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Domination Numbers on the Boolean Function Graph $B(\overline{K_p}, NINC, \overline{L}(G))$ of a Graph

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Abstract: *For any graph G, let V(G) and E(G) denote the vertex set and edge set of G respectively. The Boolean function graph $B(\overline{K_p}, NINC, \overline{L}(G))$ of G is a graph with vertex set $V(G)\cup E(G)$ and two vertices in $B(\overline{K_p}, NINC, \overline{L}(G))$ are adjacent if and only if they correspond to two nonadjacent edges of G or to a vertex and an edge not incident to it in G, where L(G) is the line graph of G. For brevity, this graph is denoted by $B_3(G)$. In this paper, various domination numbers of $B_3(G)$ are determined.*

Keywords: *Boolean Function Graph, domination number.*

1. Introduction

Graphs discussed in this paper are undirected and simple graphs. For a graph G, let V(G) and E(G) denote its vertex set and edge set respectively. For a connected graph G, the *eccentricity* $e_G(v)$ of a vertex v in G is the distance to a vertex farthest from v. Thus, $e_G(v) = \{d_G(u, v) : u\in V(G)\}$, where $d_G(u, v)$ is the distance between u and v in G. We denote the eccentricity of vertex v in G as e(v) and the distance between two vertices u, v in G as d(u, v),. The minimum and maximum eccentricities are the *radius and diameter* of G, denoted r(G) and diam(G) respectively. The *neighborhood* $N_G(v)$ of a vertex v is the set of all vertices adjacent to v in G. The set $N_G[v] = N_G(v)\cup\{v\}$ is called the closed neighborhood of v. A set S of edges in a graph G is said to be independent, if no two of the edges in S are adjacent. A set of independent edges covering all the vertices of a graph G is called *perfect matching*. An edge e = (u, v) is a *dominating edge* in a graph G, if every vertex of G is adjacent to at least one of u and v.

The concept of domination in graphs was introduced by Ore [17]. A set $D \subseteq V(G)$ is said to be a *dominating set* of G, if every vertex in $V(G)-D$ is adjacent to some vertex in D. D is said to be a minimal dominating set if $D-\{u\}$ is not a dominating

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set, for any $u \in D$. The *domination number* $\gamma(G)$ of G is the minimum cardinality of a dominating set. We call a set of vertices a γ-set, if it is a dominating set with cardinality $\gamma(G)$. Different types of dominating sets have been studied by imposing conditions on the dominating sets. A dominating set D is called a *connected* (*independent*) *dominating* set, if the induced subgraph <D> is connected [21] (independent) . D is called a total dominating set, if every vertex in V(G) is adjacent to some vertex in D [1]. A dominating set D is called a *cycle dominating set*, if the subgraph <D> has a Hamiltonian cycle and is called a *perfect dominating set*, if every vertex in $V(G)-D$ is adjacent to exactly one vertex in D [2]. D is called a *restrained dominating set*, if every vertex in $V(G)-D$ is adjacent to another vertex in $V(G)-D$ [3]. By γ_c, γ_i, γ_t, γ_0, γ_p and γ_r, we mean the minimum cardinality of a connected dominating set, independent dominating set, total dominating set, cycle dominating set, perfect dominating set and restrained dominating set respectively.

Sampathkumar and Pushpalatha [20] introduced the concept of point-set domination number of a graph. A set $D \subseteq V(G)$ is called a *point-set dominating set* (psd-set), if for every set $T \subseteq V(G)-D$, there exists a vertex $v \in D$ such that the subgraph $\langle T \cup \{v\} \rangle$ induced by $T \cup \{v\}$ is connected. The point-set domination number $\gamma_{ps}(G)$ is the minimum cardinality of a psd-set of G. Kulli and Janakiram introduced the concept of split [15] and non-split [16] domination in graphs. A dominating set D of a connected graph G is a *split* (*non-split*) *dominating set*, if the induced subgraph $\langle V(G)-D \rangle$ is disconnected (connected). The split (non-split) domination number $\gamma_s(G)$ ($\gamma_{ns}(G)$) of G is the minimum cardinality of a split(non-split) dominating set.

Sampathkumar[19] introduced the concept of global domination in graphs. Kulli and Janakiram [14] introduced the concept of total global domination in graphs. Pushpalatha [18] introduced the concept of global point-set domination in graphs.

A dominating set of G is a *global dominating set* [19], if it is a dominating set of both G and its complement $\overline{G}$.

A total dominating set T of G is a *total global dominating set* [14] (t.g.d. set), if T is also a total dominating set of $\overline{G}$. For a co-connected graph G = (V, E), a set $D \subseteq V$ is said to be a *global point set dominating set* [18], if it is a psd-set of both G and $\overline{G}$. The *global domination number* γ_g of G *is* defined as the minimum cardinality of a global dominating set. The *total global dominating number* $\gamma_{tg}(G)$ of G and *global point set domination number* γ_{pg} of G is defined similarly.

Using L(G), the line graph of G, G, incident and non-incident, complementary operations, complete and totally disconnected structures, one can get thirty-two graph operations. As already total graphs, semi-total edge graphs, semi-total vertex graphs and quasi-total graphs and their complements (8 graphs) are defined and studied, Janakiraman, Muthammai and Bhanumathi [5 – 13] studied all other similar remaining graph operations and called as Boolean Function Graphs.

The *Boolean function graph* B($\overline{K_p}$, NINC, $\overline{L}$(G)) G is a graph with vertex set V(G)∪E(G) and two vertices in B($\overline{K_p}$, NINC, $\overline{L}$(G)) are adjacent if and only if they correspond to two nonadjacent edges of G or to a vertex and an edge not incident to it in G. For brevity, this graph is denoted by $B_3(G)$, where L(G) is the line graph of G. The vertices of G and L(G) in $B_3(G)$ are referred as point and line vertices respectively. The line vertex in $B_3(G)$ corresponding to an edge e in G is denoted by e′. In this paper, we determine the various domination numbers for the graph $B_3(G)$.

2. Prior Results

In this section, we list some results with indicated references, which will be used in the subsequent main results. Let G be any (p, q) graph.

Theorem 2.1 [20]:

Let G = (V, E) be a graph. A set S ⊆ V is a point-set dominating set of G if and only if for every independent set W in V−S, there exists a vertex u in S such that W ⊆ N_G(u)∩(V−S).

Theorem 2.2 [14]:

A total dominating set T of G is a total global dominating set if and only if for each vertex v∈V, there exists a vertex u∈T such that v is not adjacent to u.

Theorem 2.3 [18]:

For a graph G, a set S ⊆ V(G) is a global point-set dominating set if and only if the following conditions are satisfied.

(i). For every independent set W in V−S, there exists u in S such that W ⊆ N(u)∩(V−S) in G; and

(ii). For every set D ⊆ V−S such that <D> is complete in G, there exists v in S such that D∩N(v) = ϕ in G.

Observation 2.1 [13]:

1. $\overline{L}$(G) is an induced subgraph of $B_3(G)$ and subgraph of $B_3(G)$ induced by point vertices is totally disconnected.
2. If $d_i = \deg_G(v_i)$, $v_i \in V(G)$, then the number of edges in $B_3(G)$ is $(q/2)(2p + q - 3) - (1/2)\Sigma_{1\le i\le p} d_i^2$.

3. The degree of a point vertex v in $B_3(G)$ is q - $deg_G(v)$ and the degree of a line vertex e' in $B_3(G)$ is $\deg_{\overline{L(G)}}(e') + p - 2 = p + q - \deg_{L(G)}(e') - 3$ and hence $\delta(B_3(G)) = q - \Delta(G)$.
4. $B_3(G)$ contains isolated vertices if and only if G is one of the following graphs. K_2, $K_{1,n}$, $n \geq 2$, nK_1, $K_2 \cup nK_1$, $K_{1,m} \cup nK_1$, where $n \geq 1$ and $m \geq 2$.
5. $B_3(G)$ is totally disconnected if and only if $G \cong nK_1$ or K_2, $n \geq 1$.
6. $B_3(G)$ is disconnected if and only if $G \cong nK_1$, $K_2 \cup mK_1$, $C_3 \cup mK_1$ and $K_{1,n} \cup mK_1$, for $m \geq 0$ and $n \geq 2$.

3. Main results

Domination, Cycle, Connected and Total domination numbers in $B_3(G)$

In the following, we find the graphs G for which the domination number γ of $B_3(G)$ is 2 or 3.

Proposition 3.1:

For any graph G having at least one edge, $\gamma(B_3(G)) \geq 2$.

Proof:

Since, there is no vertex of degree p + q – 1 in $B_3(G)$, the proposition follows.

Lemma 3.2:

Any 2-set of $B_3(G)$ containing either two point vertices or a point vertex and a line
vertex is not a dominating set of $B_3(G)$.

Proof: Let D be a 2-set of $B_3(G)$.

Case(i). D contains two point vertices.

Let D = $\{v_1, v_2\}$, where v_1, v_2 are any two point vertices in $B_3(G)$. Then $v_1, v_2 \in V(G)$. If v_1 and v_2 are not adjacent in G, then since G contains at least one edge and the subgraph of $B_3(G)$ induced by point vertices is totally disconnected, D is not a dominating set of $B_3(G)$. If v_1 and v_2 are adjacent in G, then the line vertex in $B_3(G)$ corresponding to the edge joining v_1 and v_2 is not adjacent to any of the vertices in D. Hence, D is not a dominating set of $B_3(G)$.

Case(ii): D contains one point and one line vertex.

Let e' be the line vertex in D and e be the corresponding edge in G. Then the point vertex in $B_3(G)$ corresponding to at least one of the end vertices of e is not adjacent to any of the vertices in D and hence D is not a dominating set of $B_3(G)$. Hence, the lemma follows.

Theorem 3.3:

$\gamma(B_3(G)) = 2$ if and only if there exists a set of two independent edges e_1 and e_2 in G such that no edge in G is adjacent to both e_1 and e_2.

Proof:

Assume $\gamma(B_3(G)) = 2$. Then there exists a minimal dominating set for $B_3(G)$ containing two vertices. By Lemma 3.2., if there is a dominating 2-set for $B_3(G)$, then that set contains only line vertices of $B_3(G)$, the edges in G corresponding to these line vertices are nonadjacent in G. If the edges are adjacent, then the point vertex in $B_3(G)$ corresponding to the common vertex is not dominated. Let $D = \{e_1', e_2'\}$ be a minimal dominating set for $B_3(G)$, where the edges e_1, e_2 in G corresponding to these line vertices are independent edges. If there exists an edge e in G adjacent to both e_1 and e_2, then the line vertex in $B_3(G)$ corresponding to the edge e will not be adjacent to any of the vertices e_1', e_2' in D, which is a contradiction. Hence, there exists no edge in G adjacent to both e_1 and e_2. Converse follows easily.

Remark 3.4:

The set D is also a connected (total) dominating set of $B_3(G)$.

Proposition 3.5:

If G is not totally disconnected, then $\gamma(B_3(G)) \leq 3$.

Proof:

Since G is not totally disconnected, there exists an edge e = (u, v) in G, where u, $v \in V(G)$. Let e' be the line vertex in $B_3(G)$ corresponding to the edge e. Then $D = \{u, v, e'\} \subseteq V(B_3(G))$ is a dominating set of $B_3(G)$. Hence, $\gamma(B_3(G)) \leq 3$.

Remark 3.6:

(i) The set D is also an independent dominating set of $B_3(G)$ and hence $\gamma_i(B_3(G)) \leq 3$. Also any 2-set containing independent vertices in $B_3(G)$ is not a dominating set of $B_3(G)$ and $\gamma_i(B_3(G)) \geq 3$. Thus, $\gamma_i(B_3(G)) = 3$ if and only if G is not totally disconnected.

(ii) By Theorem 3.3. and Proposition 3.5., it follows that $\gamma(B_3(G)) = 2$ or 3.

Next, the cycle domination number γ_0 of $B_3(G)$ is determined. Here, the graphs G such that $B_3(G)$ is connected and contain cycles are considered.

Proposition 3.7:

For any (p, q) graph G with $p \geq 5$ and $\beta_1(G) \geq 2$, $\gamma_0(B_3(G)) = 3$.

Proof:

Let e_1, e_2 be any two independent edges in G and v be a vertex in G not incident with both e_1 and e_2. If e_1' and e_2' are the line vertices in $B_3(G)$ corresponding to the edges e_1 and e_2 respectively, then $D = \{v, e_1', e_2'\} \subseteq V(B_3(G))$ is a 3-cycle dominating set of $B_3(G)$ and hence $\gamma_0(B_3(G)) = 3$.

Note :

If $\beta_1(G) = 1$, then $B_3(G)$ is disconnected

Proposition 3.8:

If $p = 4$ and $\beta_1(G) = 2$, then $\gamma_0(B_3(G)) = 5$.

Proof:

Since $B_3(G)$ is connected, $p = 4$ and $\beta_1(G) = 2$, G contains P_4 as a sub graph. Then the set D of line vertices in $B_3(G)$ corresponding to the edges in P_4 together with the end vertices in P_4 is a cycle dominating set of $B_3(G)$ and

$\langle D\rangle \cong C_5$ in $B_3(G)$. Thus, $\gamma_0(B_3(G)) \leq 5$. Also, $B_3(G)$ contains neither C_3 nor C_4, when $p = 4$. Hence, $\gamma_0(B_3(G)) = 5$.

Remark 3.9:

1. From Proposition 3.8., it follows that, if G is any (p, q) graph with $p \geq 5$ and $\beta_1(G) \geq 2$,

then $\gamma_c(B_3(G)) \leq 3$. This bound is attained, if G is a cycle on five vertices.

2. If $p = 4$ and $\beta_1(G) = 2$, then $\gamma_c(B_3(G))$ = 3, if $G \cong P_4$ or $K_{1,3} + e$.

= 4, otherwise.

3. Let G be any graph such that for every pair of independent edges e_1 and e_2 in G, there exist at least one edge adjacent to both e_1 and e_2. Then $\gamma_c(B_3(G)) = 3$ if and only if G contains $2K_2 \cup K_1$ as a sub graph, G is a path on four vertices or $G \cong K_{1,3} + e$.

In the following, the total domination number γ_t of $B_3(G)$ is obtained. The following propositions are stated without proof, since the proofs are similar to above theorems.

Proposition 3.10:

$\gamma_t(B_3(G)) = 2$ if and only if there exists two independent edges e_1 and e_2 in G such that no edge in G is adjacent to both e_1 and e_2.

Proposition 3.11:

$\gamma_t(B_3(G)) = 3$ if and only if either G contains $2K_2 \cup K_1$ as a sub graph or G contains three edges e_1, e_2 and e_3 such that $<\{e_1, e_2, e_3\}> \neq K_{1,3}$.

Proposition 3.12:

Let G be any graph having a perfect matching containing at least three edges. Then the set of all line vertices in $B_3(G)$ corresponding to the edges in the perfect matching is a total dominating set of $B_3(G)$.

Proposition 3.13:

Let G be a graph that is neither a star nor a cycle on three vertices and let D be a line cover for G with $|D| = \alpha_1(G)$. Then the set of all line vertices in $B_3(G)$ corresponding to the edges in D is a total dominating set of $B_3(G)$.

Perfect, Point set and Restrained domination numbers in $B_3(G)$

In the following, the graphs G for which the perfect domination number γ_p of $B_3(G)$ is 2 or 3 are obtained.

Theorem 3.14:

Let G be any graph such that there exists a pair of independent edges in G such that no edge in G is adjacent to both of these independent edges. Then $\gamma_p(B_3(G)) = 2$ if and only if $G \cong 2K_2$.

Proof:

Assume $\gamma_p(B_3(G)) = 2$. Then there exists a perfect dominating set D of $B_3(G)$ containing two vertices. Since D is a dominating set of $B_3(G)$, D contains line vertices only such that the corresponding edges, say e_1 and e_2 in G are independent and there exists no edge in G adjacent to both e_1 and e_2. Let

e_1', e_2' be the line vertices in $B_3(G)$ corresponding to e_1 and e_2. Then $D = \{e_1', e_2'\}$ is a dominating set of $B_3(G)$. If there exists a vertex in G adjacent to at least one of the end vertices of e_1 and e_2, then the corresponding point vertex is adjacent to both e_1' and e_2'. Thus, D is not a perfect dominating set of $B_3(G)$ and hence $G \cong 2K_2$. Converse follows easily.

Theorem 3.15:

Let G be any graph, not totally disconnected. Then $\gamma_p(B_3(G)) = 3$ if and only if one of the following holds.

(i). G is a graph on three vertices.

(ii). There exists an edge e in G such that all the edges of G are adjacent to e. That is, r(L(G)) = 1.

(iii). There exists two independent edges e_1 and e_2 and a vertex v incident with one of e_1 and e_2, say e_1 such that the edges of G incident with v are not adjacent to e_2 and the edges not incident with v are adjacent to both e_1 and e_2.

(iv). G contains $2K_2 \cup K_1$ as a sub graph such that all the edges of G are incident with the vertex in K_1 and adjacent to one of the edges of $2K_2$.

(v). G is $G_1 \cup K_1$, where G_1 is a graph on four vertices with $\beta_1(G_1) \geq 2$.

Proof:

Assume $\gamma_p(B_3(G)) = 3$. Then there exists a perfect dominating set D of $B_3(G)$ having three vertices.

Case(i). All the vertices of D are point vertices.

Since the set of point vertices in $B_3(G)$ is independent, $V(B_3(G)) - D$ does not contain any point vertex. Also, D is a perfect dominating set implies that the end vertices of the edges of G must be one of the three vertices in D. Hence, G is any graph on three vertices.

Case(ii). D contains two point vertices and one line vertex.

Let v_1, v_2 be the vertices and e be the edge in G corresponding to two point vertices and one line vertex in D respectively. If v_1 and v_2 are adjacent in G, then the corresponding edge must be e, otherwise D cannot be a dominating set of $B_3(G)$. Also since D is perfect dominating set, all the edges of G must be adjacent to e. Thus (ii) holds. Assume v_1 and v_2 are not adjacent and e = (u, v), where u, $v \in V(G)$. Then u, $v \in V(B_3(G)) - D$ are adjacent to none of the vertices in D, which is a contradiction.

Case(iii). D contains two line vertices and one point vertex.

Let e_1 and e_2 be the edges and v be a vertex in G corresponding to the two line vertices and one point vertex in D respectively. If e_1 and e_2 are adjacent, then v must be the common vertex, otherwise D cannot be a dominating set of $B_3(G)$. Since D is a perfect dominating set, $G \cong C_3$ or P_3. Assume e_1 and e_2 are nonadjacent.

Subcase(i). v is incident with at least one of e_1 and e_2, say e_1.

Then since D is a perfect dominating set of $B_3(G)$, the edges of G incident with v are not adjacent to e_2 and the edges not incident with v are adjacent to both e_1 and e_2. Hence, (iii) holds.

Subcase(ii). v is not incident with both e_1 and e_2.

Then either the edges of G are incident with v and adjacent to one of e_1 and e_2 or $G \cong G_1 \cup K_1$, where G_1 is a graph on four vertices with $\beta_1(G_1) \geq 2$. Thus, (iv) and (v) hold.

Case(iv). All the vertices of D are line vertices.

If the sub graph of G induced by the edges corresponding to the line vertices in D is C_3, then G $\cong C_3$.

The other cases are not possible since D is a perfect dominating set of $B_3(G)$. Converse can be proved easily.

Remark 3.16:

(i). Let G be a graph having at least four vertices. Then the set of all point vertices in $B_3(G)$ is not a perfect dominating set of $B_3(G)$.

(ii). Let G be a graph other than a star. If the set D of line vertices in $B_3(G)$ such that at least three of the corresponding edges are independent in G, then D is not a perfect dominating set of $B_3(G)$.

In the following, the point set domination number γ_{ps} of $B_3(G)$ is obtained by using Theorem 2.1. Here, the graphs G for which $B_3(G)$ is connected are considered.

Theorem 3.17:

The set of all point vertices in $B_3(G)$ is a point set dominating set (psd-set) of $B_3(G)$ if and only if $1 < r(G) \leq \infty$.

Proof:

Assume $1< r(G) \leq \infty$.

Case(i). G is connected with r(G) > 1.

Let D be the set of all point vertices in $B_3(G)$. Then D = V(G) and $V(B_3(G))–D = V(L(G))$. If W is an independent set in $V(B_3(G))–D$, then W contains line vertices in $B_3(G)$ such that the edges corresponding to these line vertices are mutually adjacent in G. Since $r(G) > 1$, there exists a vertex $v \in D$ such that $W \subseteq N(v) \cap (V(B_3(G))–D)$.

Case(ii). G is disconnected.

Then G contains at least two components. If W is an independent set in $V(B_3(G))–D$, then the edges corresponding to the line vertices in W belong to the same component of G. Hence, there exists a vertex $u \in D$, where u is one of the vertices in the remaining components of G such that $W \subseteq N(u) \cap (V(B_3(G))–D$. Thus, D is a psd-set of $B_3(G)$.

Conversely, assume r(G) = 1 and the set D of all point vertices in $B_3(G)$ is a psd-set of $B_3(G)$. Since r(G) = 1, there exists a vertex $v_1 \in V(G)$ such that $e_G(v_1) = 1$. Then the set W of all line vertices in $V(B_3(G))–D$ corresponding to the edges incident with v_1 is an independent set in $V(B_3(G))–D$ and there exists no vertex in D adjacent to all the line vertices in W, which is a contradiction. Hence, $1< r(G) \leq \infty$.

Proposition 3.18:

A sub set of $B_3(G)$ containing line vertices only is not a psd-set of $B_3(G)$.

Proof:

Let $D \subseteq V(B_3(G))$ contain only line vertices and

$W = \{v \in V(G)$. v is an end vertex of the edge corresponding to a line vertex in D$\}$. Then W is an independent set in $V(B_3(G))-D$ and there exists no vertex in D such that $W \subseteq N(u) \cap (V(B_3(G))-D$, which is a contradiction. Hence, the proposition follows.

Theorem 3.19:

Let G be a disconnected graph but not totally disconnected. Then $\gamma_{ps}(B_3(G)) = 3$ if and only if G contains K_2 as one of its components.

Proof:

By Theorem 3.3., $\gamma(B_3(G)) = 2$ if and only if there exists two independent edges e_1 and e_2 in G such that no edge in G is adjacent to both e_1 and e_2. Also, by Proposition 3.18., any subset of $V(B_3(G))$ containing line vertices only is not a psd-set of $B_3(G)$. Hence, $\gamma_{ps}(B_3(G)) \geq 3$. Let G contain K_2 as one of its components. Then the set of vertices of $B_3(G)$ corresponding to the vertices and the edge in K_2 is a psd-set of $B_3(G)$. Thus, $\gamma_{ps}(B_3(G)) = 3$. Conversely, assume $\gamma_{ps}(B_3(G)) = 3$. Then there exists a point set dominating set D of $B_3(G)$ containing three vertices.

Case(i). All the vertices of D are point vertices.

Since no two point vertices in $B_3(G)$ are adjacent, D = V(G) and r(G) >1. Also, D is a psd-set of $B_3(G)$ implies that $G \cong K_2 \cup K_1$.

Case(ii). D contains two point vertices and one line vertex.

Then D is a psd-set of $B_3(G)$ implies that G contains K_2 as one of its components, since G is disconnected.

In the remaining two cases, D is not a psd-set of $B_3(G)$. Thus, the theorem follows.

Remark 3.20:

If G is connected, then there exists no psd-set in $B_3(G)$ containing 3 vertices.

Theorem 3.21:

Let G contain $2K_2$ as an induced sub graph. Then $\gamma_{ps}(B_3(G)) \leq 4$, if for every v in $V(G)-V(2K_2)$, $N(v) \cap V(2K_2) \subseteq \{$end vertices of exactly one of the edges in $2K_2\}$

Proof:

Let $e_{12} = (v_1, v_2)$, $e_{34} = (v_3, v_4)$ be the edges in $2K_2$, where $v_1, v_2, v_3, v_4 \in V(2K_2)$ and e_{12}', e_{34}' be the corresponding line vertices in $B_3(G)$. Then

D = {v_1, v_2, e_{12}', e_{34}'} $\subseteq$ V(B_3(G)) is a dominating set of B_3(G). Let W be an independent set in V(B_3(G))–D. Let $N_{B3(G)}$(v) denote the neighborhood of a vertex v in B_3(G).

(i). Let W $\subseteq$ V(G)–{v_1, v_2}, then W $\subseteq$ $N_{B3(G)}(e_{12}')\cap$(V(B_3(G))–D).

(ii). If W contains both point and line vertices in $B_{3(}$G), then by the given conditions, W $\subseteq$ $N_{B3(G)}(e')\cap$(V(B_3(G))–D), where $e' = e_{12}'$ or e_{34}', since $2K_2$ is an induced sub graph of G.

(iii). If W contains line vertices only, then W $\subseteq$ $N_{B3(G)}(v)\cap$(V(B_3(G))–D), where v = v_1 or v_2. Thus, D is a psd- set of B_3(G) and $\gamma_{ps}(B_3(G)) \leq 4$.

This bound is attained, if G $\cong$ C_n, for n $\geq$ 8.

Similarly, the following theorem can be proved.

Theorem 3.22:

$\gamma_{ps}(B_3(G)) \leq 5$, if G contains P_3 as a sub graph such that

(i). The degree of the center vertex of P_3 is 2; and

(ii). Any vertex in V(G)–V(P_3) is either nonadjacent or adjacent to exactly one of the end vertices of P_3 or N(v)$\cap$V(P_3) $\subseteq$ {exactly one of the end vertices of P_3},

for all v$\in$N(G)–V(P_3) .

Proof:

Assume G contains P_3 as a subgraph satisfying conditions (i) and (ii). Then the subset of V(B_3(G)) containing the vertices of P_3 and the line vertices in B_3(G) corresponding to the edges in P_3, is a psd-set of B_3(G). Hence, $\gamma_{ps}(B_3(G)) \leq 5$

This bound is attained, if G $\cong$ C_n, for n = 5, 6.

Remark 3.23:

In Theorem 3.22., either if the degree of the center vertex of P_3 is at least three or if there exists a vertex in V(G)–V(P_3) adjacent to both the end vertices of P_3, then D cannot be a psd-set of B_3(G).

Similarly, the following theorem can be proved.

Theorem 3.24:

$\gamma_{ps}(B_3(G)) \leq 6$, if G contains $2K_2$ as a sub graph such that (N_G(v)–V($2K_2$)) $\subseteq$ {end vertices of exactly one of the edges in $2K_2$}, for all v$\in$V(G)–V($2K_2$). This bound is attained, if G $\cong$ K_4.

Proposition 3.25:

Let G be a graph and v be a vertex of minimum degree $\delta(G)$ in G, where $\delta(G) \geq 2$. If radius of the sub graph<N(v)>of G induced by N(v) is at least 2, then $\gamma_{ps}(B_3(G)) \leq \delta(G) + 3$.

Proof:

Let v be a vertex of minimum degree $\delta(G)$ in G and v_1 and v_2 be any two vertices in N(v). Let D be a set of line vertices in $B_3(G)$ corresponding to the edges in G adjacent to v. Then $D \cup \{v, v_1, v_2\} \subseteq V(B_3(G))$ is a psd-set of $B_3(G)$ and hence $\gamma_{ps}(B_3(G)) \leq \delta(G) + 3$.

Remark 3.26:

If $\delta(G) = 1$, then $\gamma_{ps}(B_3(G)) \leq \delta_s(G) + 3$, where $\delta_s(G)$ is the second minimum degree of G such that the radius of the subgraph of G induced by the vertices in the second neighborhood of v is at least 2, where $deg_G(v) = \delta_s(G)$, $v \in V(G)$.

Proposition 3.27:

$\gamma_{ps}(B_3(G)) \leq q + 2$.

Proof:

Since $D = V(L(G)) \cup \{v_1, v_2\}$ is a psd-set of $B_3(G)$, where v_1 and v_2 are any two adjacent vertices in G, $\gamma_{ps}(B_3(G)) \leq q + 2$.

Proposition 3.28:

Let G contain a triangle such that the edges of this triangle do not lie on any C_3 in G. Then $\gamma_{ps}(B_3(G)) \leq 6$.

Proof:

The line and point vertices corresponding to edges and vertices of C_3 respectively is a psd-set of $B_3(G)$.

In the following, the restrained domination number γ_r of $B_3(G)$ is obtained.

Theorem 3.29:

$\gamma_r(B_3(G)) = 2$ if and only if there exists two independent edges e_1 and e_2 in G such that no edge in G is adjacent to both e_1 and e_2 in G and for each $v \in V(G)$ there exists at least one edge in $G - \{e_1, e_2\}$ not incident with v.

Proof:

Assume $\gamma_r(B_3(G)) = 2$. Then there exists a restrained dominating set D of $B_3(G)$ containing two vertices. Since D is a dominating set of $B_3(G)$, the vertices of D must be line vertices and the corresponding edges in G are independent and no edge in G is adjacent to both of these edges. If all the edges in

$<G-\{e_1, e_2\}>$ are incident with $v\in V(G)$, then $v\in V(B_3(G))$ is not adjacent to any of the vertices in $V(B_3(G))-D$, which is a contradiction. Converse follows easily.

The following propositions are stated without proof.

Proposition 3.30:

Let G be any graph having at least five vertices and $e_{12} = (v_1,v_2)\in E(G)$, where $v_1, v_2\in V(G)$. If e_{12}' is the corresponding line vertex in $B_3(G)$, then $\{v_1, v_2, e_{12}'\}$ is a restrained dominating set of $B_3(G)$ if and only if for every vertex v in $V(G)-\{v_1, v_2\}$ there exists at least one edge in $E(<V(G)-\{v_1, v_2\}>)\cup \{e\in E(G) . e$ is incident with v_1 or v_2, $e \neq e_{12}\}$, not incident with v in G.

Proposition 3.31:

Let G be a graph having at least five vertices and e_1 and e_2 be any two independent edges in G and v be vertex in G not incident with both e_1 and e_2. If e_1', e_2' be the line vertices in $B_3(G)$ corresponding to e_1 and e_2, then $\{v, e_1', e_2'\}$ is a restrained dominating set of $B_3(G)$ if and only if for each vertex $u\in V(G)-v$ there exists at least one edge e, different from e_1 and e_2 in $E(<V(G)-v>)\cup\{e\in E(G) : e$ is incident with $v\}$, not incident with v in G.

Proposition 3.32:

$\gamma_r(B_3(G)) \leq \alpha_0(G) + 1$, if there exists a point cover D for G having at least two vertices with $|D| = \alpha_0(G)$, $|V(G)-D| \geq 2$ and D is not independent in G.

Remark 3.33:

If $|V(G)-D| = 1$, then $G \cong K_n$, $n \geq 3$. If $G \cong K_4$, then the sub set of $B_3(K_4)$ containing three point vertices and the line vertices corresponding to any two independent edges in K_4 is a restrained dominating set of $B_3(G)$. Hence, $\gamma_r(B_3(K_4)) \leq 5$.

Proposition 3.34:

$\gamma_r(B_3(G)) \leq \alpha_0(G) + 2$, if there exists a point cover D of G with $|D| = \alpha_0(G)$ and two independent edges e_1 and e_2 in G such that $<E(G)-\{e_1, e_2\}> \neq K_{1,n} \cup mK_1$, where $n \geq 1$ and $m \geq 0$.

Remark 3.35:

If $<E(G)-\{e_1, e_2\}> \cong K_{1,n} \cup mK_1$, ($n \geq 1$ and $m \geq 0$) for all independent edges in e_1 and e_2 in G, then $\gamma_r(B_3(G)) \leq \alpha_0(G) + 3$.

Proposition 3.36:

$\gamma_r(B_3(G)) \leq \gamma(G)$, if there exists a dominating set D of G with $|D| = \gamma(G)$ and $|E(<V(G)-D>)| \geq 1$.

Remark 3.37:

The set of all point vertices is a restrained dominating set of $B_3(G)$, if $r(L(G)) \geq 2$.

Split and Non split domination numbers in $B_3(G)$

In the following, split and non split domination numbers are determined. Here, the graphs G for which $B_3(G)$ is connected are considered.

Theorem 3.38:

$\gamma_s(B_3(G)) = 2$ if and only if there exists a pair of independent edges e_1 and e_2 in G such that no edge in G is adjacent to both e_1 and e_2 and $<G-\{e_1, e_2\}> \cong K_{1,n} \cup mK_1$, for $n \geq 1$ and $m \geq 0$.

Proof:

We have proved $\gamma(B_3(G)) = 2$ if and only if there exists a apir of independent edges e1 and e2 in G such that no edge in G is adjacent to both e_1 and e_2.

Since $G - \{e_1, e_2\} \cong K_{1,n} \cup mK_1$, for $n \geq 1$ and $m \geq 0$, the centre vertex of $K_{1,n}$ is isolated in $<V(B_3(G) - D>$ and hence D is a split dominating set of $B_3(G)$.

Conversely, if $G - \{e_1, e_2\} \neq K_{1,n} \cup mK_1$, for $n \geq 1$ and $m \geq 0$, then $<V(B_3(G) - D>$ is connected, which is a contradiction.

Theorem 3.39:

Let G be a graph with at least four vertices. If there exists an edge e in G with $<G-e> \cong K_{1,n} \cup mK_1$, for $n \geq 1$ and $m \geq 0$, then $\gamma_s(B_3(G)) \leq 3$.

Proof:

Let $e = (u, v) \in E(G)$ be an edge in G with $<G-e> \cong K_{1,n} \cup mK_1$, for $n \geq 1$ and $m \geq 0$, where $u, v \in V(G)$. If e' is the line vertex in $B_3(G)$ corresponding to the edge e, then $D = \{u, v, e'\} \subseteq V(B_3(G))$ is a dominating set of $B_3(G)$. Further, the point vertex corresponding to the center vertex of $K_{1,n}$ is isolated in $V(B_3(G))-D$. Thus, D is a split dominating set of $B_3(G)$ and $\gamma_s(B_3(G)) \leq 3$.

Theorem 3.40:

If G is any (p, q) graph with $\Delta(G) < p-1$, then $\gamma_s(B_3(G)) \leq p + q - 2\Delta(G)$.

Proof:

Let $v \in V(G)$ be such that $deg_G(v) = \Delta(G)$, where $\Delta(G) < p-1$ and let D' be the set of line vertices in $B_3(G)$ corresponding to the edges not incident with v in G.

Case(i). $<V(G)-v> \neq K_{1,n}$, for $n \geq 1$.

Then $D = D' \cup (V(G)-N[v])$ is a dominating set of $B_3(G)$. Also v is isolated in $V(B_3(G))-D$. Thus, D is a split dominating set of $B_3(G)$ and

$\gamma_s(B_3(G)) \leq (p + q) - (2\Delta(G) + 1)$.

Case(ii). $<V(G)-v> \cong K_{1,n}$, for $n \geq 1$.

Then $D \cup \{$center vertex of $K_{1,n}\}$ is a split dominating set of $B_3(G)$. Thus,

$\gamma_s(B_3(G)) \leq p + q - 2\Delta(G)$.

This bound is attained, if $G \cong C_3$.

Theorem 3.41:

Let G be a (p, q) graph with $p \geq 5$. Then $\gamma_s(B_3(G)) \leq p + q - k - 1$, where $k = \max \{deg_G(u) + deg_G(v) : uv \in E(G)$ and $<V(G)-\{u, v\}>$ is not totally disconnected$\}$.

Proof:

Let $u, v \in V(G)$ be such that $e = (u, v) \in E(G)$, $<V(G)-\{u, v\}>$ is not totally disconnected and $deg_G(u) + deg_G(v) = k$ is maximum. If D' is the set of line vertices in $B_3(G)$ corresponding to the edges not incident with u or v, then $D = <V(G)-\{u, v\}> \cup D'$ is a dominating set of $B_3(G)$ and $<V(B_3(G))-D> \cong K_{1,n} \cup K_{1,m} \cup K_1$, where $m, n \geq 1$. Thus, D is a split dominating set of $B_3(G)$ and $\gamma_s(B_3(G)) \leq p + q - k - 1$.

Remark 3.42:

(i). Let G be a graph other than a star. Then the set of all line vertices is a split dominating set of $B_3(G)$. Thus $\gamma_s(B_3(G)) \leq q$.

(ii). Let G be a graph having at least three vertices. If $\overline{L(G)}$ is disconnected, then the set of all point vertices is a split dominating set of $B_3(G)$. If $\overline{L(G)}$ is connected with connectivity $\kappa''(G)$, then $\gamma_s(B_3(G)) \leq p + \kappa''(G)$.

Next, the non split domination number γ_{ns} of $B_3(G)$ is obtained. In view of Theorem 3.38 and Theorem 3.39, the following theorems are stated without proof.

Theorem 3.43:

$\gamma_{ns}(B_3(G)) = 2$ if and only if there exists a pair of independent edges e_1 and e_2 in G such that no edge in G is adjacent to both e_1 and e_2 and $G-\{e_1, e_2\} \neq K_{1,n} \cup mK_1$, for $n \geq 1$ and $m \geq 0$.

Theorem 3.34:

Let e = (u, v) be an edge in G, where $u, v \in V(G)$ and e' be the corresponding line vertex in $B_3(G)$.

(i). If $\langle G-e \rangle \cong K_{1,n} \cup mK_1$, for $n \geq 1$ and $m \geq 0$, then $\{u, v, e'\}$ is a non split dominating set of $B_3(G)$.

(ii). If for each edge e in G, $\langle G-e \rangle \cong K_{1,n} \cup mK_1$, for $n \geq 1$ and $m \geq 0$, then $\{u, v, e'$, center vertex of $K_{1,n}$, for $n \geq 1\}$ is a non split dominating set of $B_3(G)$ and hence $\gamma_{ns}(B_3(G)) \leq 4$.

(ii). If both G and $\overline{L(G)}$ are connected graphs, then $\gamma_{ns}(B_3(G)) \leq p$.

Global, total global and global point set domination numbers in $B_3(G)$

In the following, various bounds for the global domination number γ_g of $B_3(G)$ are discussed. The graphs G for which $\gamma_g(B_3(G))$ is 2 or 3 are obtained. Here, $\overline{B_3(G)}$ denotes the complement of $B_3(G)$.

Theorem 3.35:

Let G be a graph having at least three vertices. Then $\gamma_g(B_3(G)) = 2$ if and only if $G \cong 2K_2$.

Proof:

By Theorem 3.3., $\gamma(B_3(G)) = 2$ if and only if there exists two independent edges e_1 and e_2 in G such that no edge in G is adjacent to both e_1 and e_2. Let e_1' and e_2' be the corresponding line vertices in $B_3(G)$. If $D = \{e_1', e_2'\}$ is a global dominating set of $B_3(G)$, then D must be a line cover for G. Thus,

$G \cong 2K_2$. Converse is obvious.

Theorem 3.36:

Let G be any graph having no isolated vertices and at least four vertices. Then $\gamma_g(B_3(G)) = 3$ if and only if one of the following holds.

(i). radius of L(G) is 1;

(ii). There exists a dominating edge in L(G);

(iii). There exists at least two adjacent edges e_1 and e_2 and a vertex v in G incident with e_1 or e_2 such that each edge in G is adjacent to at least one of e_1 and e_2 and there exists no edge in G incident with v and adjacent to both e_1 and e_2;

(iv). There exists at least two independent edges e_1 and e_2 and a vertex v in G not incident with both e_1 and e_2 such that each edge in G is either incident with v or adjacent to at least one of e_1 and e_2 or both; and

(v). There exists a line cover D' for G with $|D'| = 3$ and $\beta_0(\langle D' \rangle) \geq 2$ satisfying, for every $e \in E(G) - D'$, there exists at least one edge e_1 in D' such that $d(e', e_1') \geq 2$ in L(G), where e' and e_1' are the vertices in L(G) corresponding to the edges e and e_1 respectively.

Proof:

Assume $\gamma_g(B_3(G)) = 3$. Then there exists a global dominating set D of $B_3(G)$ with $|D| = 3$. Since G contains at least four vertices, there exists no dominating set of $B_3(G)$ containing point vertices only, since D is a dominating set of $B_3(G)$.

Case(i). D contains two point vertices v_1 and v_2 and one line vertex, say e'.

Then $v_1, v_2 \in V(G)$. Let e be the edge in G corresponding to the line vertex e'. If v_1 and v_2 are non-adjacent and e is incident with one of the vertices v_1 and v_2 in G, then $D = \{v_1, v_2, e'\}$ is not a dominating set of $B_3(G)$. Similarly, if e is incident with none of the vertices v_1 and v_2 in G, then also D is not a dominating set of $B_3(G)$. Therefore, v_1 and v_2 must be adjacent in G and $e = (v_1, v_2)$. Then $D = \{v_1, v_2, e'\}$ is a global dominating set of $B_3(G)$ implies that $r(L(G)) = 1$.

Case(ii). D contains two line vertices e_1', e_2' and one point vertex v (say).

Then $v \in V(G)$. Let e_1 and e_2 be the edges in G corresponding to the line vertices e_1' and e_2' respectively.

(a). If e_1 and e_2 are adjacent and v is incident with e_1 or e_2 (not both), then since

$D = \{e_1', e_2', v\}$ is a global dominating set of $B_3(G)$, either each edge in G is adjacent to at least one of e_1 and e_2 and there exists no edge incident with v and adjacent to both e_1 and e_2.

(b). If e_1 and e_2 are adjacent edges in G and v is the common vertex of both e_1 and e_2, then $D = \{e_1', e_2', v\}$ is a dominating set of both $B_3(G)$ and $\overline{B_3}(G)$ implies that each edge in G is adjacent to at least one of e_1 and e_2. That is, there exists a dominating edge in L(G).

(c). If e_1 and e_2 are independent edges in G and v is not incident with both e_1 and e_2, then either each edge in G is incident with v or adjacent to at least one of e_1 and e_2 or both.

Case(iii). D contains three line vertices e_1', e_2' and e_3', (say).

Let e_1, e_2 and e_3 be the respective edges in G. Then $D = \{e_1', e_2', e_3'\}$ is a dominating set of $B_3(G)$ implies that $\beta_0(<\{e_1, e_2, e_3\}>) \geq 2$ in G and for every $e \in E(G)-\{e_1, e_2, e_3\}$, there exists at least one edge e_i, i = 1, 2, 3 with $d(e', e_i') \geq 2$ in L(G), where e' is the vertex in L(G) corresponding to the edge e. D is also a dominating set of $\overline{B_3}(G)$ implies that $\{e_1, e_2, e_3\}$ is a line cover for G. Converse can be verified easily.

The following propositions are stated without proof.

Proposition 3.37:

Let G be a graph having no isolated vertices and at least six vertices. Then $\gamma_g(B_3(G)) \leq p/2$, if there exists a perfect matching in G.

Proposition 3.38:

Let G be a graph, which is not bipartite. Then

$\gamma_g(B_3(G)) \leq \alpha_0(G) + 1$.

Proposition 3.39:

If G is a bipartite graph with bipartition [A, B], then $\gamma_g(B_3(G)) \leq \min(m, n) + 2$, where m and n are the number of vertices in A and B respectively.

Proposition 3.40:

Let G be a graph having no isolated vertices. Then $\gamma_g(B_3(G)) \leq \gamma_g(L(G)) + 1$ if and only if there exists a global dominating set of L(G) containing at least two independent vertices.

Proposition 3.41:

$\gamma_g(B_3(G)) \leq \gamma_g(L(G))$ if and only if there exists a global dominating set D of L(G) such that D is a line cover for G and $\beta_0(<D>) \geq 2$.

In the following, the total global domination number γ_{tg} of $B_3(G)$ is determined by applying Theorem 2.2. Here, the graphs G for which both $\delta(B_3(G)) \geq 1$ and $\delta(\overline{B_3(G)}) \geq 1$ are considered.

Theorem 3.42:

Let $\beta_1(G) = 2$ and $e_{12} = (v_1, v_2)$, $e_{34} = (v_3, v_4)$ be two independent edges in G, where $v_1, v_2, v_3, v_4 \in V(G)$. Let e_{12}' and e_{34}' be the line vertices in $B_3(G)$ corresponding to the edges e_{12} and e_{34} respectively. Then $\{v_1, v_3, e_{12}', e_{34}'\}$ is a total global dominating set of $B_3(G)$ if and only if there exists no edge in G joining the vertices v_1 and v_3.

Proof:

Assume there exists no edge in G joining the vertices v_1 and v_3. Then $D = \{v_1, v_3, e_{12}', e_{34}'\}$ is a dominating set of $B_3(G)$ and $\langle D \rangle \cong P_4$ in $B_3(G)$. Hence, D is a total dominating set of $B_3(G)$. Also for each point vertex v in $V(B_3(G))–D$, there is a vertex $u \in D$ such that uv is not an edge in $B_3(G)$. Similarly, since $\beta_1(G) = 2$, each line vertex in $V(B_3(G))–D$ is not adjacent to at least one of the vertices in D. Hence, D is a total global dominating set of $B_3(G)$. Conversely, assume $\{v_1, v_3, e_{12}', e_{34}'\}$ is a total global dominating set of $B_3(G)$. If v_1 and v_3 are adjacent in G, then the line vertex in $V(B_3(G))–D$ corresponding to the edge (v_1, v_3) is not adjacent to any of the vertices in D, which is a contradiction.

The following theorem relates $\gamma_{tg}(B_3(G))$ with the maximum number of edges $\beta_1(G)$.

Theorem 3.43:

If $\beta_1(G) \geq 3$, then $\gamma_{tg}(B_3(G)) \leq 2\beta_1(G)$.

Proof:

Let $\beta_1(G) = n$, where $n \geq 3$ and $e_1, e_2, \ldots, e_n$, $(n \geq 3)$ be the independent edges in G. Let v_i be a vertex in G incident with e_i, $i = 1, 2, \ldots, n$ and e_i' be the corresponding line vertex in $B_3(G)$. Then $\{v_1, v_2, \ldots, v_n, e_1', e_2', \ldots, e_n'\}$ is a total global dominating set of $B_3(G)$ and hence $\gamma_{tg}(B_3(G)) \leq 2\beta_1(G)$, where $\beta_1(G) \geq 3$.

The following theorem relates $\gamma_{tg}(B_3(G))$ with the point covering number $\alpha_0(G)$ of G.

Theorem 3.44:

$\gamma_{tg}(B_3(G)) \leq [3\alpha_0(G)/2]$, if there exists a point cover D for G with $|D| = \alpha_0(G)$ and $\beta_1(<D>_G) \geq 2$.

Proof:

Let D be a point cover for G with $|D| = \alpha_0(G)$ and $\beta_1(<D>_G) = n$, where $n \geq 2$. Let $e_1, e_2, \ldots, e_n$ $(n \geq 2)$ be the independent edges in $<D>_G$.

Then $D \cup \{e_i', i = 1,2, \ldots, n\}$ is a total global dominating set of $B_3(G)$, where e_i' is the line vertex in $B_3(G)$ corresponding to e_i. Thus,

$$\begin{aligned}\gamma_{tg}(B_3(G)) &\leq \alpha_0(G) + \beta_1(<D>_G)\\ &\leq \alpha_0(G) + [|D|/2])\\ &= \alpha_0(G) + [\alpha_0(G)/2]\\ &= [3\alpha_0(G)/2].\end{aligned}$$

The following propositions 3.45. and 3.46. are stated without proof.

Proposition 3.45:

Let G be any graph having no isolated vertices. Then $\gamma_{tg}(B_3(G)) \leq \gamma_{tg}(L(G)) + 1$ if and only if there exists a total global dominating set D' of L(G) with $\beta_0(<D'>_{L(G)}) \geq 2$.

Proposition 3.46:

$\gamma_{tg}(B_3(G)) \leq \gamma_{tg}(L(G))$ if and only if there exists a total global dominating set D' of L(G) such that D is a line cover for G and $\beta_0(<D'>_{L(G)}) \geq 2$.

Next, the global point set domination number γ_{pg} of $B_3(G)$ is obtained by using Theorem 2.2. Here, the graphs G for which both $B_3(G)$ and is complement $\overline{B_3(G)}$ are connected are considered. An upper bound of $\gamma_{pg}(B_3(G))$ is given in terms of maximum degree of G.

Proposition 3.47:

$\gamma_{pg}(B_3(G)) \leq p + q - \Delta(G)$, where $\Delta(G) < p-1$.

Proof;

Let v be a vertex of maximum degree in G and D' be the set of line vertices in $B_3(G)$ corresponding to the edges not incident with v. Then $V(G) \cup D'$ is a global point set dominating set (psd-set) of $B_3(G)$. Hence, $\gamma_{pg}(B_3(G)) \leq p + q - \Delta(G)$.

Proposition 3.48:

The set of all point vertices in $B_3(G)$ is not a global psd-set of $B_3(G)$.

Proof:

By Theorem 3.17., the set of all point vertices is a psd- set of $B_3(G)$ if and only if $1 < r(G) \leq \infty$. Let D be the set of all point vertices in $B_3(G)$ and $S \subseteq V(B_3(G))-D$ be such that <S> is complete. Then S will contain line vertices only and the corresponding edges in G are independent and there exists no vertex v in D such that $S \cap N(v) = \phi$ in $B_3(G)$. Thus, D is not a global psd-set of $B_3(G)$.

The following propositions are stated without proof.

Proposition 3.49:

Let G be a disconnected graph but not totally disconnected. Then $\gamma_{pg}(B_3(G)) = 3$ if and only if G contains K_2 as one of its components.

Proposition 3.50:

If G contains P_3 as one of its components, then $\gamma_{pg}(B_3(G)) \leq 5$.

Conclusion:

We have obtained domination, cycle, connected and total domination numbers in $B_3(G)$. Also, perfect, Point set and restrained domination numbers are found for this graph. Further, Split and Non split domination numbers, global, total global and global point set domination numbers are determined.

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