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k-Even Mean Labeling of Some Graph Operations

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Abstract: *Mean labeling of graphs was discussed in [24-26] and the concept of odd mean labeling was introduced in [22]. k-odd mean labeling and (k, d)-odd mean labeling are introduced and discussed in [1, 6-8]. k-even mean and (k, d)-even mean labeling introduced and discussed in [9-17]. In this paper, we discuss the k-even mean labeling of some graph operations.*

Keywords: *k-even mean labeling, k-even mean graph.*

1. Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [20]. The symbols $V(G)$ and $E(G)$ will denote the vertex set and edge set of a graph. Labeled graphs serve as useful models for a broad range of applications [2-4].

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling).

Graph labeling was first introduced in the late 1960's. Many studies in graph labeling refer to Rosa's research in 1967 [23]. Labeled graphs serve as useful models for a broad range of applications such as X-ray crystallography, radar, coding theory, astronomy, circuit design and communication network addressing. Particularly interesting applications of graph labeling can be found in [5].

Mean labeling of graphs was discussed in [24-26]. Vaidya [29-32] and et al. have investigated several new families of mean graphs. Nagarajan [31] and et al. have found some new results on mean graphs.

Ponraj, Jayanthi and Ramya extended the notion of mean labeling to super mean labeling in [21]. Gayathri and Tamilselvi [18-19, 27] extended super mean labeling to k-super mean, (k, d)-super mean, k-super edge mean and (k, d)-super edge mean labeling. Manickam and Marudai [22] introduced the concept of odd mean graph. Gayathri and Amuthavalli [1, 6-8] extended this concept to k-odd mean and (k, d)-odd mean graphs.

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Gayathri and Gopi[9-17] extended this concept to k-even mean and (k, d)-Even mean graphs.

In this paper, we have found the k-even mean labeling of some graph operations. Throughout this paper, k denotes any positive integer greater than or equal to 1. For brevity, we use *k-EML* for k-even mean labeling.

2. Main Results

Definition2.1.1:

A (p, q) graph G is said to have a ***k*-even mean labeling** if there exists a injection $f: V \longrightarrow \{0, 1, 2, ..., 2k + 2(q-1)\}$ such that the induced map $f^*: E(G) \longrightarrow \{2k, 2k+2, ..., 2k + 2(q-1)\}$ defined by

$$f^*(uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$ is a bijection.

A graph that admits a k-even mean labeling is called a ***k*-even mean graph**.

Definition 2.1.2:

The **shadow graph $D_2(G)$** of a connected graph G is constructed by taking two copies of G say G' and G''. Join each vertex u' in G' to the neighbours of the corresponding vertex v' in G''.

Theorem 2.1.3:

The graph $D_2(K_{1,n})(n \geq 2)$ is a k—even mean graph for k and n.

Proof:

Let $\{v_i, 1 \leq i \leq 2n, u, v\}$ be the vertices and $\{a_i, 1 \leq i \leq n, a_i', 1 \leq i \leq n, b_i, 1 \leq i \leq n, b_i', 1 \leq i \leq n\}$ be the edges which are denoted as in Figure 2.1.

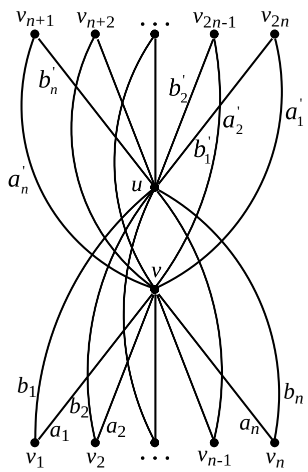

Figure 2.1: Ordinary labeling of $D_2(K_{1,n})$

First we label the vertices as follows:

Define $f: V \rightarrow \{0, 1, 2, ..., 2k + 2q - 2\}$ by

For $1 \leq i \leq 2n$, $f(v_i) = 2k + 4i - 2$

$f(u) = 2k + 8n - 3$; $f(v) = 2k - 2$

Then the induces edge labels are:

For $1 \leq i \leq n$, $f^*(a_i) = 2k + 2(i - 1)$; $f^*(a_i^{'}) = 2k + 2n + 2(i - 1)$

$f^*(b_i) = 2k + 4n + 2(i - 1)$; $f^*(b_i^{'}) = 2k + 6n + 2(i - 1)$

Therefore, $f^*(E) = \{2k, 2k + 2, ..., 2k + 2q - 2\}$. So, f is a k–even mean labeling and hence, the graph $D_2(K_{1,n})(n \geq 2)$ is a k—even mean graph for any k and n.

3—*EML* of $D_2(K_{1,4})$ is shown in Figure 2.2.

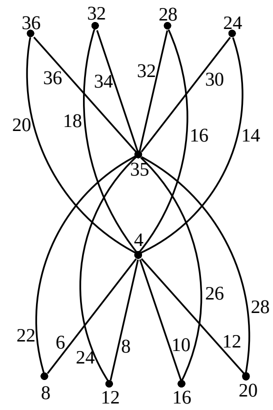

Figure 2.2: 3-*EML* of $D_2(K_{1,4})$

Definition 2.1.4:

Consider a cycle C_n and let $e_k = v_k v_{k+1}$ be an edge in it with $e_{k-1} = v_{k-1} v_k$ and $e_{k+1} = v_{k+1} v_{k+2}$ be its incident edges and $e_k^{'} = v_k^{'} v_{k+1}^{'}$ be a new edge. The duplication of an edge e_k by an edge $e_k^{'}$ produces a new graph G in such a way that $N(v_k) \cap N(v_k^{'}) = \{v_{k-1}\}$ and $N(v_{k+1}) \cap N(v_{k+1}^{'}) = \{v_{k+2}\}$ which is called edge duplication of C_n and denoted by $ED(C_n)$ where $N(v_k)$ denotes the set of vertices adjacent to v_k.

Theorem 2.1.5:

The graph $ED(C_n)$ $(n \geq 4)$ is a k–even mean graph for any k and n.

Proof:

Let $\{v_i, 0 \leq i \leq n-1, v_1^{'}, v_2^{'}\}$ be the vertices and $\{e_i, 0 \leq i \leq n-1, e_1^{'}, e_2^{'}, e_3^{'}\}$ be the edges of C_n. The duplication of an edge e_1 of C_n by an edge $e_1^{'}$ is denoted as $ED(C_n)$ and its ordinary labeling is given in Figure 2.3.

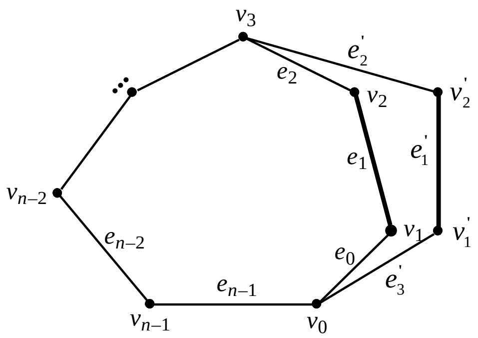

Figure 2.3: Ordinary labeling of $ED(C_n)$

We observe that,

$$e_1 = v_1 v_2 ; \qquad e_0 = v_0 v_1$$

$$e_2 = v_2 v_3 ; \qquad e_1' = v_1' v_2'$$

and $N(v_1) \cap N(v_1') = \{v_0\}$, $N(v_2) \cap N(v_2') = \{v_3\}$

Case (i): *n* is even

First we label the vertices as follows:

Define $f: V \longrightarrow \{0, 1, 2, ..., 2k + 2q - 2\}$ by

$$f(v_0) = 2k + 2n + 1$$

For $1 \leq i \leq \frac{n+2}{2}$, $f(v_i) = 2k + 2i - 3$

For $\frac{n+4}{4} \leq i \leq n - 1$, $f(v_i) = 2k + 2i + 1$

$$f(v_1') = 2k + 2n + 3 ; \ f(v_2') = 2k + 2n + 4$$

Then the induced edge labels are:

For $1 \leq i \leq \frac{n}{2}$, $f^*(e_i) = 2k + 2(i - 1)$; $f^*\left(e_{\frac{n+2}{2}}\right) = 2k + n + 2$

For $\frac{n+4}{2} \leq i \leq n - 1$, $f^*(e_i) = 2k + 2i + 2$

$$f^*(e_0) = 2k + n \quad ; \quad f^*(e_1') = 2k + 2n + 4$$

$$f^*(e_2') = 2k + n + 4 \quad ; \quad f^*(e_3') = 2k + 2n + 2.$$

Case (ii): *n* is odd

First we label the vertices as follows:

Define $f: V \longrightarrow \{0, 1, 2, ..., 2k + 2q - 2\}$ by

$$f(v_0) = 2k + 2n ; \ f(v_1) = 2n + 2k - 1$$

For $2 \leq i \leq \frac{n+3}{2}$, $f(v_i) = \begin{cases} 2k+2i-6 & i \text{ is even} \\ 2k+2i-5 & i \text{ is odd} \end{cases}$

For $\frac{n+5}{2} \leq i \leq n-1$, $f(v_i) = \begin{cases} 2k+2i-2 & i \text{ is even} \\ 2k+2i-1 & i \text{ is odd} \end{cases}$

$$f(v_1') = 2k + 2n + 3 \;;\; f(v_2') = 2k + 2n + 4$$

Then the induced edge labels are:

$$f^*(e_0) = 2k + 2n \;;\; f^*(e_1) = 2k + n - 1$$

For $2 \leq i \leq \frac{n+1}{2}$, $f^*(e_i) = 2k + 2i - 4$; $f^*\left(e_{\frac{n+3}{2}}\right) = 2k + n + 1$

For $\frac{n+5}{2} \leq i \leq n-1$, $f^*(e_i) = 2k + 2i$; $f^*(e_1') = 2k + 2n + 4$

$$f^*(e_2') = 2k + n + 3 \;;\; f^*(e_3') = 2k + 2n + 2.$$

Therefore, $f^*(E) = \{2k, 2k + 2, ..., 2k + 2q - 2\}$. So, f is a k—even mean labeling and hence, the graph $ED(C_n)$ $(n \geq 4)$ is a k–even mean graph for any k and n.

4—EML of $ED(C_6)$ is shown in Figure 2.4.

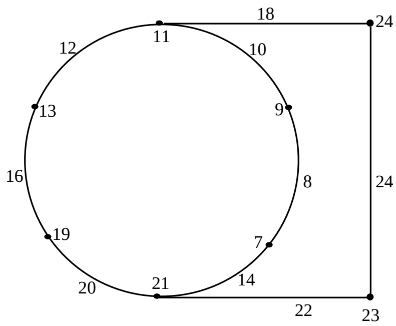

Figure 2.4: 4—*EML* of *ED*(C_6)

2—EML of $ED(C_9)$ is shown in Figure 2.5.

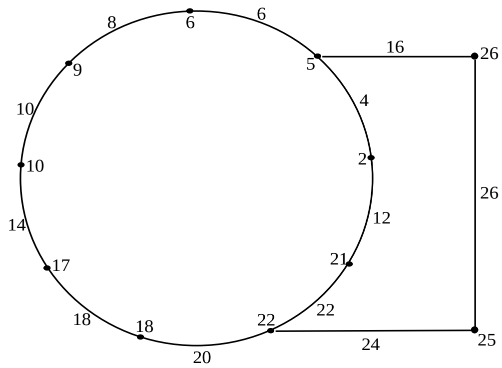

Figure 2.5: 2—*EML* of *ED*(C_9)

Definition 2.1.6:

Duplication of a vertex v_k of a graph G produces a new graph by adding a new vertex v_k' in such a way that $N(v_k) = N(v_k')$. It is called vertex duplication of graph and denoted by $VD(G)$.

Theorem 2.1.7:

The graph $VD(P_n)(n \geq 4)$ is a k—even mean graph for any k and n.

Proof:

Let P_n denoted a path on n vertices $\{v_1, 1 \leq i \leq n, v_2^{'}\}$ be the vertices and $\{e_i, 1 \leq i \leq n-1, e_1^{'}, e_2^{'}\}$ be the edges of $VD(P_n)$ obtained by the duplication of the vertex v_2 which are denoted as in Figure 2.6.

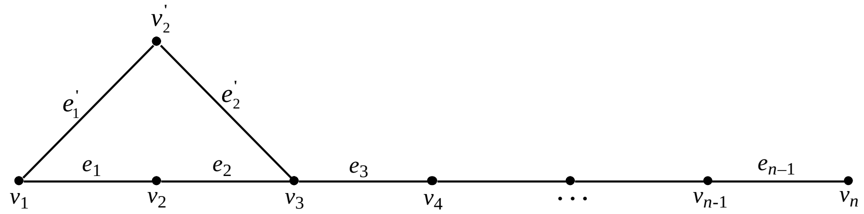

Figure 2.6: Ordinary labeling of $VD(P_n)$

We observe that $N(v_2) = N(v_2^{'})$.

First we label the vertices as follows:

Define $f: V \longrightarrow \{0, 1, 2, ..., 2k + 2q - 2\}$ by

$f(v_1) = 2k - 1$; $f(v_2) = 2k + 1$

For $3 \leq i \leq n - 1$,

$f(v_i) = 2k + 2i + 1$

$f(v_n) = 2k + 2n$; $f(v_2^{'}) = 2k + 5$.

Then the induced edge labels are:

$f^*(e_1) = 2k$; $f^*(e_2) = 2k + 4$

For $3 \leq i \leq n - 1$,

$f^*(e_i) = 2k + 2i + 2$

$f^*(e_1^{'}) = 2k + 2$; $f^*(e_2^{'}) = 2k + 6$

Therefore, $f^*(E) = \{2k, 2k + 2, ..., 2k + 2q - 2\}$. So, f is a k—even mean labeling and hence, the graph $VD(P_n)(n \geq 4)$ is a k–even mean graph for any k and n.

2—*EML* of $VD(P_7)$ is shown in Figure 2.7.

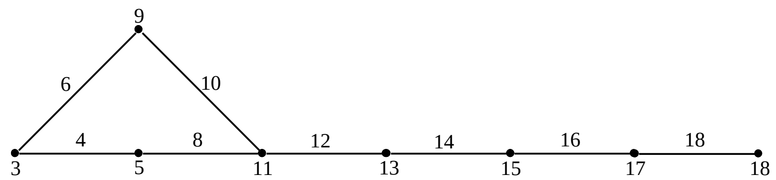

Figure 2.7: 2—*EML* of $VD(P_7)$

Theorem 2.1.8:

The graph $VD(C_n)(n \geq 4)$ is a k—even mean graph for any k and n.

Proof:

Let $\{v_i,\ 1 \leq i \leq n,\ v_n^{'}\}$ be the vertices and $\{e_i,\ 1 \leq i \leq n,\ e_1^{'},\ e_2^{'}\}$ be the edges of $VD(C_n)$ obtained by the duplication of the vertex v_n which are denoted as in Figure 2.8.

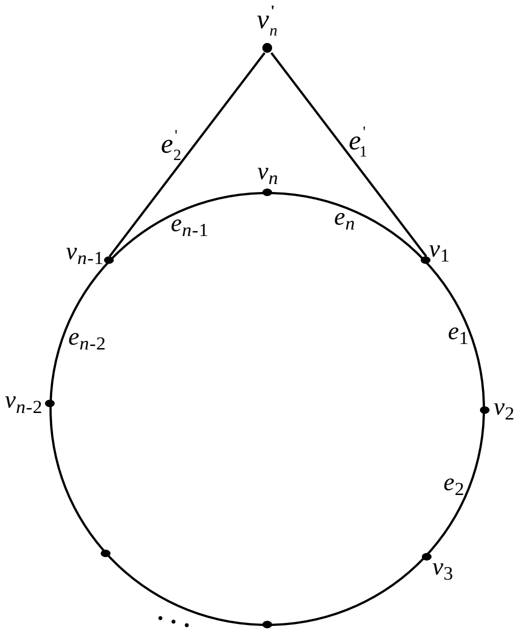

Figure 2.8: Ordinary labeling of $VD(C_n)$

We observe that, $N(v_n) = N(v_n^{'})$.

Case (i): $n \equiv 0 \pmod 4$

First we label the vertices as follows:

Define $f: V \rightarrow \{0, 1, 2, ..., 2k + 2q - 2\}$ by

For $1 \leq i \leq \frac{n}{2}$, $f(v_i) = 2k + 2i - 3$

For $\frac{n+2}{2} \leq i \leq n$, $f(v_i) = \begin{cases} 2k+2i-3 & i \text{ is even} \\ 2k+2i+4 & i \text{ is odd} \end{cases}$

$f(v_n^{'}) = 2k + 2n + 1.$

Then the induced edge labels are:

$f^*(e_1^{'}) = 2k + n + 2$; $f^*(e_2^{'}) = 2k + 2n + 2$

For $1 \leq i \leq \frac{n-2}{2}$, $f^*(e_i) = 2k + 2(i - 1)$

For $\frac{n}{2} \leq i \leq n-1$, $f^*\left(e_i\right) = 2k+2i+2$; $f^*(e_n) = 2k+n-2$

Case (ii): $n \equiv 1 \pmod 4$

First we label the vertices as follows:

Define $f: V \rightarrow \{0, 1, 2, ..., 2k+2q-2\}$ by

$$f\left(v_n'\right) = 2k+2n+1$$

For $1 \leq i \leq n$, i odd, $f\left(v_i\right) = 2k+2i$

For $1 \leq i \leq \frac{n-1}{2}$, i even

$$f\left(v_i\right) = \begin{cases} 2k+2i-5 & \text{if } i \leq \frac{n-1}{2} \\ 2k+2i+3 & \text{if } i > \frac{n-1}{2} \\ 2k+2n+2 & \text{if } i = n-1. \end{cases}$$

Then the induced edge labels are:

For $1 \leq i \leq \frac{n-1}{2}$, $f^*\left(e_i\right) = 2k+2i-2$

For $\frac{n+1}{2} \leq i \leq n-1$,

$$f^*\left(e_i\right) = 2k+2i+2 \ ; \ f^*\left(e_n\right) = 2k+n-1$$

$$f^*\left(e_1'\right) = 2k+2n+2 \ ; \ f^*\left(e_2'\right) = 2k+n+2$$

Case (iii): $n \equiv 2 \pmod 4$

First we label the vertices as follows:

Define $f: V \rightarrow \{0, 1, 2, ..., 2k+2q-2\}$ by

$$f\left(v_1\right) = 2(k-1)$$

For $2 \leq i \leq \frac{n-2}{2}$, $f\left(v_i\right) = \begin{cases} 2k+2i-2 & i \text{ is even} \\ 2k+2i-5 & i \text{ is odd} \end{cases}$

For $\frac{n}{2} \leq i \leq n-2$, $f\left(v_i\right) = \begin{cases} 2k+2i-1 & i \text{ is even} \\ 2k+2i+3 & i \text{ is odd} \end{cases}$

$$f\left(v_{n-1}\right) = 2k+2n+1 \ ; \ f\left(v_n\right) = 2k+2n-6$$

$$f\left(v_n'\right) = 2k+2n+2.$$

Then the induced edge labels are:

For $1 \leq i \leq \frac{n-4}{2}$, $f^*\left(e_i\right) = 2k+2i-2$; $f^*\left(e_{\frac{n-2}{2}}\right) = 2k+n-2$

For $\frac{n}{2} \le i \le n-3$, $f^*(e_i) = 2k + 2i + 2$; $f^*(e_{n-2}) = 2k + 2n$

$f^*(e_{n-1}) = 2k + 2n - 2$; $f^*(e_n) = 2k + n - 4$

$f^*(e_1') = 2k + 2n + 2$; $f^*(e_2') = 2k + n$

Case (iv): $n \equiv 3 \pmod 4$

First we label the vertices as follows:

Define $f: V \to \{0, 1, 2, ..., 2k + 2q - 2\}$ by

For $1 \le i \le \frac{n-1}{2}$, $f(v_i) = \begin{cases} 2k+2i-2 & i \text{ is odd} \\ 2k+2i-5 & i \text{ is even} \end{cases}$

For $\frac{n+1}{2} \le i \le n-3$, $f(v_i) = \begin{cases} 2k+2i-1 & i \text{ is even} \\ 2k+2i+3 & i \text{ is odd} \end{cases}$

$f(v_{n-2}) = 2k + 2n - 2$; $f(v_{n-1}) = 2k + 2n + 2$

$f(v_n) = 2k + 2n - 6$; $f(v_n') = 2k + 2n + 1$.

Then the induced edge labels are:

For $1 \le i \le \frac{n-3}{2}$, $f^*(e_i) = 2k + 2i - 2$; $f^*\left(e_{\frac{n-1}{2}}\right) = 2k + n - 1$

For $\frac{n+1}{2} \le i \le n-3$, $f^*(e_i) = 2k + 2i + 2$

$f^*(e_{n-2}) = 2k + 2n$; $f^*(e_{n-1}) = 2k + 2n - 2$; $f^*(e_n) = 2k + n - 3$

$f^*(e_1') = 2k + 2n + 2$; $f^*(e_2') = 2k + n + 1$

Therefore, $f^*(E) = \{2k, 2k + 2, ..., 2k + 2q - 2\}$. So, f is a k—even mean labeling and hence, the graph $VD(C_n)(n \ge 4)$ is a k—even mean graph for any k and n. 6—*EML* of $VD(C_8)$ is shown in Figure 2.9.

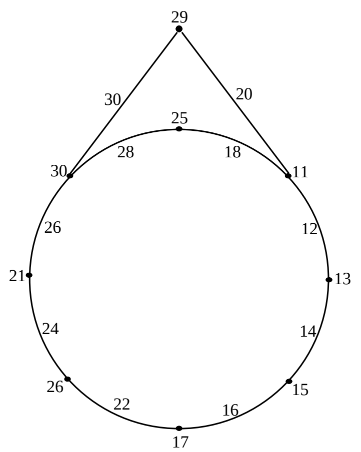

Figure 2.9: 6—*EML* of $VD(C_8)$

3−*EML* of $VD(C_5)$ is shown in Figure 2.10.

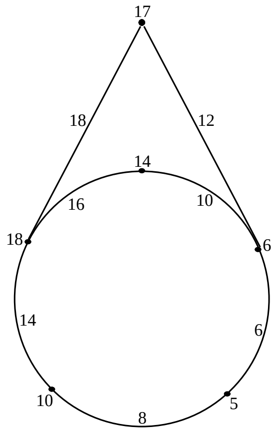

Figure 2.10: 3-*EML* of $VD(C_5)$

5−*EML* of $VD(C_{10})$ is shown in Figure 2.11.

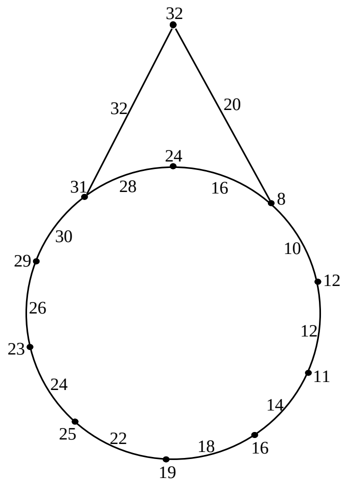

Figure 2.11: 5−*EML* of $VD(C_{10})$

2−*EML* of $VD(C_7)$ is shown in Figure 2.12.

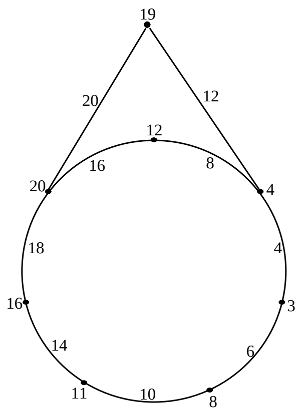

Figure 2.12: 2−*EML* of $VD(C_7)$

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