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Graceful Labeling of a Special Class of Fan Graph

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Abstract: *In this paper, a graceful labeling of a special class of fan graph denoted by* $F_{2,m}^{n}$ *which is obtained from merging all* K_1*'s of n copies of* $K_1 + \overline{K_2} + \overline{K_m}$ *to an apex or central vertex.*

Key words: *graceful labeling, graceful graph.*

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1. Introduction

Let G = (V, E) be a simple undirected graph with p vertices and q edges. Graph labeling was first introduced in the late 1960's. Many studies in graph labeling refer to Rosa's research in 1967. Rosa [4] called a function a **β-valuation** of a graph G with q edges if f is an injection from the vertices of G to the set {0, 1, 2,…, q} such that when each edge xy is assigned the label f*(xy) =|f(x) – f(y)|, the resulting edge labels are distinct. Independently, Golomb [3] studied the same type of labeling and called this as graceful, which is now, the popular term.

A graph that admits a graceful labeling is called a **graceful labeling**.

Gallian [2] gives the extensive survey of contributions to graceful labeling of variety of graphs. The notation and terminology used in this paper are taken from [2]. There are many works relating to graceful labeling of graphs with cycles. Barrientos [1] has given graceful arbitrary super subdivisions of graphs, . Shee and Ho[5,6] shown the cordiality of one-point union of n-copies of a graph and the cordiality of the path-union of n copies of a graph, lastly Shiu, Lam [7] given super-edge-graceful labeling of multi-level wheel graphs, fan graphs and actinia graphs.

The above contributions motivated us to give a graceful labeling of a special type of fan graph denoted by $F_{2,m}^{n}$.

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In this paper, we introduce a new type of graph called a special type of fan graph $F^n_{2,m}$ and investigate its gracefulness.

2.Main result

Definition: 2.1

The graph $F^n_{2,m}$ $(m \geq 2,\ n \geq 2)$ is obtained by merging all K_1's of n copies of $K_1 + \overline{K_2} + \overline{K_m}$ to an apex or central vertex.

Theorem 2.2

The graph $F^n_{2,m}$ $(m \geq 2,\ n \geq 2)$ is graceful for all m and n.

Proof:

Let $V(F^n_{2,m}) = \{u\} \cup \{u_{ik}: 1 \leq i \leq n,\ 1 \leq k \leq 2\} \cup \{v_{ij}: 1 \leq i \leq n,\ 1 \leq j \leq m\}$ and

$E(F^n_{2,m}) = \{g_{ik} = (u, u_{ik}): 1 \leq i \leq n,\ 1 \leq k \leq 2\} \cup \{e_{ijk} = (u_{ik}, v_{ij}) : 1 \leq i \leq n,\ 1 \leq j \leq m,$ $1 \leq k \leq 2\}$ denote the vertex and edge sets of $F^n_{2,m}$. Let u denote an apex or central vertex .

The ordinary labeling of $F^n_{2,m}$ is given in figure 2.1

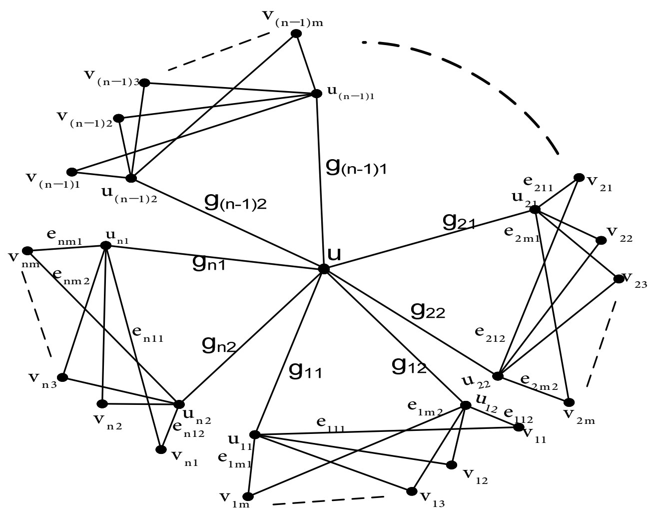

Figure 2.1.

Proof: <u>Case 1</u>: n is odd

First, we label the vertices of $F^{n}_{2,m}$ as follows.

$f(u_{ik}) = (2m+2)(i-1)+(k-1),\ 1 \le i \le n,\ 1 \le k \le 2.$

$$f(v_{ij}) = \begin{cases} (2m+2)(1-i)+q-2-2(j-1), \text{ where } 1 \le i < \dfrac{n+3}{2},\ \ 1 \le j \le m \\ (2m+2)(1-\dfrac{n+1}{2})+q-2-2(j-1)-(2m+1), \text{ where } i = \dfrac{n+3}{2},\ \ 1 \le j \le m \\ (2m+2)(1-\dfrac{n+1}{2})+q-2-2(j-1)-(2m-1)-(2m+2)(i+1-(\dfrac{n+5}{2}))-2, \\ \text{where } \dfrac{n+5}{2} \le i \le n,\ \ 1 \le j \le m \end{cases}$$

and $f(u) = q = n\ (2m + 2)$.

Now induced edge labels are

$f^*(g_{ik}) = q+(2m+2)(1-i)-(k-1),\ 1 \le i \le n,\ 1 \le k \le 2.$

$$f^*(e_{ijk}) = \begin{cases} q-2-4(m+1)(i-1)-(k-1)-2(j-1), \\ \text{where } 1 \le i \le \dfrac{n+1}{2},\ 1 \le j \le m,\ 1 \le k \le 2 \\ (2j-1)+(k-1)+2(m+1), \text{ where } i = \dfrac{n+3}{2},\ 1 \le j \le m,\ 1 \le k \le 2 \\ (2j-1)+(k-1)+4(m+1)(i-(\dfrac{n}{2}+3))+6(m+1), \\ \text{where } \dfrac{n+5}{2} \le i \le n,\ 1 \le j \le m,\ 1 \le k \le 2 \end{cases}$$

Now it follows that f* is induced a graceful labeling as its range set is {1, 2,…, q}. Hence in this case $F^{n}_{2,m}$ is a graceful graph.

<u>Case 2</u>: n is even

First, we label the vertices of $F^{n}_{2,m}$ as follows.

$f(u_{ik}) = (2m+2)(i-1)+(k-1),\ 1 \le i \le n,\ 1 \le k \le 2.$

$$f(v_{ij}) = \begin{cases} (2m+2)(1-i)+q-2-2(j-1), \text{ where } 1 \le i < \frac{n+2}{2},\ 1 \le j \le m \\ (2m+2)(1-\frac{n+1}{2})+q-2-2(j-1)-(2m+1), \text{ where } i = \frac{n+2}{2},\ 1 \le j \le m \\ (2m+2)(1-\frac{n+1}{2})+q-2-2(j-1)-(2m-1)-(2m+2)(i+1-(\frac{(n+4)}{2}))-2, \\ \text{where } \frac{n+4}{2} \le i \le n,\ 1 \le j \le m \end{cases}$$

and $f(u) = q$.

Now the induced edge labels are

$f^*(g_{ik}) = q+(2m+2)(1-i)-(k-1),\ 1 \le i \le n,\ 1 \le k \le 2.$

$$f^*(e_{ijk}) = \begin{cases} q-2-4(m+1)(i-1)-(k-1)-2(j-1), \\ \text{where } 1 \le i \le \frac{n}{2}+1,\ 1 \le j \le m,\ 1 \le k \le 2 \\ (2j-1)+(k-1), \text{ where } i = \frac{n}{2}+1,\ 1 \le j \le m,\ 1 \le k \le 2 \\ (2j-1)+(k-1)+4(m+1)(i-(\frac{n}{2}+2))+4(m+1), \\ \text{where } \frac{n}{2}+2 \le i \le n,\ 1 \le j \le m,\ 1 \le k \le 2 \end{cases}$$

Now it follows that f^* is a induced graceful labeling as its range set is $\{1, 2, \ldots, q\}$. Hence in this case $F^n_{2,m}$ is a graceful graph.

Thus from both the cases it follows that $F^n_{2,m}$ is graceful for all m, n.

The general edge assignment in the descending order of values from q to 1is as follows.

labeling of two edges g_{i1}, g_{i2} in the ith fan	labeling of 2m edges e_{i11} to e_{im1} and e_{i12} to e_{im2}.
$f^*(g_{11}) = q$; $f^*(g_{12}) = q-1$.	$f^*(e_{1i1}) = q-2i$, i = 1 to m; $f^*(e_{1j2}) = q-(2j+1)$, j = 1 to m.
$f^*(g_{21}) = q-(2m+2)$; $f^*(g_{22}) = q-((2m+2)+1)$	$f^*(e_{ni1}) = q-2i-(2m+2)$, i = 1 to m; $f^*(e_{nj2}) = q-(2j+1)-(2m+2)$, j = 1 to m.
$f^*(g_{31}) = q-2(2m+2)$; $f^*(g_{32}) = q-(2(2m+2)+1)$	$f^*(e_{2i1}) = q-2i-2(2m+2)$, i = 1 to m; $f^*(e_{2j2}) = q-(2j+1)-2(2m+2)$, j = 1 to m.
$f^*(g_{41}) = q-3(2m+2)$; $f^*(g_{42}) = q-(3(2m+2)+1)$	$f^*(e_{(n-1)i1}) = q-2i-3(2m+2)$, i = 1 to m; $f^*(e_{(n-1)j2}) = q-(2j+1)-3\ (2m+2)$, j = 1 to m.
incremental value (2m+2) for any i	incremental value (2m+2) for any i follows alternatively(i.e. 1, n, 2, n-1, 3, n-2,…).
$f^*(g_{n1}) = 2m+1$; $f^*(g_{n2}) = 2m+2$	for n is odd $i = \frac{n+1}{2}$, i = 1 to m , $f^*(e_{ii1}) = 2i$; $f^*(e_{ij2}) = 2j-1$, j = 1 to m. for n is even $i = \frac{n+2}{2}$, i= 1 to m, $f^*(e_{ii1}) = 2i$ $f^*(e_{ij2}) = 2j-1$, j = 1 to m.

Table 2.1

The graceful labeling of $F^{6}_{2,5}$ and $F^{9}_{2,6}$ are shown in table 2.2

q	vertex values(u_{ik}: $1 \le i \le 6$, $1 \le k \le 2$)	vertex values(v_{ij}: $1 \le i \le 6$, $1 \le j \le 5$)
72	(0, 1)(12, 13)(24, 25)(36, 37)(48, 49)(60, 61)	(70, 68, 66, 64, 62)(58, 56, 54, 52, 50)(46, 44, 42, 40, 38)(35, 33, 31, 29, 27)(23, 21, 19, 17, 15)(11, 9, 7, 5, 3)
q	vertex values(u_{ik}: $1 \le i \le 9$, $1 \le k \le 2$)	vertex values(v_{ij}: $1 \le i \le 9$, $1 \le j \le 6$)
126	(0, 1)(14, 15)(28, 29)(42, 43)(56, 57)(70, 71)(84, 85)(98, 99)(112, 113)	(124, 122, 120, 118, 116, 114)(110, 108, 106, 104, 102, 100)(96, 94, 92, 90, 88, 86)(82, 80, 78, 76, 74, 72)(68, 66, 64, 62, 60,

		58)(55, 53, 51, 49, 47, 45)(41, 39, 37, 35, 33, 31)(27, 25, 23, 21, 19, 17)(13, 11, 9, 7, 5, 3)

Table 2.2

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