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Even Sequential Harmonious Labeling of Some Tree Related Graphs

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Abstract: *Graham and Sloane introduced the harmonious graphs and Singh & Varkey introduced the odd sequential graphs. Gayathri et al introduced even sequential harmonious labeling of graphs. In this paper, we investigate even sequential harmonious labeling of some tree related graphs.*

1. Introduction

All graphs in this paper are finite, simple and undirected. The symbols V (G) and E(G) denote the vertex set and the edge set of a graph G. The cardinality of the vertex set is called the **order** of G. The cardinality of the edge set is called the **size** of G. A graph with p vertices and q edges is called a **(p, q) graph**.

Graham and Sloane[2] introduced the harmonious graphs and Singh & Varkey [6] introduced the odd sequential graphs. Harmonious and related graphs are dealt in [3 - 5]. We refer to the excellent survey by Gallian [1] for varieties of labeling and graphs.

Gayathri et al [1] say that a labeling is an **even sequential harmonious labeling** if there exists an injection f from the vertex set V to {0,1,2,…,2q} such that the induced mapping f^{+} from the edge set E to {*2,4,6,…,2q*} defined by $f^{+}(uv) = \begin{cases} f(u)+f(v), \text{if } f(u)+f(v) \text{ is even} \\ f(u)+f(v)+1, \text{if } f(u)+f(v) \text{ is odd} \end{cases}$ are distinct. A graph G is said to be an **even sequential harmonious graph** if it admits an even sequential harmonious labeling. In this paper, we investigate even sequential harmonious labeling of some tree related graphs.

2. Main Results

Theorem 2.1:

The path $P_{n,}$ $(n \geq 2)$ is an even sequential harmonious graph.

Proof:

Let the vertices of P_n , $V(P_n)$ $=\{v_1,v_2,v_3,\ldots,v_n\}$ and the edges of P_n , $E(P_n) = \{e_1,e_2,e_3,\ldots,e_{n-1}\}$defined as follows, $e_i = (v_i,v_{i+1})$ for $i = 1,2,3,\ldots,n-1$ which are denoted as in Figure 2.1.

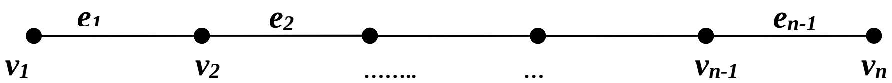

Figure 2. 1: P_n with ordinary labeling

We, now label the vertices of P_n by: $f(v_i) = i-1 \quad 1\leq i \leq n$.

Then the induced edge labels are: $f^{+}(e_i) = 2i \quad 1\leq i \leq n-1$.

Clearly, the edge labels are even and distinct, $f^{+}(E) = \{2,4,6,\ldots,2q\}$.

Hence, P_n is an even sequential harmonious graph.

ESHL of P_6 is shown in Figure 2. 2.

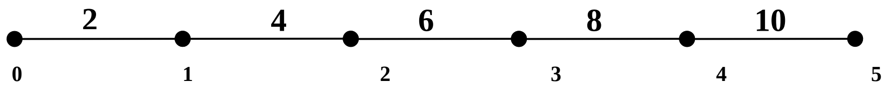

Figure 2. 2: ESHL of P_6.

Theorem 2.2:

The graph P_n^+, $n \geq 2$ is an even sequential harmonious graph.

Proof:

Let the vertices of P_n^+ be $v_1,v_2,v_3,\ldots,v_n$ and $u_1,u_2,u_3,\ldots,u_n$ and the edges of P_n^+ be

$e_1,e_2,e_3,\ldots,e_n$ and $e_1',e_2',e_3',\ldots,e_{n-1}'$ are defined as follows, $e_i = (v_i,u_i)$ for $1\leq i \leq n$. $e_i' = (v_i,v_{i+1})$ for $1\leq i \leq n-1$ which are denoted as in Figure 2. 3.

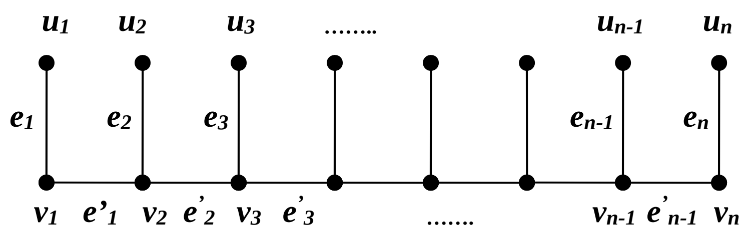

Figure 2. 3: P_n^+ with ordinary labeling.

We, first label the vertices of P_n^+ as follows:

For $1 \le i \le n$

$$f(u_i)=\begin{cases} 2n+i-2, & i \text{ is odd} \\ 2n+i, & i \text{ is even} \end{cases}$$

$$f(v_i)=\begin{cases} i & ;\text{ when } i \text{ is odd} \\ i-2 & ;\text{ when } i \text{ is even} \end{cases}$$

Then the induced edge labels are:

$f^+(e_i') = 2i \qquad 1 \le i \le n-1$

$f^+(e_i) = 2n+2i-2 \qquad 1 \le i \le n$

Clearly, the edge labels are even and distinct, $f^+(E)=\{2,4,6,\ldots,2q\}$.

Hence P_n^+ is an even sequential harmonious graph. ESHL of P_9^+ is shown in Figure 2. 4.

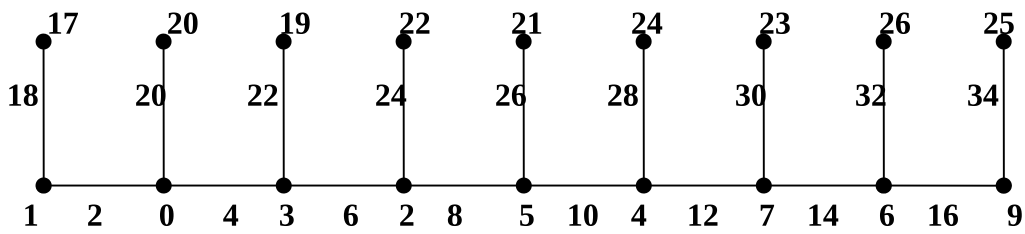

Figure 2.4: ESHL of P_9^+

Theorem 2.3:

The graph Bistar $B_{n,n}$, $(n \ge 2)$ is an even sequential harmonious graph.

Proof:

Let the vertices of Bistar $B_{n,n}$ be $\{u, w, u_i, w_i ;\ 1 \le i \le n\}$ and the edges be $\{e_i ;\ 1 \le i \le 2n+1\}$ which are denoted as in Figure 2. 5.

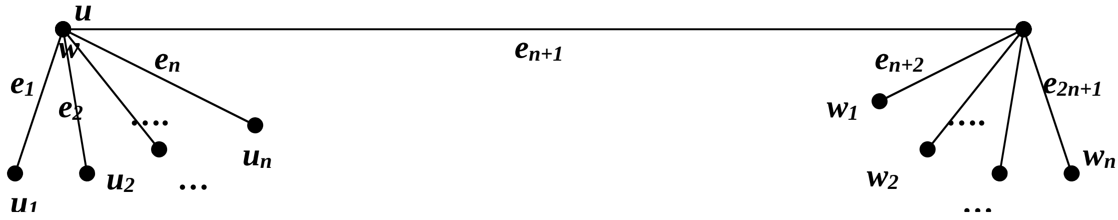

Figure 2.5: $B_{n,n}$ with ordinary labeling

First we label the vertices of Bistar as follows:

$f(u) = 0$, $f(w) = 2(n+1)$

$f(u_i) = 2i$, $1 \le i \le n$; $f(w_i) = 2i-1$, $1 \le i \le n$

Then the induced edge labels are: $f^+(e_i) = 2i$, $1 \le i \le 2n+1$

Clearly, the edge labels are even and distinct, $f^+(E) = \{2, 4, 6, \ldots, 2q\}$.

Hence, the graph Bistar $B_{n,n}$,($n \ge 2$) is an even sequential harmonious graph.

ESHL of $B_{4,4}$ is shown in Figure 2. 6.

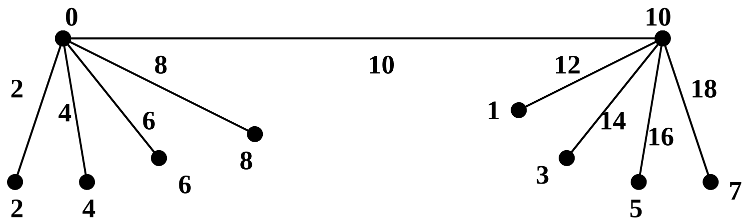

Figure 2. 6: ESHL OF $B_{4,4}$

2.1 Definition:

The graph $\langle K_{1,n} : m \rangle$,$(m,n \ge 2)$ is obtained by taking m disjoint copies of $K_{1,n}$ and joining a new vertex to the centers of the copies of $K_{1,n}$.

2.4 Theorem:

The graph $\langle K_{1,n} : m \rangle$,$(m,n \ge 2)$ is an even sequential harmonious graph.

Proof:

Let the vertices be$\{u_i, w_j : 1 \le i \le m$ and $1 \le j \le m-1\}$and $\{u_{ij} : 1 \le i \le m$ and $1 \le j \le n\}$ and the edges be $\{e_i : 1 \le i \le mn+2(m-1)\}$ which are denoted as in Figure 2. 7.

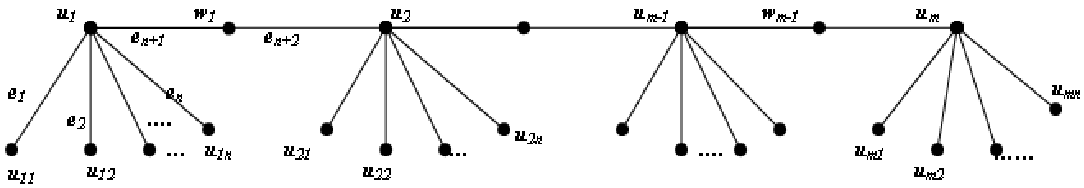

Figure 2. 7: ‹$K_{1,n}$: *m*› with ordinary labeling

First we label the vertices as follows:

$f(u_1) = 0$; $f(u_i) = 2i-3$, $2 \le i \le m$

$f(w_j) = 2n+2+(2n+2)(j-1)$, $1 \le j \le m-1$

$f(u_{ij}) = 2j+(2n+2)(i-1)$, $1 \le i \le m$ and $1 \le j \le n$

Then the induced edge labels are: $f^+(e_i) = 2i$, $1 \le i \le mn+2(m-1)$

Clearly, the edge labels are even and distinct, $f^+(E) = \{2, 4, 6, \ldots, 2q\}$.

Hence, the graph ‹$K_{1,n}$: *m*› is an even sequential harmonious graph.

ESHL of ‹$K_{1,3}$: 3› is shown in Figure 2. 8.

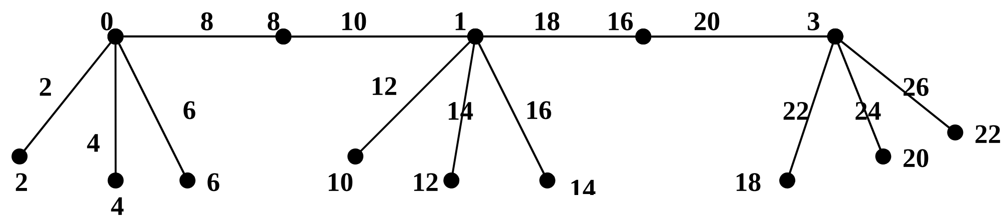

Figure 2. 8: ESHL OF ‹$K_{1,3}$: 3›

2.5 Theorem:

The Festoon graph $P_m \odot K_{1,n}$ ($m \ge 2$, $n \ge 1$) is an even sequential harmonious graph.

Proof:

Let the vertices of $P_m \odot K_{1,n}$ be $\{u_i, u_{ij} : 1 \le i \le m$ and $1 \le j \le n\}$ and the edges of be $\{(u_i\, u_{i+1}); 1 \le i \le m-1\} \cup \{(u_i, u_{ij}) : 1 \le i \le m$ and $1 \le j \le n\}$ which are denoted as in Figure 2. 9.

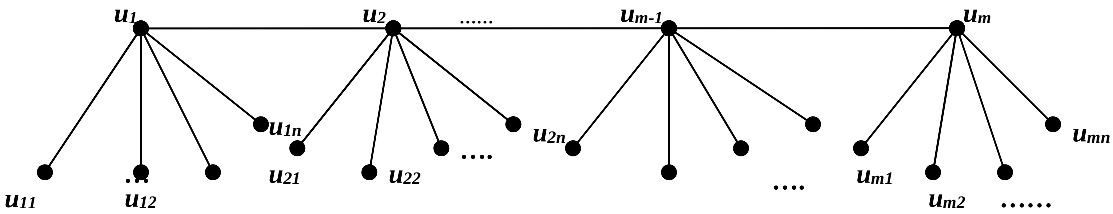

Figure 2. 9 : $P_m \Theta K_{1,n}$ with ordinary labeling

First we label the vertices as follows:

$f(u_i) = i\text{-}1 \qquad 1 \le i \le m$

$f(u_{ij}) = 2m+2(j\text{-}1)+(2n\text{-}1)(i\text{-}1) \qquad 1 \le i \le m$ and $1 \le j \le n$

Then the induced edge labels are:

$f^+(u_i u_{i+1}) = 2i \qquad 1 \le i \le (m\text{-}1)$

$f^+(u_i u_{ij}) = 2m+2(j\text{-}1)+2n(i\text{-}1) \qquad 1 \le i \le m$ and $1 \le j \le n$

Clearly, the edge labels are even and distinct, $f^+(E) = \{2,4,6,\ldots,2q\}$.

Hence, the graph $P_m \Theta K_{1,n}$ $(m \ge 2, n \ge 1)$ is an even sequential harmonious graph.

ESHL of $P_4 \Theta K_{1,3}$ is shown in Figure 2. 10.

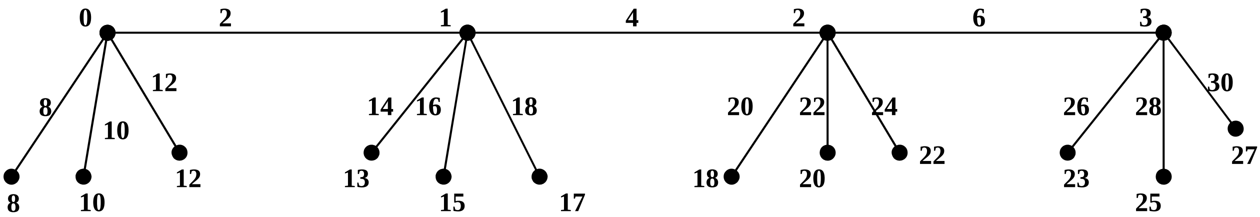

Figure 2.10 : ESHL OF $P_4 \Theta K_{1,3}$

2.6 Theorem:

The graph Banana tree $B_{m,n}$ $(m \ge 1, n \ge 1)$ is an even sequential harmonious graph.

Proof:

Let the vertices of $B_{m,n}$ be $\{u, v, w, u_i ; 1 \le i \le m+n\}$ and the edges of $B_{m\,n}$ be $\{e_i ; 1 \le i \le m+n+2\}$ which are denoted as in Figure 2.11.

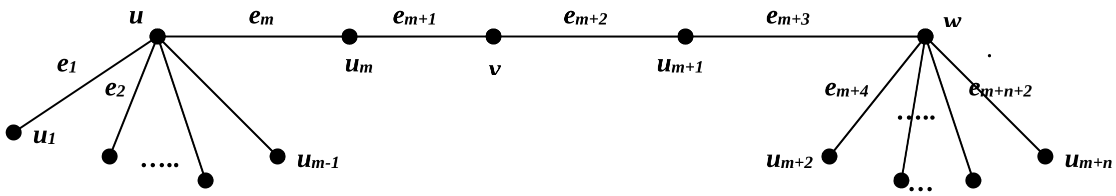

Figure 2. 11: $B_{m,n}$ with ordinary labeling

Now, we label the vertices as follows:

$$f(u) = 0\ ;\ f(v) = 1\ ;\ f(w) = 3$$

$$f(u_i) = 2i \quad 1 \leq i \leq m+n$$

Then the induced edge labels are: $f^{+}(e_i) = 2i \quad 1 \leq i \leq m+n+2$

Clearly, the edge labels are even and distinct, $f^{+}(E) = \{2, 4, 6, \ldots, 2q\}$.

Hence the graph Banana tree $B_{m,n}$ $(m \geq 1, n \geq 1)$ is an even sequential harmonious graph.

ESHL of $B_{5,4}$ is shown in Figure 2. 12.

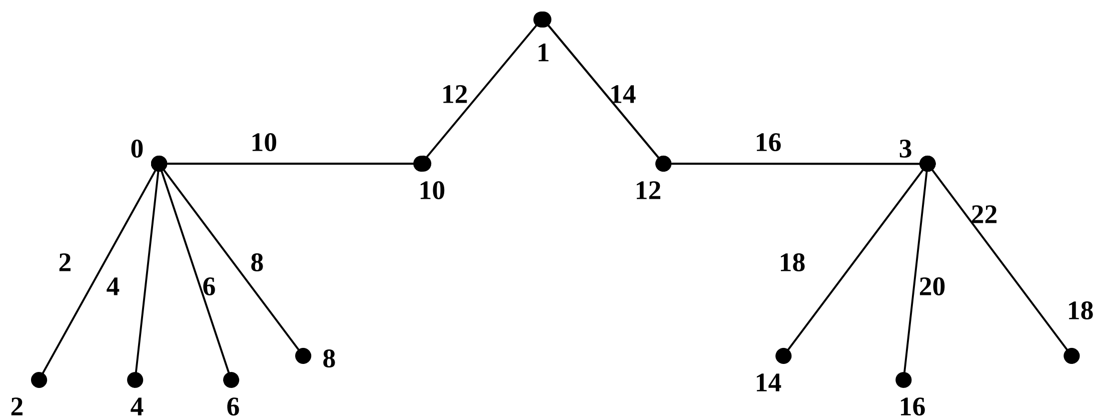

Figure 2. 12: ESHL OF $B_{5,4}$

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