International Journal of Engineering Science, Advanced Computing and Bio-Technology
Vol. 4, No. 1, January – March 2013, pp. 19 -31

Effect of Viscous Dissipation and Non-Uniform Double Slot Injection on a Steady Water Boundary Layer Flow over Rotating Sphere with Non-Uniform Mass Transfer

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Abstract: *An analysis is performed to study the influence of viscous dissipation and non-uniform double slot injection on a steady laminar boundary layer flow over a rotating sphere when the fluid properties such as viscosity and Prandtl number are inverse linear functions of temperature. Non-similar solutions have been obtained from the starting point of the streamwise co-ordinate to the exact point of separation. The difficulties arising at the starting point of the streamwise co-ordinate, at the edges of the slot and at the point of separation have been overcome by applying an implicit finite difference scheme in combination with the quasi-linearization technique and an appropriate selection of the finer step sizes along the stream-wise direction. The present investigation shows that the point of ordinary separation moved upstream by non-uniform double slot injection and by moving the slot downstream. But the point of separation delayed by rotation and viscous dissipation with the influence of non-uniform double slot injection.*

Keywords: *Unsteady flow; rotating sphere; boundary layer; variable properties; non-similar solution; forced convection flow; non-uniform mass transfer; injection*

1. Introduction

A detailed analysis of boundary layer flow problems taking non-similarity into account has become significantly important in recent past. In an earlier study, a review on the non-similarity solution methods along with the relevant publications is given by Dewey and Gross [1]. Subsequently, many attempts have been made to provide non-similar solutions of boundary layer flow problems by finite difference method [2], [3] and an implicit finite difference method in combination with quasi-linearization technique [4], [5]. Fluid viscosity and thermal conductivity are the main governing fluid properties in the laminar water boundary layer forced flow and hence their variations can be expected to affect separation. Further, mass transfer through a slot strongly influences the development of a boundary layer along a surface and in particular can prevent or at least delay separation of the viscous region. Different studies [6], [7], [8], [9] show the effect of single slot suction (injection) into steady compressible and water boundary layer flows over two dimensional and axi-symmetric bodies. Moreover, Roy [10] and Subhashini et.al [11] have investigated the influence of non-uniform multiple slot suction (injection) on

Received: 18 December, 2013; Revised: 11 January, 2013; Accepted: 28 January, 2013

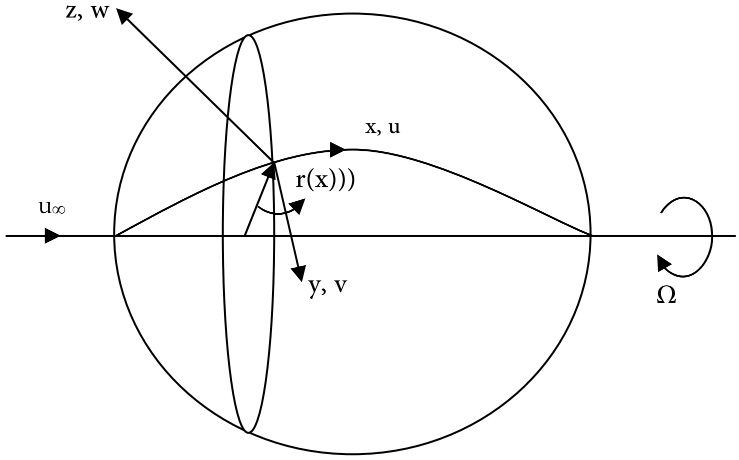

Figure 1: Flow model and coordinate

compressible boundary layer flows over cylinder and yawed cylinder, respectively. The studies on the flows over rotating sphere have been made by some researchers in [12], [13], [14]. Also, in more recent studies, Roy et.al. [15], [16] have reported the influence of non-uniform double slot suction (injection) on an incompressible boundary layer flow over a slender cylinder and sphere, respectively. But the effect of non-uniform double slot injection and the simultaneous effects of viscous dissipation and rotation with the influence of non-uniform injection, on velocity and temperature gradients are not yet studied by any researchers. In the present investigation, the effect of non-uniform double slot injection and its performance with viscous dissipation and rotation of sphere in the steady laminar non-similar boundary layer flow is considered.

2. Mathematical Formulation

Consider a steady laminar non-similar boundary layer forced convection flow (of water) with temperature-dependent viscosity and Prandtl number over a rotating sphere when the non-uniform mass transfer (suction/injection in a slot) vary with the axial distance (x) along the surface. The sphere, rotating with the constant angular velocity ω is placed in a uniform stream with its axis of rotation parallel to the free stream velocity. An orthogonal curvilinear coordinate system (see Figure.1) has been chosen in which coordinate x measures the distance from the forward stagnation point along a meridian, y represents the distance in the direction of rotation and z is the distance normal to the body surface. The radius of a section normal to the axis of the sphere at a distance x along the meridian from the pole is $r(x)$ and it is assumed that it is large compared with the boundary layer thickness. The fluid is assumed to flow with moderate velocities, and the temperature difference between the wall and the free stream is small $(< 40^\circ C)$. In the range of temperature considered (i.e., $(0^\circ C - 40^\circ C)$), the variation of both density

(ρ) and specific heat (C_p), of water, with temperature is less than 1% [8] and hence they are taken as constants. However, since the variations of viscosity μ thermal conductivity κ [and hence Prandtl number (Pr)] with temperature are quite significant, the viscosity and Prandtl number are assumed to vary as an inverse function of temperature (T) [8]:

$$\mu = \frac{1}{b_1 + b_2 T}, \quad Pr = \frac{1}{c_1 + c_2 T} \tag{1}$$

The numerical data, used for these correlations, are taken from [17]. The relations in equation (1) are reasonably good approximations for liquids such as water, particularly for moderate temperature differences between the wall and ambient fluid. The fluid at the edge of the boundary layer is maintained at a constant temperature T_∞ and the body has a uniform temperature T_w $(T_w > T_\infty)$. The blowing rate of the fluid is assumed to be small and it does not affect the inviscid flow at the edge of the boundary layer. It is also assumed that the injected fluid possesses the same physical properties as the boundary layer fluid and has a static temperature equal to the wall temperature. Now with the above assumptions the dimensionless governing equations are [8],

$$(NF_\eta)_\eta + \{fF_\eta + \beta(\xi)(1 - F^2)\} + \alpha(\xi)S^2 = 2\xi(FF_\xi - f_\xi F_\eta) \tag{2}$$

$$(NS_\eta)_\eta + fS_\eta - \alpha_1(\xi)SF = 2\xi(FS_\xi - f_\xi S_\eta) \tag{3}$$

$$(N\,Pr^{-1} G_\eta)_\eta + fG_\eta + NEc(u_e / u_\infty)^2 (F_\eta^2 + \lambda S_\eta^2) = 2\xi(FG_\xi - f_\xi G_\eta) \tag{4}$$

with boundary conditions

$$\begin{aligned} &F(\xi,0) = 0 \quad S(\xi,0) = 1 \quad G(\xi,0) = 0 \\ &F(\xi,\infty) = 1 \quad S(\xi,\infty) = 0 \quad G(\xi,\infty) = 1 \end{aligned} \tag{5}$$

$$N = \frac{\mu}{\mu_\infty} = \frac{b_1 + b_2 T_\infty}{b_1 + b_2 T} = \frac{1}{a_1 + a_2 G}, \qquad Pr = \frac{1}{c_1 + c_2 T} = \frac{1}{a_3 + a_4 G}$$

$$\Delta T_w = T_w - T_\infty, \qquad u/u_e = f_\eta = F, \qquad \beta(\xi) = \frac{2\xi}{u_e}\frac{du_e}{d\xi}, \qquad \alpha_1(\xi) = \frac{4\xi}{r}\frac{dr}{d\xi}$$

$$a_1 = \frac{b_1 + b_2 T_w}{b_1 + b_2 T_\infty}, \quad a_2 = \frac{b_2 (T_\infty - T_w)}{b_1 + b_2 T_\infty}, \quad a_3 = c_1 + c_2 T_w, \quad a_4 = c_2 (T_\infty - T_w)$$

$$\alpha(\xi) = \frac{2\xi}{r}\lambda r_\xi, \qquad \lambda = \left(\frac{\Omega_0 r}{u_e}\right)^2, \qquad Ec = \frac{u_\infty^2}{C_p (\Delta T_w)}$$

$$w = (r/L)(2\xi\, Re_L)^{1/2} u_e \left[f + 2\xi f_\xi + (\beta(\xi) + \alpha_1(\xi)/2 \quad 1)\eta F \right]$$

$$f = \int_0^\eta F d\eta + f_w , \quad f_w$$

$$= (\xi)^{1/2} (Re_L/2)^{1/2} \int_0^x (w_w/u_\infty)(r/L) d(x/L) \qquad (6)$$

To achieve the dimensionless governing equation the following non-similarity transformation is used,

$$\xi = \int (u_e/u_\infty)(r/L)^2 d(x/L), \; \eta = (Re_L/2\xi)^{1/2} (u_e/u_\infty)(r/L)(z/L), \; G = \frac{T \quad T_w}{T_\infty \quad T_w}$$

$$\psi(x,z) = u_\infty L (2\xi/Re_L)^{1/2} f(\xi,\eta), \qquad u = (L/r)\psi_z , \qquad w = \quad (L/r)\psi_x$$

$$v(x,z) = \Omega_0 r(x) s(\xi,\eta)$$

Here f is dimensionless stream function; f_w is the surface stream function value; F and S are dimensionless velocity components in x and y directions respectively; N is the viscosity ratio; $\beta(\xi)$ and Ec are pressure gradient and dissipation (Eckert number) parameters respectively; λ is rotation parameter. For a sphere, the unsteadiness, as well as non-similarity is both due to the external velocity at the edge of the boundary layer, angular velocity of the sphere and curvature of the body. The set of equations (2)-(4) reduces to that of the classical non-similar unsteady flow over a stationary sphere for $\lambda = 0$. Hence equation (3) becomes redundant as the velocity component in the y-direction is zero $v = 0$ (i.e. $S = 0$) for $\lambda = 0$.

The free-stream velocity distribution for the case of axi-symmetric flow over a rotating sphere and the distance from the axis of the body are given by,

$$u_e/u_\infty = \frac{3}{2} \sin \bar{x}, \qquad \bar{x} = \frac{x}{L}, \qquad \frac{r}{L} = \sin \bar{x} \qquad (7)$$

The expressions for $\xi, \beta, \alpha, f_w, \lambda$ and α_1 can be written, respectively, as

$$\xi = \left[(1 - \cos \bar{x})^2 (2 + \cos \bar{x}) \right] / 2, \qquad \beta = (2/3) \left[\cos \bar{x} (2 + \cos \bar{x}) / (1 + \cos \bar{x})^2 \right] \qquad (8)$$

$$\lambda = \frac{4}{9} (\Omega_0 L/u_\infty)^2, \qquad \alpha_1 = 2\beta, \qquad \alpha = \lambda\beta$$

Here α_1 and α are the dimensionless parameters.

Assume $M\left(\bar{x},\bar{x}_0\right)=\dfrac{\sin\left\{\left(\omega^*-1\right)\bar{x}-\omega^*\bar{x}_0\right\}+\sin\bar{x}_0}{\omega^*-1}-\dfrac{\sin\left\{\left(\omega^*+1\right)\bar{x}-\omega^*\bar{x}_0\right\}-\sin\bar{x}_0}{\omega^*+1}$,

$$P_1=1-\cos\bar{x},\qquad P_2=1+\cos\bar{x},\qquad P_3=2+\cos\bar{x} \tag{9}$$

$$f_w=\left\{\begin{array}{ll} 0 & \\ AP_1^{-1}\left(P_3\right)^{-1/2}M\left(\bar{x},\bar{x}_0\right), & \bar{x}_0\le\bar{x}\le\bar{x}_0^* \\ AP_1^{\ 1}\left(P_3\right)^{-1/2}M\left(\bar{x}_0^*,\bar{x}_0\right), & \bar{x}_0^*\le\bar{x}\le\bar{x}_1 \\ AP_1^{\ 1}\left(P_3\right)^{-1/2}\left[M\left(\bar{x}_0^*,\bar{x}_0\right)+M\left(\bar{x},\bar{x}_1\right)\right], & \bar{x}_1\le\bar{x}\le\bar{x}_1^* \\ AP_1^{\ 1}\left(P_3\right)^{-1/2}\left[M\left(\bar{x}_0^*,\bar{x}_0\right)+M\left(\bar{x}_1^*,\bar{x}_1\right)\right], & \bar{x}\ge\bar{x}_1^* \end{array}\right\} \tag{10}$$

Here $w_w\left(\bar{x}\right)$ is taken

$$\text{as } w_w=\left\{\begin{array}{ll} 0 & \bar{x}\le\bar{x}_0 \\ u_\infty\left(Re_L/2\right)^{-1/2}2^{1/2}A\omega^*\sin\left\{\omega^*\left(\bar{x}\quad\bar{x}_0\right)\right\}, & \bar{x}_0\le\bar{x}\le\bar{x}_0^* \\ 0 & \bar{x}_0\le\bar{x}\le\bar{x}_0^* \\ u_\infty\left(Re_L/2\right)^{-1/2}2^{1/2}A\omega^*\sin\left\{\omega^*\left(\bar{x}\quad\bar{x}_1\right)\right\}, & \bar{x}_1\le\bar{x}\le\bar{x}_1^* \\ 0 & \bar{x}\ge\bar{x}_1^* \end{array}\right\}$$

where ω^*, $\left(\bar{x}_0,\bar{x}_1\right)$ are the free parameters which determine the slot length and slot locations, respectively. The function $w_w\left(\bar{x}\right)$ is a continuous function for all values of $\bar{x}$ representing velocity at wall (i.e. $\eta=0$) and it has non-zero values only in the intervals $\left[\bar{x}_0,\bar{x}_0^*\right],\left[\bar{x}_1,\bar{x}_1^*\right]$ and zero value at all points outside the interval. The reason for taking such a function is that it allows the mass transfer to change slowly in the neighborhood of the leading and the trailing edges of the slots. The parameter $A>0$ or $A<0$ according to whether there is a suction or an injection. It is convenient to express eqns. (2) - (4) in terms of $\bar{x}$ instead of ξ. Equation (8) gives the relation between ξ and $\bar{x}$ as

$$\xi\frac{\partial}{\partial\xi}=B\left(\bar{x}\right)\frac{\partial}{\partial\bar{x}},\qquad \text{where } B\left(\bar{x}\right)=3^{\ 1}\tan\left(\bar{x}/2\right)P_3P_2^{\ 1} \tag{11}$$

The skin-friction and heat transfer coefficients at the wall can be expressed in the form

$$C_f\left(Re_L\right)^{1/2} = \frac{9}{2}\sin \bar{x} P_2 P_3^{-1/2} N_w \left(F_\eta\right)_w \tag{12}$$

$$\bar{C}_f\left(Re_L\right)^{1/2} = \frac{9}{2}\lambda^{1/2}\sin \bar{x} P_2 P_3^{-1/2} N_w \left(S_\eta\right)_w \tag{13}$$

$$Nu\left(Re_L\right)^{-1/2} = \frac{3}{2} P_2 P_3^{-1/2}\left(G_\eta\right)_w \tag{14}$$

where,

$$C_f = \frac{2\left(\mu\left(\partial u/\partial z\right)\right)_w}{\rho u_\infty^2},\quad \bar{C}_f = \frac{2\left(\mu\left(\partial v/\partial z\right)\right)_w}{\rho u_\infty^2},\quad N_w = \frac{1}{a_1 + a_2 G_w} = \frac{1}{a_1},\; Nu = \frac{L\left(\partial T/\partial z\right)_w}{T_\infty - T_w}.$$

3. Solution Method and Results and Discussions

The set of equations (2)-(4) simplified in terms of $\bar{x}$, with the boundary conditions (5), have been solved numerically using an implicit finite difference scheme in combination with the quasilinearization method as discussed by Bellman and Inouye and Tate in [18] and [19] respectively. Then it has been further simplified to a system of linear algebraic equations with block tri-diagonal structure which have been solved using Vargas's algorithm [20]. The step size in the $\eta-$direction has been chosen as $\Delta\eta = 0.01$ throughout the computations as it has been found that a further decrease $\Delta\eta$ does not change the results up to the fourth decimal place. In the $\bar{x}$-direction, $\Delta\bar{x} = 0.01$ has been used for small values of $\bar{x} < 0.5$, then it has been decreased $\Delta\bar{x} = 0.005$. This value of $\Delta\bar{x}$ has been used for $\Delta\bar{x} < 1.2$, thereafter the step size has been reduced further, ultimately choosing a value $\Delta\bar{x} = 0.0001$ in the neighborhood of the point of zero skin friction. The solutions is assumed to have converged and the iterative process is terminated when,

$$Max\left\{\left|\left(F_\eta\right)_w^{k+1} - \left(F_\eta\right)_w^k\right|,\ \left|\left(S_\eta\right)_w^{k+1} - \left(S_\eta\right)_w^k\right|,\ \left|\left(G_\eta\right)_w^{k+1} - \left(G_\eta\right)_w^k\right|\right\} < 10^{-4}.$$

The solution has been obtained starting from the origin of streamwise coordinate to the exact point of separation. Computations were carried out for various values of $A\left(0 \le A \le -0.4\right), \lambda\left(0 \le \lambda \le 1\right)$ and viscous dissipation $Ec\left(0 \le Ec \le 1\right)$. The effect of non-uniform double slot injection is discussed in **Figure. 2.** The double slots are positioned at $\left(\bar{x}_0 = 0.3,\ \bar{x}_0^* = 0.8\right)$ and $\left(\bar{x}_1 = 0.9,\ \bar{x}_1^* = 1.4\right)$ with $Ec = 0, \lambda = 0$. It is observed form the figure that the separation point moves upstream with the increasing value of injection. And also the effect of injection is just opposite to the effect suction which is already discussed by Roy and Saikrishnan [8] for non-uniform single slot case. Also Saikrishnan and Roy [21] studied that the point of separation moves upstream when rotation and viscous dissipation values increase when there is no mass transfer $(A = 0)$.

The effects of rotation parameter λ on velocity gradients $\left((F_\eta)_w, (-S_\eta)_w\right)$ and temperature gradient $(G_\eta)_w$ with the influence of non-uniform double slot injection $(A = \ \ 0.2)$. and viscous dissipation $Ec = 1$ in the flow have been investigated in **Figure.3.** It is observed from the figure that, as rotation increases, the point of separation moves downstream with the influence of injection while the point of separation moves upstream without the influence of injection (i.e. $(A = 0)$).

In **Figure. 4.,** the effect of viscous dissipation on velocity gradients $\left((F_\eta)_w, (-S_\eta)_w\right)$ and temperature gradient $(G_\eta)_w$ with the influence of non-uniform double slot injection $(A = -0.2)$ and rotation $(\lambda = 1)$ is presented. It is noted from the figure that when the viscous dissipation value increases the point of separation moves downstream with the influence of injection while the point of separation moves upstream without the influence of injection (i.e., $(A = 0)$.).

It is noticed from **Figure.5,** that the non-uniform double slot injection $(A = -0.4)$ is applied in the slots at the positions $\overline{x_0} = 0.3$, $\overline{x_1} = 0.9$ with the rotation parameter $\lambda = 1$. The point of separation moves upstream when the positions of the slots are moved further downstream. Therefore the downstream movements of the double slot injection speed up separation with rotation.

4. Conclusions:

- Increasing value of injection and downstream movement of the double slots speed up the separation.
- The separation is delayed with the increasing value of viscous dissipation with the influence of non-uniform double slot injection.
- The separation is delayed with the increasing value of rotation with the influence of non-uniform double slot injection.

5. Figures:

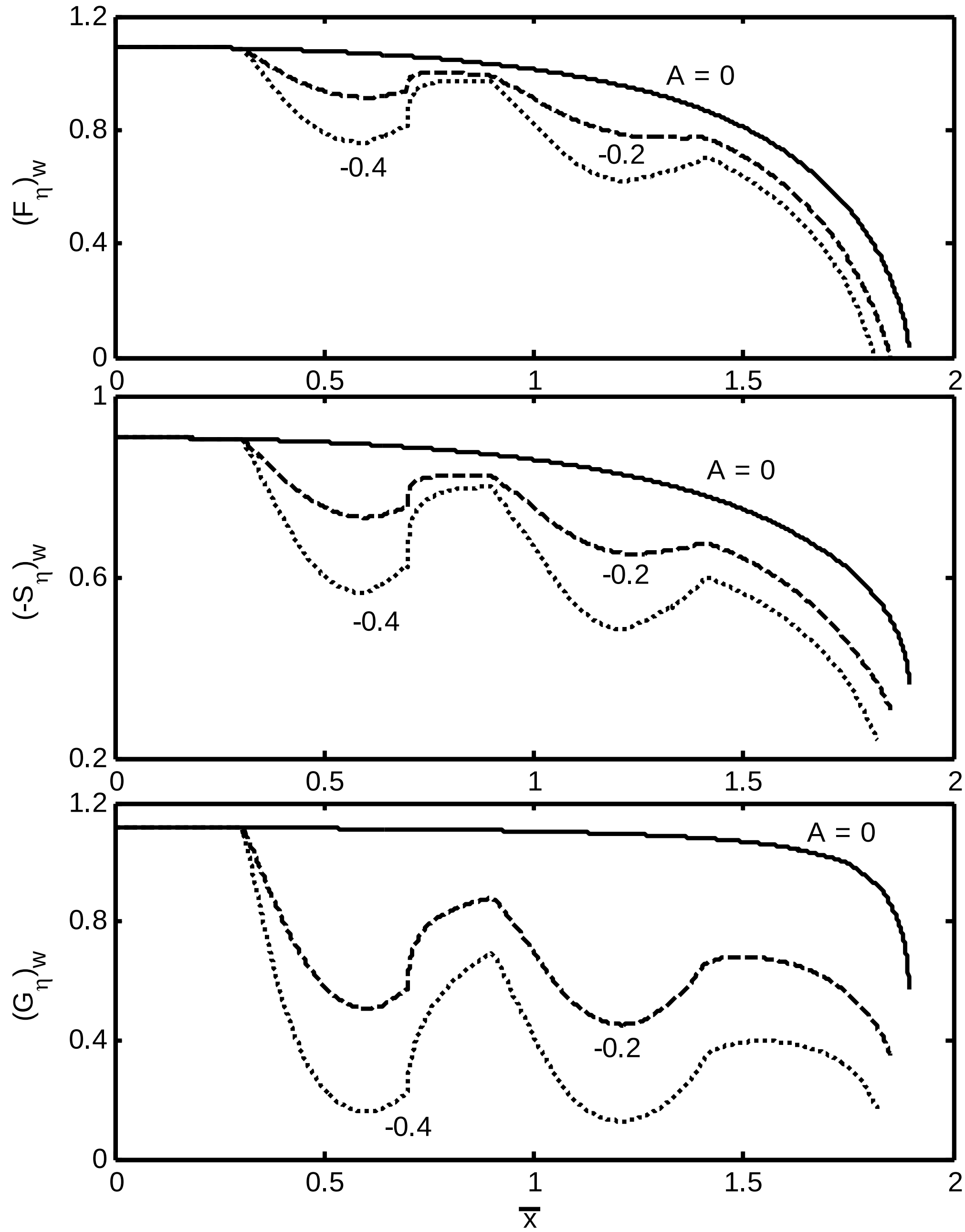

Figure 2: Effect of non-uniform injection $(A = 0)$ through double slot on velocity and temperature gradients $\left((F_\eta)_w, (-S_\eta)_w, (G_\eta)_w\right)$ when $T_\infty = 18.7$, $T_w = 28.7$, $Ec = 0.0$, $\lambda = 0$ and $\omega^* = 2\pi$. Slot locations at $\bar{x}_0 = 0.3$, $\bar{x}_1 = 0.9$.

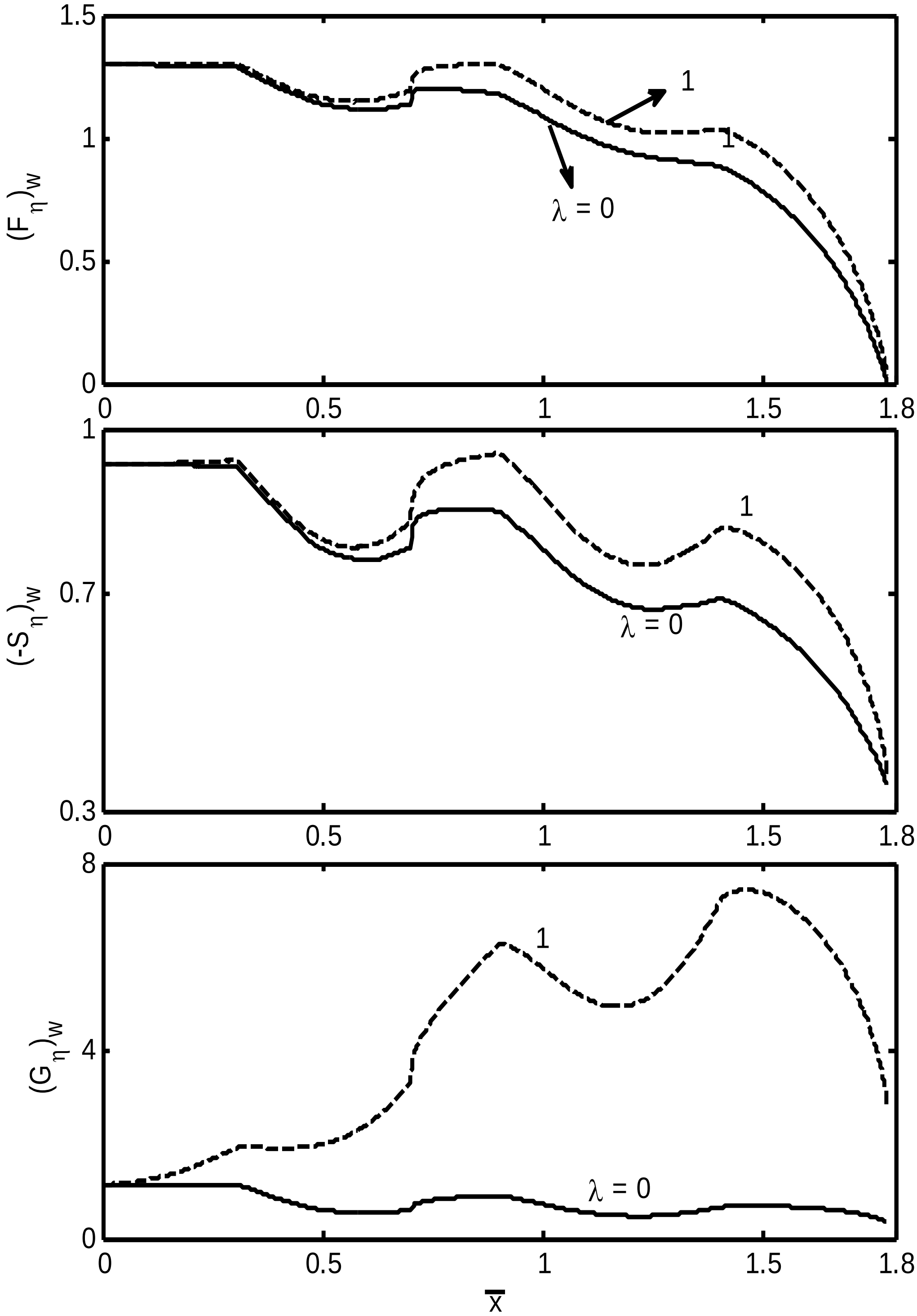

Figure 3: Effect of rotation (λ) on velocity and temperature gradients $\left((F_\eta)_w, (-S_\eta)_w, (G_\eta)_w\right)$ when $T_\infty = 18.7, T_w = 28.7, Ec = 1.0$ with the influence of non-uniform injection $(A = -0.2)$. through double slot. Slot locations at $\bar{x}_0 = 0.3$, $\bar{x}_1 = 0.9$, $\omega^* = 2\pi$.

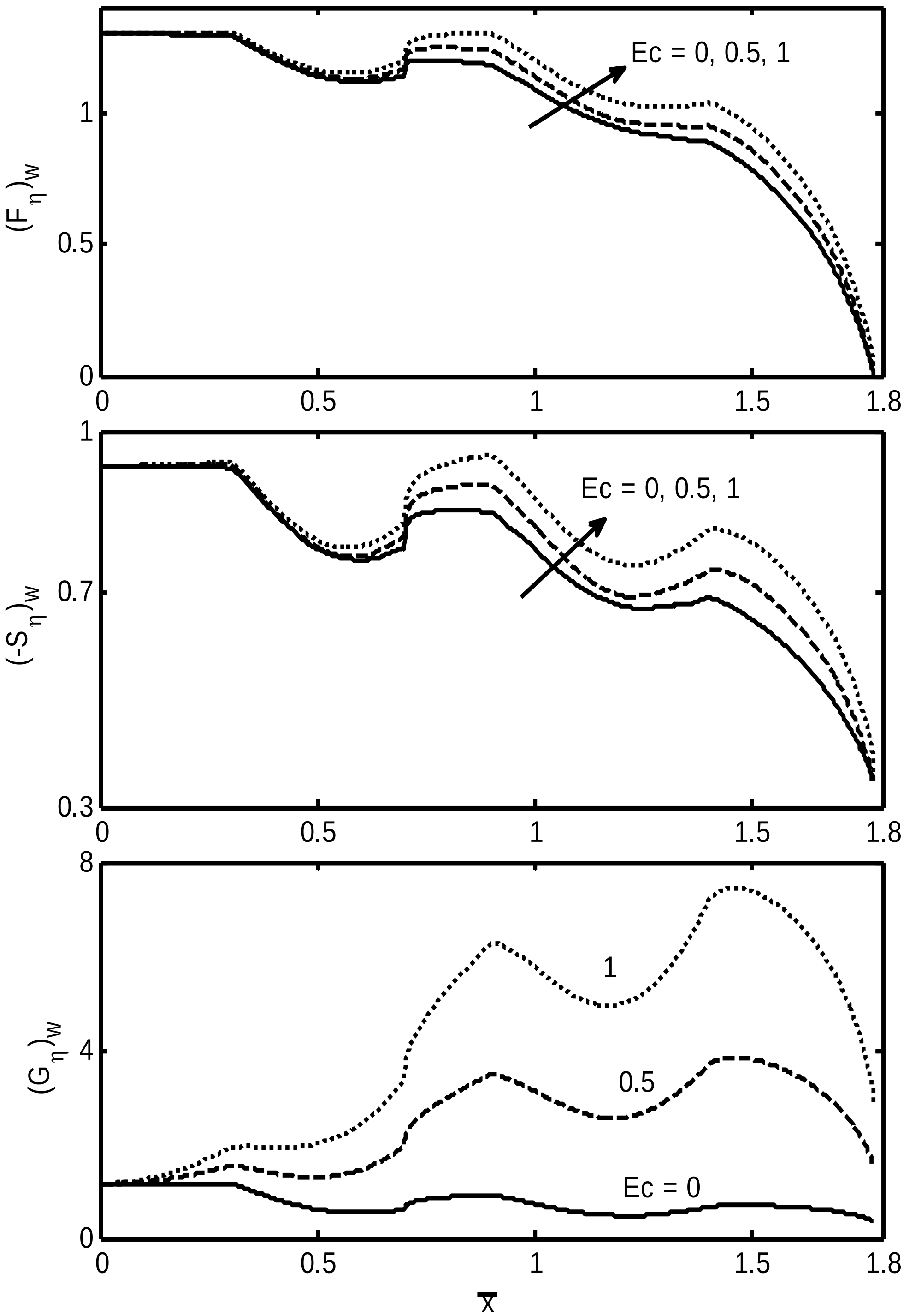

Figure 4: Effect of viscous dissipation (Ec) on velocity and temperature gradients $(F_\eta)_w$, $(-S_\eta)_w$, (G_η) when $T_\infty = 18.7, T_w = 28.7, \lambda = 1.0$ and with the influence of non-uniform injection $(A = -0.2)$ through double slot. Slot locations at $\overline{x_0} = 0.3, \overline{x_1} = 0.9, \omega^* = 2\pi$.

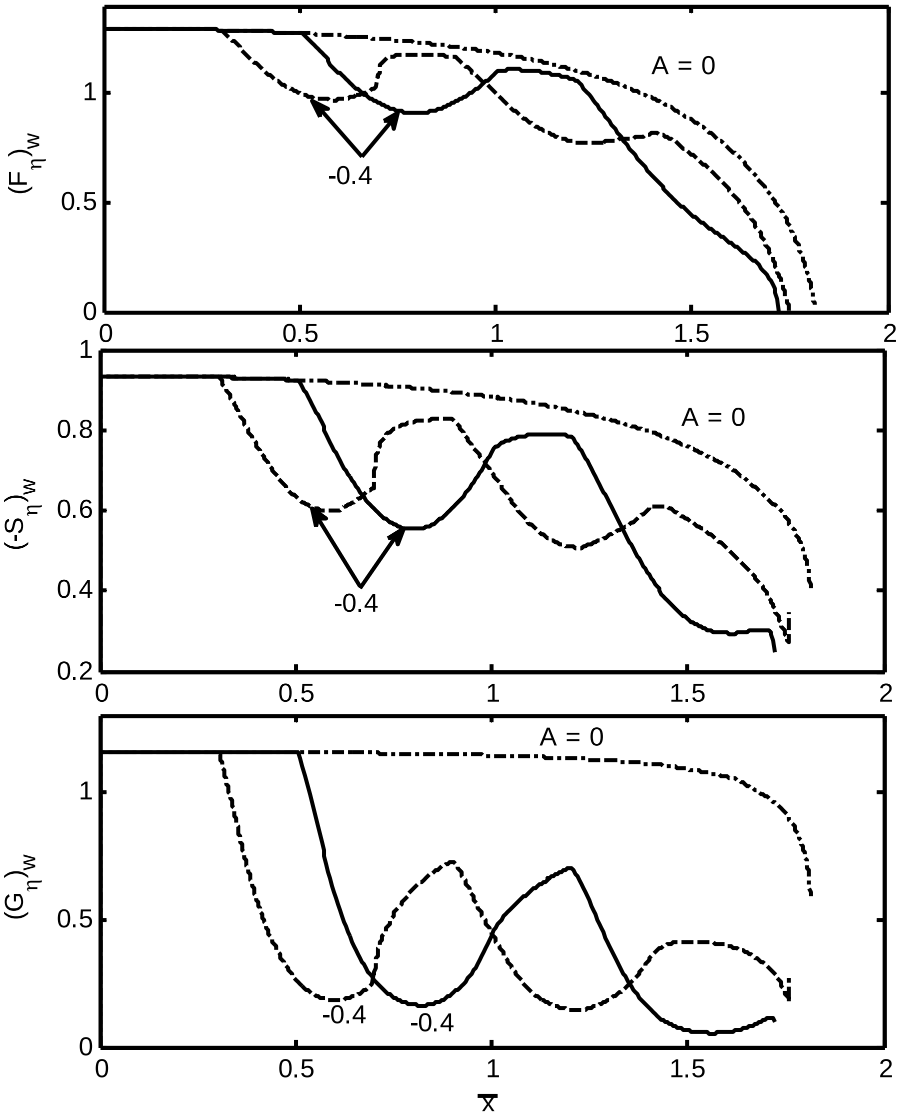

Figure 5: Effect of downstream movement of the slot (through which non-uniform injection have been applied) on velocity and temperature gradients $(F_\eta)_{w}$, $(-S_\eta)_{w}$, (G_η) when $T_\infty = 18.7$, $T_w = 28.7$, $\lambda = 1.0$, $\omega^* = 2\pi$, $Ec = 0$, $A = -0.4$ and $t^* = 0$. Slot locations at $\overline{x_0} = 0.3$, $\overline{x_1} = 0.9$ and $\overline{x_0} = 0.5$, $\overline{x_1} = 1.2$.

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