = \{u_{21}, v_{13}\}$, ..., $N(u_n) \cap S = \{u_{21}, v_{1n}\}$.Hence $N(u) \cap S \ne N(v) \cap S$, for any u, $v \in V - S$. Therefore, S is a cild-set of $P_2 \times K_n$. Hence, $\gamma_{cild}(P_2 \times K_n) \le n$. All the vertices in $P_2 \times K_n$is of degree n and there should be atleast one isolated vertex in V – S. Thus, any γ_{cild} - set of $P_2 \times K_n$ contains atleast n vertices. Therefore, $\gamma_{cild}(P_2 \times K_n) \ge n$. Hence, $\gamma_{cild}(P_2 \times K_n) = n$. This completes the proof of the theorem.

Theorem 3. 7: For $n \ge 2$; $\gamma_{cild}(P_3 \times K_n) = \begin{cases} 3; & \text{if } n = 2 \\ 2n-2; & \text{if } n \ge 3 \end{cases}$

Proof: Let $G \cong P_3 \times K_n$.

If n = 2, then G ≅



and $\gamma_{cild}(G) = 3$.Let S be a co –isolated locating dominating set of G. Three copies of K_n are subgraphs of $P_3 \times K_n$. Let $V_1(K_n) = \{v_{11}, v_{12}, \ldots, v_{1n}\}$, $V_2(K_n) = \{u_{21}, u_{22}, \ldots, u_{2n}\}$and $V_3(K_n) = \{w_{31}, w_{32}, \ldots, w_{3n}\}$ in $P_3 \times K_n$.

For a co-isolated locating dominating set, there should exist atleast one isolated vertex in V− S. Let v_{11} be isolated in V − S. Then, $N(v_{11}) \subseteq S$.Let $S' = N(v_{11}) = \{v_{12}, v_{13}, ..., v_{1n}, u_{21}\}$. Here S' cannot be a co-isolated locating dominating set of $P_3 \times K_n$, since $N(w_{3i}) \cap S' = \varphi$, for i = 1, 2, ..., n. Let$S = S' \cup \{w_{33}, w_{34}, \ldots, w_{3n}\}$.Then, $V - S = \{v_{11}, u_{21}, u_{23}, \ldots, u_{2n}, w_{31}, w_{32}\}$ and $|S| \leq 2n - 2$. Also, $N(u) \cap S \neq N(v) \cap S$ for any u, v ∈ V − S. Therefore, S is a cild-set of $P_3 \times K_n$. Hence, $\gamma_{cild}(P_3 \times K_n) \leq 2n - 2$. All the vertices in $P_3 \times K_n$ is of degree n and there should be atleast one isolated vertex in V − S. Hence, any γ_{cild} - set of $P_3 \times K_n$ contains atleast n vertices. Therefore, $\gamma_{cild}(P_3 \times K_n) \geq 2n - 2$. Hence, $\gamma_{cild}(P_3 \times K_n) = 2n - 2$. This completes the proof of the theorem.

Note 3.8: For $n \geq 3$, $\gamma_{cild}(P_4 \times K_n) = \gamma_{cild}(P_3 \times K_n)$.

In the following, $\gamma_{cild}(K_n \times K_m)$ is found.

Theorem 3. 9: For m, n ≥ 3; $\gamma_{cild}(K_n \times K_m) = n + m - 2$.

Proof: Let S be a co–isolated locating dominating set of $K_n \times K_m$. It is to be observed that, m copies of K_n are subgraphs of $K_n \times K_m$. Let the vertex sets of those copies of K_n be $V_1(K_n) = \{v_{11}, v_{12}, \ldots, v_{1n}\}$, $V_2(K_n) = \{v_{21}, v_{22}, \ldots, v_{2n}\}$, …, $V_m(K_n) = \{v_{m1}, v_{m2}, \ldots, v_{mn}\}$ in $K_n \times K_m$.

For S to be a co-isolated locating dominating set, there should exist atleast one isolated vertex in V−S. Without loss of generality, it is assumed that $v_{11}$is isolated in V− S. Then, $N(v_{11}) \subseteq S$. That is, $v_{12}, v_{13}, \ldots, v_{1n}, v_{21}, v_{31}, \ldots, v_{m1} \in S$ which implies that $|S| \leq n + m - 2$. Also, $N(v_{22}) \cap S = \{v_{21}, v_{12}\}$, $N(v_{23}) \cap S = \{v_{21}, v_{13}\}$,…, $N(v_{2n}) \cap S = \{v_{21}, v_{1n}\}$, $N(v_{32}) \cap S = \{v_{31}, v_{12}\}$, …,$N(v_{3n}) \cap S = \{v_{31}, v_{1n}\}$,…, $N(v_{m2}) \cap S = \{v_{m1}, v_{12}\}$, $N(v_{m3}) \cap S = \{v_{m1}, v_{13}\}$, …,$N(v_{mn}) \cap S = \{v_{m1}, v_{1n}\}$.

Hence $N(u) \cap S \neq N(v) \cap S$, for any u, v ∈ V − S. Therefore, S is a cild-set of $K_n \times K_m$. Hence, $\gamma_{cild}(K_n \times K_m) \leq n+m-2$. All the vertices in $K_n \times K_m$ is of degree n + m − 2 and there should be at least one isolated vertex in V − S. Thus, any γ_{cild} - set of $K_n \times K_m$ has at least n + m − 2 vertices. Therefore, $\gamma_{cild}(K_n \times K_m) \geq n + m - 2$. Hence, $\gamma_{cild}(K_n \times K_m) = n+m-2$. This completes the proof of the theorem.

References:

[1] Benjamin N. Omamalin., Sergio R. Canoy Jr. and Helen M. Rara,Locating Total Dominating sets in the Join, Corona and composition of Graphs, Applied Mathematical Sciences, Vol.8, 2014, no. 48, p. 2363 – 2374.

[2] M. Blidia, M. Chellai, F. Maffray, J. Moncel and A. Semri, Locating-domination and identifying codes in trees, Australasian J. Combin. 39 (2007), 219 – 232.

[3] M. Blidia, O. Favaron and R. Lounes, Locating-domination, 2-domination and independence in trees, Australasian J. Combin. 42(2008), 309 – 316.

[4] M. Chellai, M. Mimouni and P.J. Slater, On Locating-domination in graphs, Discuss. Math. Graph Theory, 30 (2010), 223 – 235.

[5] F. Harary, Graph Theory, Addison – Wesley, Reading Mass, 1969.

[6] T. W. Haynes, M.A. Henning and J. Howard, Locating and total dominating sets in trees, Discrete Appl. Math., 154 (2006), 1293 – 1300.

[7] V. R. Kulli, Theory of Domination in Graphs, Vishwa International Publications, Gulbarga, India, 2010.

[8] S. Muthammai and N. Meenal, Co-isolated Locating Domination Number for some standard Graphs, National conference on Applications of Mathematics & Computer Science (NCAMCS-2012), S.D.N.B Vaishnav College for Women(Autonomous), Chennai, February 10, 2012, p. 60 – 61.

[9] S. Muthammai and N. Meenal, Isolated Locating Domination Number of a Graph, Proceedings of the UGC sponsored National Seminar on Applications in Graph Theory, Seethalakshmi Ramaswamy College (Autonomous), Tiruchirappalli, 18th&19thDecember 2012,p. 7 – 9.

[10] Nasrin Soltankhah, On The Total Domination Subdivision Numbers Of Grid Graphs, International Journal of Contemporary Mathematical Sciences, Vol. 5, 2010, no. 49, p. 2419 – 2432.

[11] O. Ore, Theory of Graphs, Amer. Math. Soc. Coel. Publ. 38, Providence, RI, 1962.

[12] P.J. Slater, Domination and location in acyclic graphs, Networks 17 (1987), 55 – 64.

[13] P. J. Slater, Domination and reference sets in graphs, J. Math. Phys. Sci. 22 (1988), 445 – 455.

[14] T. Tamil Chelvam and G. S. Grace Prema, Inverse Domination in Grid Graphs, Algebra, Graph theory and their applications, Eds. T. Tamizh Chelvam, S. Somasundaram and R. Kala, Narosa Publishing House Pvt. Ltd., Chennai, 2010, p. 93 – 103.