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On the Boolean Function Graph $B(\overline{K}_p, NINC, \overline{L}(G))$ of a Graph

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Abstract: *For any graph G, let V(G) and E(G) denote the vertex set and edge set of G respectively. The Boolean function graph $B(\overline{K}_p, NINC, \overline{L}(G))$ of G is a graph with vertex set $V(G)\cup E(G)$ and two vertices in $B(\overline{K}_p, NINC, \overline{L}(G))$ are adjacent if and only if they correspond to two nonadjacent edges of G or to a vertex and an edge not incident to it in G, where L(G) is the line graph of G. For brevity, this graph is denoted by $B_3(G)$. In this paper, structural properties of $B_3(G)$ including traversability and eccentricity properties are studied. Also the graphs G for which $B_3(G)$ contains C_n, for $n \geq 4$ are obtained. Further, decomposition of $B_3(G)$ for some known graphs are given.*

Key Word: *Boolean Function Graph.*

1. Introduction

Graphs discussed in this paper are undirected and simple graphs. For a graph G, let V(G) and E(G) denote its vertex set and edge set respectively. *Eccentricity* of a vertex $u\in V(G)$ is defined as $e_G(u) = \max \{d_G(u, v): v\in V(G)\}$, where $d_G(u, v)$ is the distance between u and v in G. We denote the eccentricity of vertex v in G as e(v) and the distance between two vertices u, v in G as d(u, v). The minimum and maximum eccentricities are the *radius* and *diameter* of G, denoted r(G) and diam(G) respectively. When diam(G) = r(G), G is called a *self-centered* graph with radius r, equivalently G is r-self-centered. A vertex u is said to be an eccentric point of v in a graph G, if d(u, v) = e(v). In general, u is called an *eccentric point*, if it is an eccentric point of some vertex. We also denote the i^{th} neighborhood of v as $N_i(v) =\{u\in V(G) : d_G(u , v) = i\}$ and denote the cardinality of the set H as $|H|$. If $|N_{e(v)}(v)|$ is m for each point $v\in V(G)$, then G is called an *m- eccentric point graph*. If m = 2, we call the graph G as *bi-eccentric point graph*. A connected graph G is said to be *geodetic*, if a unique shortest path joins any two of its vertices.

Whitney[17] introduced the concept of the line graph L(G) of a given graph G in 1932. The first characterization of line graphs is due to Krausz. The Middle graph M(G) of a graph G was introduced by Hamada and Yoshimura[5]. Chikkodimath and Sampathkumar[3] also studied it independently and they called it, the semi-total graph $T_1(G)$ of a graph G. Characterizations were presented for middle graphs of any graph, trees and complete graphs in [1]. The concept of total graphs was introduced by Behzad[2] in 1966. Sastry and Raju[16] introduced the concept of quasi-total graphs and they solved the graph equations for line graphs, middle graphs, total graphs and quasi-total graphs. These graphs are very much useful in the construction of various related networks from the underlying graphs of networks. This motivates us to define and study other graph operations. Using L(G), G, incident and non-incident, complementary operations, complete and totally disconnected structures, one can get thirty-two graph operations. As already total graphs, semi-total edge graphs, semi-total vertex graphs and quasi-total graphs and their complements (8 graphs) are defined and studied, we have studied all other similar remaining graph operations.

The points and lines of a graph are called its elements. Two elements of a graph are neighbors, if they are either incident or adjacent. The *Total graph* T(G) of G has vertex set $V(G)\cup E(G)$ and vertices of T(G) are adjacent, whenever they are neighbors in G. The *Quasi- total graph* P(G) of G is a graph with vertex set as that of T(G) and two vertices are adjacent if and only if they correspond to two nonadjacent vertices of G or to two adjacent edges of G or to a vertex and an edge incident to it in G. The *Middle graph* M(G) of G is one whose vertex set is as that of T(G) and two vertices are adjacent in M(G), whenever either they are adjacent edges of G or one is a vertex of G and the other is an edge of G incident with it. Clearly, $E(M(G)) = E(T(G)) - E(G)$.

The *Boolean function graph* $B(\overline{K_p}, NINC, \overline{L}(G))$ G is a graph with vertex set $V(G)\cup E(G)$ and two vertices in $B(\overline{K_p}, NINC, \overline{L}(G))$ are adjacent if and only if they correspond to two nonadjacent edges of G or to a vertex and an edge not incident to it in G. For brevity, this graph is denoted by $B_3(G)$, where L(G) is the line graph of G. The vertices of G and L(G) in $B_3(G)$ are referred as point and line vertices respectively. In this paper, the properties of the Boolean function graph $B_3(G)$ are studied The line vertex in $B_3(G)$ corresponding to an edge e in G is denoted by e'.

2. Properties

In this section, properties of $B_3(G)$ including traversability, eccentricity properties are studied. Also the graphs G for which $B_3(G)$ contains C_n, for $n \geq 4$ are obtained. Further, decomposition of $B_3(G)$ for some known graphs are given.

Observation 2.1.

1. $\overline{L}(G)$ is an induced subgraph of $B_3(G)$ and subgraph of $B_3(G)$ induced by point vertices is totally disconnected.
2. If $d_i = deg_G(v_i)$, $v_i \in V(G)$, then the number of edges in $B_3(G)$ is $(q/2)(2p + q - 3) - (1/2)\sum_{1 \le i \le p} d_i^2$.
3. The degree of a point vertex v in $B_3(G)$ is $q - deg_G(v)$ and the degree of a line vertex e' in $B_3(G)$ is $deg_{\overline{L(G)}}(e') + p - 2 = p + q - deg_{L(G)}(e') - 3$ and hence $\delta(B_3(G)) = q - \Delta(G)$.
4. $B_3(G)$ contains isolated vertices if and only if G is one of the following graphs. K_2, $K_{1,n}$, $n \ge 2$, nK_1, $K_2 \cup nK_1$, $K_{1,m} \cup nK_1$, where $n \ge 1$ and $m \ge 2$.
5. $B_3(G)$ is totally disconnected if and only if $G \cong nK_1$ or K_2, $n \ge 1$.
6. $B_3(G)$ is disconnected if and only if $G \cong nK_1$, $K_2 \cup mK_1$, $C_3 \cup mK_1$ and $K_{1,n} \cup mK_1$, for $m \ge 0$ and $n \ge 2$.
7. Let G be any graph having at least one edge and $G \ne K_n$, $n \ge 2$. Then $B_3(G)$ is biregular if and only if G is regular.
8. Both G and $B_3(G)$ are regular if and only if either G is totally connected or G is complete.

Proposition 2.2.

For any (p, q) graph G, $B_3(G)$ contains cut-vertices if and only if there exists a vertex in G of degree q-1.

Proof:

Assume there exists a vertex $v \in V(G)$ such that $deg_G(v) = q - 1$. Therefore, there exists exactly one edge e in G, which is not incident with v and hence $deg_{B3(G)}(v) = 1$. Also, $B_3(G) - e'$ disconnected with an isolated vertex v in $B_3(G)$ and hence e' is a cut-vertex in $B_3(G)$.

Conversely, assume $B_3(G)$ contains cut- vertices. Suppose there exists no vertex $v \in V(G)$ such that $deg_G(v) = q - 1$. Therefore, all the point vertices of $B_3(G)$ have degree at least two. That is, $deg_{B3(G)}(v) \ge 2$, for all $v \in V(B_3(G))$ and each line vertex in B3(G) is adjacent to exactly (p- 2) point vertices and hence no vertex of B3(G) is a cut-vertex, which is a contradiction. Hence, there exists at least one vertex $v \in V(G)$ such that $deg_G(v) = q - 1$.

Proposition 2.3.

For any graph G with at least five vertices and $\beta_1(G) \geq 2$, $B_3(G)$ contains triangles if and only if G contains $2K_2 \cup K_1$ (with $2K_2$ induced) as a subgraph, where $\beta_1(G)$ is the line independence number of G.

Proof:

Assume $B_3(G)$ contains triangles. Since, neither G nor $\overline{G}$ is a subgraph of $B_3(G)$, atleast two vertices of the triangle must be line vertices.

(i) If the two vertices of the triangle are line vertices and the third vertex is a point vertex, then G contains $2K_2 \cup K_1$ (with $2K_2$ induced) as a subgraph.

(ii) If all the vertices of the triangle are line vertices, then $\beta_1(G) \geq 3$.

Converse follows from the construction of $B_3(G)$.

In the following, we find the girth of $B_3(G)$.

Proposition 2.4.

If G is graph with at least four vertices, then the girth of $B_3(G)$ is 3, 4 or 5.

Proof:

Case (i): $\beta_1(G) \geq 2$

If G contains atleast five vertices, then by Proposition 2.3, $B_3(G)$ contains triangles. Assume G contains exactly four vertices. Then, $B_3(G)$ contains either no cycles or contains C_5 as an induced subgraph.

Case (ii): $\beta_1(G) = 1$

If G contains $P_3 \cup 2K_1$ as a subgraph, then $B_3(G)$ contains either C_4 as an induced subgraph or contains no cycles.

Hence, girth of $B_3(G)$ is 3, 4 or 5.

Remark 2.5.

If G contains two or three vertices, then $B_3(G)$ is cycle-free.

Proposition 2.6.

Let G be a connected graph with at least four vertices other than a star. Then $B_3(G)$ is geodetic if and only if G is a path on four vertices.

Proof:

Case(i): G contains atleast five vertices

Since G is connected, G contains $P_3 \cup K_2$ as a subgraph and $B_3(G)$ contains C_4 as an induced subgraph and hence, $B_3(G)$ is not geodetic.

Case (ii): G contains four vertices and is not a path on four vertices.

If G is not a path on four vertices, then $B_3(G)$ contains C6 as an induced subgraph and hence $B_3(G)$ is not geodetic.

By Case (i) and Case (ii), we see that G is a path on four vertices.

Conversely, if G is a path on four vertices, then $B_3(G)$ is geodetic.

Proposition 2.7.

For any graph G with at least four vertices, $B_3(G)$ contains $K_{1,3}$ as an induced subgraph if and only if either $\beta_1(G) \geq 2$ or G contains $C_3 \cup K_1$ or $K_{1,3} \cup K_1$ as a subgraph.

Proof:

Assume $B_3(G)$ contains $K_{1,3}$ as an induced subgraph. If $\beta_1(G) = 1$ and if G is star on four vertices, then $B_3(G)$ is isomorphic to $C_7 \cup K_1$. Hence, either $\beta_1(G) \geq 2$ or g contains $C_3 \cup K_1$ or $K_{1,3} \cup K_1$ as a subgraph.

Conversely, if either $\beta_1(G) \geq 2$ or G contains $C_3 \cup K_1$ or $K_{1,3} \cup K_1$ as a subgraph, then $B_3(G)$ contains $K_{1,3}$ as an induced subgraph

In the following, a necessary and sufficient condition for $B_3(G)$ to be Eulerian is given. For simplicity, the degree of a vertex v in $B_3(G)$ is denoted by $d_3(v)$.

Theorem 2.8.

Let G be any (p, q) graph such that $B_3(G)$ is connected. Then $B_3(G)$ is Eulerian if and only if one of the following holds.

(i). p is odd, q is even and G or each component of G is Eulerian; and

(ii). q is odd and each vertex in G is of odd degree.

Proof.

Assume $B_3(G)$ is Eulerian. Therefore, each vertex in $B_3(G)$ is of even degree. If v is a point vertex in $B_3(G)$, then $d_3(v) = q - deg_G(v)$ is even and hence q and $deg_G(v)$ are of same parity. Similarly, if e' is a line vertex in $B_3(G)$, then $d_3(e') = deg_{\overline{L}(G)}(e') + p - 2 = p + q - deg_{L(G)}(e') - 3$ is even. Therefore, if q is even, then p must be odd, since $deg_{L(G)}(e')$ is even. Assume q is odd. Since q and $deg_G(v)$ are of same parity, $deg_G(v)$ is odd. Since the number of odd degree vertices in G is even, p is even. Thus, condition (i) or (ii) holds. Converse follows easily.

In the following, a necessary condition that $B_3(G)$ to be Hamiltonian is given.

Theorem 2.9.

Let G be any (p, q) with $p \geq 5$ and $\Delta(G) \leq q - 2$. If $\Delta(G) \leq (q-p)/2$, then $B_3(G)$ is Hamiltonian.

Proof.

Let $v \in V(G)$ be such that $deg_G(v) = \Delta(G)$. Then $v \in V(B_3(G))$. If $d_3(v)$ is the degree of v in $B_3(G)$, then $d_3(v) = q - \Delta(G) = \delta(B_3(G))$. Since $\Delta(G) \leq (q - p)/2$, $\delta(B_3(G)) \geq q - ((q - p)/2)$ and hence $\delta(B_3(G)) \geq (p + q)/2$. Thus, $B_3(G)$ is Hamiltonian.

Example 2.10.

(i). $B_3(K_4) \cong$ Petersen graph and hence not Hamiltonian.

(ii). $B_3(C_n)$, $B_3(C_m+)$, $B_3(P_n)$, $B_3(W_t)$ and $B_3(K_t)$ are Hamiltonian graphs, where $n \geq 4$, $m \geq 3$ and $t \geq 5$, where C_n is a cycle on n vertices, C_m+ is the graph obtained from the cycle C_m by attaching exactly one edge at each of its vertices, P_n is a path on n vertices, W_t is a wheel and K_t is a complete graph on t vertices .

Remark 2.11.

If G is any (p, q) graph other than a star and $q = p - 1$, then $B_3(G)$ contains a Hamiltonian path.

In the following, the eccentricity properties of $B_3(G)$ for any graph G are discussed. Here, the graphs G for which $B_3(G)$ are connected are considered. First, the graph G for which $B_3(G)$ is self-centered with radius 2 is characterized. For simplicity, the distance between two vertices u, v in $B_3(G)$ and the eccentricity of a vertex v in $B_3(G)$ are denoted by $d_3(u, v)$ and $e_3(v)$ respectively. Since, there is no vertex of degree $p + q - 1$ in $B_3(G)$, radius of $B_3(G)$ is atleast 2.

Theorem 2.12.

Let G be any graph with at least two edges. Then $B_3(G)$ is self-centered with radius 2 if and only if for every pair of vertices u, v in G there exists at least one edge not incident with both u and v.

Proof.

Assume for every pair of vertices u, v in G there exist at least one edge not incident with both u and v.

(i). Let v_1 and v_2 be two point vertices in $B_3(G)$. Then $v_1, v_2 \in V(G)$. By the assumption, there exists an edge in G not incident with both v_1 and v_2. Then, $d_3(v_1, v_2) = 2$.

(ii). Let v and e' be a point and line vertices in $B_3(G)$ respectively and e be the edge in G corresponding to e'. If e is not incident with v in G, then $d_3(v, e') = 1$. Let $e \in E(G)$ be incident with v in G. By the assumption, there exists at least one edge in G not adjacent to e. Then $d_3(v, e') = 2$.

(iii). Let e_1' and e_2' be two line vertices in $B_3(G)$ and e_1, e_2 be the corresponding edges in G. If e_1 and e_2 are nonadjacent edges in G, since $\overline{L}(G)$ is an induced subgraph of $B_3(G)$, $d_3(e_1', e_2') = 1$. Let e_1 and e_2 be adjacent edges in G. By the assumption, there exists at least one vertex v in G not incident with both e_1 and e_2 and hence $d_3(e_1', e_2') = 2$. Hence, it follows that all the vertices in $B_3(G)$ have eccentricity 2 and is self-centered with radius 2. Conversely, assume there exists a pair of vertices u, v in G such that each edge in G is incident with at least one of u and v. Then u, $v \in V(B_3(G))$. If $(u, v) \notin E(G)$, then $d_3(u, v) = 3$. Let $(u, v) \in E(G)$ and e = (u, v) and let e' be the corresponding line vertex in $B_3(G)$. Then v, u, $e' \in V(B_3(G))$ and $d_3(v, e') = d_3(u, e') = 3$, which is a contradiction.

From the above facts, following are immediate consequences.

Remark 2.13.

Let G be any graph such that $B_3(G)$ is connected. Then $B_3(G)$ is bi-eccentric with radius 2 if and only if there exists at least one pair of vertices u, v in G such that each edge in G is incident with at least one of u and v. Hence, by Theorem 6.1.4, it is clear that the diameter of $B_3(G)$ is at most 3.

Remark 2.14.

If $G \cong K_{1,n}$, $n \geq 2$, then $B_3(G) \cong C \cup K_1$, where the component C is self-centered with radius 3.

In the following, a necessary and sufficient condition that $B_3(G)$ contains $C_n (n \geq 4)$, as an induced subgraph is obtained, where G is any graph not totally disconnected.

Proposition 2.15.

$B_3(G)$ contains C_4 as an induced subgraph if and only if G contains $P_3 \cup 2K_1$ as a subgraph.

Proof:

Assume $B_3(G)$ contains C_4 as an induced subgraph. Since, no two point vertices in $B_3(G)$ are adjacent, any C_4 in $B_3(G)$ can have atmost two point vertices. If two vertices of C_4 in $B_3(G)$ are line vertices and the remaining two vertices are point vertices, then G contains $P_3 \cup 2K_1$ as an induced subgraph. The other cases also give rise to $P_3 \cup 2K_1$ as a subgraph. Converse follows easily.

Proposition 2.16.

$B_3(G)$ contains C_5 as an induced subgraph if and only if G contains either C_5 or P_4 as a subgraph.

Proof:

Assume $B_3(G)$ contains C_5 as an induced subgraph. A cycle on five vertices in $B_3(G)$ is possible, if the cycle contains either three line vertices and two point vertices or all line vertices. That means, G contains either P_4 or C_5 as a subgraph.

Similarly, the following propositions can be proved.

Proposition 2.17.

$B_3(G)$ contains C_6 as an induced subgraph if and only if either G contains G_1 as an induced subgraph or G contains C_4 or $K_{1,3}$ as a subgraph, where $G_1$is the graph obtained from K_4 - e by subdividing its diagonal edge exactly once.

Proposition 2.18.

$B_3(G)$ contains no C_n, $(n \geq 7)$ as an induced subgraph.

Remark 2.19.

From the above propositions, it follows that $B_3(G)$ contains cycles if and only if G contains one of the following graphs as a subgraph. $2K_2 \cup K_1$ (with $2K_2$ induced), $P_3 \cup 2K_1$, C_5, P_4, C_4, $K_{1,3}$ and the graph G_1, where G_1 is the graph obtained from K_4 - e by subdividing its diagonal edge exactly once.

In the following, the edge partition of $B_3(G)$ for some known graphs are given.

Theorem 2.20.

Let G be any connected (p, q) graph such that $G \neq K_{1,n}$ and K_3. Then the edges of $B_3(G)$ can be partitioned into $\overline{L(G)}$ and $qK_{1,p-2}$, where the center vertex of each $K_{1,p-2}$ is a line vertex.

Proof:

Follows from the construction of $B_3(G)$.

Theorem 2.21.

The edges of $B_3(C_n)$, $(n \geq 4)$ can be partitioned into $((n-2)/2)C_{2n}$, $((n-4)/2)C_n$ and $(n/2)K_2$, if n is even; and $((n-3)/2)C_{2n}$, $((n-3)/2)C_n$ and nK_2, if n is odd.

Proof:

Edges of $B_3(C_n)$ can be partitioned into $\overline{L(C_n)}$ and $nK_{1,n-2}$. But $\overline{L(C_n)}$ is a (n-3)-regular graph on n vertices This can be partitioned into $((n-4)/2)C_n$ and $(n/2)K_2$, if n is even; and $((n-3)/2)C_n$, if n is odd. The edges of $nK_{1,n-2}$ can be partitioned into $((n-2)/2)C_{2n}$, if n is even; and $((n-3)/2)C_{2n}$ and nK_2, if n is odd . Thus, the theorem follows.

Theorem 2.22.

$B_3(K_{1,n})$, $(n \geq 2)$ is disconnected with exactly two components, one of the components being K_1. If the other component is C, then the edges of C can be partitioned into $((n-2)/2)C_{2n}$ and nK_2, if n is even; and $((n-1)/2)C_{2n}$, if n is odd.

Proof:

$B_3(K_{1,n}) = K_1 \cup C$, where $E(C) = E((n-1)K_{1,n-1})$. Edges of $(n-1)K_{1,n-1}$ can be partitioned into $((n-2)/2)C_{2n}$ and nK_2, if n is even; and $((n-1)/2)C_{2n}$, if n is odd.

Theorem 2.23.

The edge set of $B_3(K_n)$, $(n \geq 4)$ can be partitioned into (n-1)/2 times (n-2)-regular graph on 2n vertices and (((n-2)(n-3))/2)-regular graph on ((n(n-1))/2 vertices, if n is odd; and ((n-2)/2) times (n-2)-regular graph on 2n vertices, $(n/2)K_{1,n-2}$ and (((n-2)(n-3))/2)-regular graph on (n(n-1))/2 vertices, if n is even.

Proof:

Edges of $B_3(K_n)$ can be partitioned into $\overline{L}(K_n)$ and $((n(n-1)/2)K_{1,n-2}$. But $\overline{L}(K_n)$ is a ((n-2)(n-3)/2)-regular graph on (n(n-1)/2 vertices. The edges of $((n(n-1)/2)K_{1,n-2}$ can be partitioned into ((n-1)/2) times (n-2)-regular graph on 2n vertices, if n is odd; and ((n-2)/2) times (n-2)-regular graph on 2n vertices and $(n/2)K_{1,n-2}$, if n is even. Thus, the theorem follows.

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