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# Brief Communications: Two-layered Model (Casson-Newtonian) for Blood Flow Through an Arterial Stenosis with Axially Variable Slip Velocity at the Wall

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***Abstract:*** *The purpose of this paper is to analyze the works done by Ponalagusamy and Tamilselvi[1]. Ponalagusamy and Tamilselvi[1] have obtained the analytic expression for an axially variable slip velocity at the wall by assuming the flow rate for the case of one-layered model with slip velocity is equal to that of two-layered model with zero slip velocity. This assumption may, in general, not be valid. Keeping this in view, the flow rate for the case of one-layered model with slip velocity is generally considered to be equal to the product of a constant($C_1$) and the flow rate of two-layered model without slip velocity, where* $0 < C_1$. *Slip velocity at the wall has been computed for different values of* $C_1$ *for tube diameters 40* $\mu m$ *and 66.6* $\mu m$.

In the paper[1], it is mentioned that the introduction of a thin solvent layer near the wall produces the same effect as that of the slip at the wall(Bennett[2] & Chaturani and Ponalagusamy[3]). In the case of one layered model ($R = R_1$) with slip at the wall, the flow rate $Q_{1L}$ is given as(for more detail, refer[1]):

$$Q_{1L} = \frac{\rho^* q(z)\mathrm{Re}}{8\beta} R^4 + R^2 u_s(z) + \frac{\mathrm{Re}\,\theta\rho^*}{3}\{R^3 - R_p{}^3\} - \frac{2\sqrt{2}\rho^*\mathrm{Re}}{7\sqrt{\beta}}\{\theta q(z)\}^{\frac{1}{2}}\{R^{7/2} - R_p{}^{7/2}\} \quad \ldots(1)$$

where $\rho^* = \dfrac{\bar{\rho}_p}{\bar{\rho}}$, $\mathrm{Re} = \dfrac{\bar{\rho}\bar{U}_0\bar{R}_0}{\bar{\mu}^*}$ and $\bar{\rho}$ and $\bar{\mu}^*$ are the density and viscosity of the fluid when the flow is one-layered. '-' over a letter denotes the corresponding dimensional quantity. For the two-layered model without slip at the wall ( $u_s = 0$), the flow rate $Q_{2L}$ is given as

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$$Q_{2L} = \frac{q(z) R_{ep} R^4}{8\beta}\left[1-(1-\frac{\delta(z)}{R})^4(1-\mu)\right] + \frac{\mu R_{ep}\theta R^3}{3}\{(1-\frac{\delta(z)}{R})^3 - (\frac{R_p}{R})^3\}$$
$$-\frac{2\sqrt{2}\mu R_{ep}}{7\sqrt{\beta}}\{\theta q(z) R^7\}^{1/2}[\{1-\frac{\delta(z)}{R}\}^{7/2} - (\frac{R_p}{R})^{7/2}]$$

.... (2)

where $\delta(z) = \bar{\delta}(\bar{z})/\bar{R}_0$ is the non-dimensional peripheral layer thickness which is a function of axial distance z. Since the two models (one-layered with slip and two-layered without slip) represent the same phenomena and reported by(Bennett[2]), the flow rates can be equated as

$$Q_{1L} = C_1 Q_{2L}$$
$$\frac{\rho^* q(z)\mathrm{Re}}{8\beta}R^4 + R^2 u_s(z) + \frac{\mathrm{Re}\,\theta\rho^*}{3}\{R^3 - R_p^{\ 3}\} - \frac{2\sqrt{2}\rho^*\mathrm{Re}}{7\sqrt{\beta}}\{\theta q(z)\}^{1/2}\{R^{7/2} - R_p^{\ 7/2}\}$$

$$=$$

$$C_1\{\frac{q(z) R_{ep} R^4}{8\beta}\left[1-(1-\frac{\delta(z)}{R})^4(1-\mu)\right] + \frac{\mu R_{ep}\theta R^3}{3}\{(1-\frac{\delta(z)}{R})^3 - (\frac{R_p}{R})^3\}$$
$$-\frac{2\sqrt{2}\mu R_{ep}}{7\sqrt{\beta}}\{\theta q(z) R^7\}^{1/2}[\{1-\frac{\delta(z)}{R}\}^{7/2} - (\frac{R_p}{R})^{7/2}]\}$$

Where $Q_{2L} = Q^*$ .

....(3)

From Eq.(3), one can obtain $u_s(z)$ as

$$u_s(z) = \frac{q(z)R^2}{8\beta}\left[C_1 R_{ep}\left[1-(1-\frac{\delta(z)}{R})^4(1-\mu)\right] - \rho^*\mathrm{Re}\right]$$
$$+\frac{R\theta}{3}[C_1\mu R_{ep}\{(1-\frac{\delta(z)}{R})^3 - (\frac{R_p}{R})^3\} - \rho^*\mathrm{Re}\{1-(\frac{R_p}{R})^3\}]$$
$$+\frac{2}{7}\{2\theta q(z)R^3/\beta\}^{1/2}[\rho^*\mathrm{Re}\{1-(\frac{Rp}{R})^{7/2}\} - C_1\mu R_{ep}\{(1-\frac{\delta(z)}{R})^{7/2} - (\frac{R_p}{R})^{7/2}\}]$$

...(4)

From Eq.(4), the dimensional form of the slip velocity $\bar{u}_s$ is obtained as

$$\bar{u}_s = \frac{\bar{q}_0(\bar{R}_0)^2}{8}\left[\frac{C_1}{\bar{\mu}_p}\{1-\{1-\frac{\bar{\delta}_0}{R_0}\}^4(1-\mu)\}-\frac{1}{\bar{\mu}^*}\right]$$
$$+\frac{\bar{\tau}_0\bar{R}_0}{3}\left[\frac{C_1}{\bar{\mu}_c}\{(1-\frac{\bar{\delta}_0}{\bar{R}_0})^3-(\frac{\bar{R}_p}{\bar{R}_0})^3\}-\frac{1}{\bar{\mu}^*}\{1-(\frac{\bar{R}_p}{\bar{R}_0})^3\}\right]$$
$$+\frac{8}{7}\{\frac{\bar{\tau}_0\bar{Q}^*\bar{\rho}_p}{\pi\bar{R}_0^2}\}^{1/2}\left[(\frac{1}{\bar{\mu}^*})^{1/2}\{1-(\frac{\bar{R}_p}{\bar{R}_0})^{7/2}\}-C_1\mu(\frac{1}{\bar{\mu}_p})^{1/2}\{(1-\frac{\bar{\delta}_0}{\bar{R}_0})^{7/2}-(\frac{\bar{R}_p}{\bar{R}_0})^{7/2}\}\right] \quad \ldots\ldots(5)$$

The slip velocities at the wall with $C_1 = 1$ for tube diameters $40\,\mu m$ and $66.6\,\mu m$ are mistakenly mentioned in the paper[1] and the corresponding corrected values are given in Table-I.

| **Tube Diameter** | $\bar{u}_s$ $cm/\sec$ | | | |
|---|---|---|---|---|
| | $C_1$ =.25 | $C_1$ =.5 | $C_1$ =.75 | $C_1$ =1 |
| 40 $\mu m$ | 0.30248 | 1.06002 | 2.01756 | 2.87511 |
| 66.6 $\mu m$ | 0.06103 | 0.36307 | 0.66512 | 0.96716 |

Table-I Slip Velocity $\bar{u}_s$ $cm/\sec$

It is observed from Table-I that the slip velocity at the wall decreases as the value of $C_1$ decreases. The value of $C_1$ should be less than unity due to the fact that the slip velocity cannot be greater than the velocity at the centre of the tube for tube diameter $40\,\mu m$. Whereas for the case of tube diameter $66.6\,\mu m$, the value of $C_1$ may be greater than unity. Hence, it is recommended that a series of experiments on blood flow is to be carried out to determine the proper values of $C_1$ under various values of flow parameters. Further, the exact information could lead to a thorough understanding of the nature of blood flow in the microvascular system[4,5,6,7,8,9] which, in turn, may shed some lights on knowing the cause factor of genesis of atherosclerosis and other diseases and thereby leads to the development of modern diagnostic tools for the effective treatment of patients.

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