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k-Even Mean Labeling of $T_{n,m,t}$

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Abstract: *Mean labeling of graphs was discussed in [10] and the concept of odd mean labeling was introduced in [9]. k-odd mean labeling and (k, d) - odd mean labeling are introduced and discussed in [5], [6], [7]. In this paper, we introduce the concept of k-even mean labeling and investigate k-even mean labeling of* $T_{n,m,t}$.

Key words: *k-even mean labeling, k-even mean graph.*

AMS (MSC) Subject Classification: 05C78

1. Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [8]. The symbols $V(G)$ and $E(G)$ will denote the vertex set and edge set of a graph. Labeled graphs serve as useful models for a broad range of applications [1-3].

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling).

Graph labeling were first introduced in the late 1960's. Many studies in graph labeling refer to Rosa's research in 1967 [11].

Labeled graphs serve as useful models for a broad range of applications such as X-ray crystallography, radar, coding theory, astronomy, circuit design and communication network addressing. Particularly interesting applications of graph labeling, can be found in [4].

Mean labeling of graphs was discussed in [10] and the concept of odd mean labeling was introduced in [9]. *k*-odd mean labeling and (*k*, *d*) – odd mean labeling are introduced and discussed in [5], [6], [7]. In this paper, we introduce the concept of k-even mean labeling and here we investigate the k-even mean labeling of $T_{n,m,t}$

Throughout this paper, k denote any positive integer ≥ 1. For brevity, we use *k*-EML for *k*-even mean labeling.

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2. Main Results

2.1 Definition: *k*-even mean labeling

A (p, q) graph G is said to have a ***k*-even mean labeling** if there exists a injection $f : V \rightarrow \{0, 1, 2, ..., 2k + 2(q\text{-}1)\}$ such that the induced map $f^* : E(G) \rightarrow \{2k, 2k + 2, 2k + 4, ..., 2k + 2(q - 1)\}$ defined by

$$f^*(uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

is a bijection.

A graph that admits a k – even mean labeling is called a k – even mean graph.

2.2. Theorem

$T_{n,m,t}$ $(n, m \geq 2$ and $t \geq 3)$ is a k -even mean graph for any k when $m = n$.

Proof

Let $V(T_{n,m,t}) = \{u_i, 1 \le i \le n\} \cup \{v_i, 1 \le i \le m\} \cup \{p_i, 1 \le i \le t\}$ and $E(T_{n,m,t}) = \{e_i, 1 \le i \le n+m+t-1\}$ (see Figure 2.1)

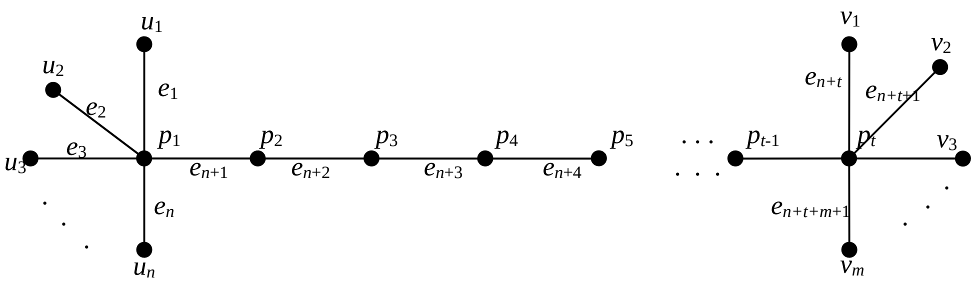

Figure 2.1: Ordinary labeling of $T_{n,m,t}$

In this graph, $p = n + m + t$ and $q = n + m + t - 1$.

First we label the vertices of $T_{n,m,t}$ as follows:

Define $f : V(T_{n,m,t}) \rightarrow \{0, 1, 2, \ldots, 2k + 2q - 2\}$ by

For $1 \le i \le n$,

$$f(u_i) = 2k + 4i - 3$$

For $1 \le i \le t-1$, $f(p_i) = \begin{cases} 2k + 2i - 3 & \text{if } i \text{ is odd} \\ 2k + 4n + 2i - 3 & \text{if } i \text{ is even} \end{cases}$

$$f(p_t) = 2k + 2(n+m+t) - 4$$

For $1 \le i \le m$,

$$f(v_i) = \begin{cases} 2k + 2t + 4(i-1) - 4 & \text{if } t \text{ is odd} \\ 2k + 2t + 4(i-1) - 1 & \text{if } t \text{ is even} \end{cases}$$

Then the induced edge labels are

For $1 \leq i \leq n + t - 2$,

$$f^*(e_i) = 2k + 2i - 2$$

$$f^*(e_{n+t-1}) = \begin{cases} 2k + 2\,(n+m+t) - 4 & \text{if } t \text{ is odd} \\ 2k + 2\,(n+t) - 4 & \text{if } t \text{ is even} \end{cases}$$

For $n + t \leq i \leq n + m + t - 1$,

$$f^*(e_i) = \begin{cases} 2k + 2\,i - 4 & \text{if } t \text{ is odd} \\ 2k + 2\,i - 2 & \text{if } t \text{ is even} \end{cases}$$

Therefore, $f^*(E(T_{n,\,m,\,t})) = \{2k,\ 2k+2,\ 2k+4,\ \ldots,\ 2k+2q-2\}$

So, f is a k-even mean labeling and hence, $T_{n,m,t}$ is a k-even mean graph for any k when $m = n$.

4-EML of $T_{6,\,6,\,7}$ is shown in Figure 2.2.

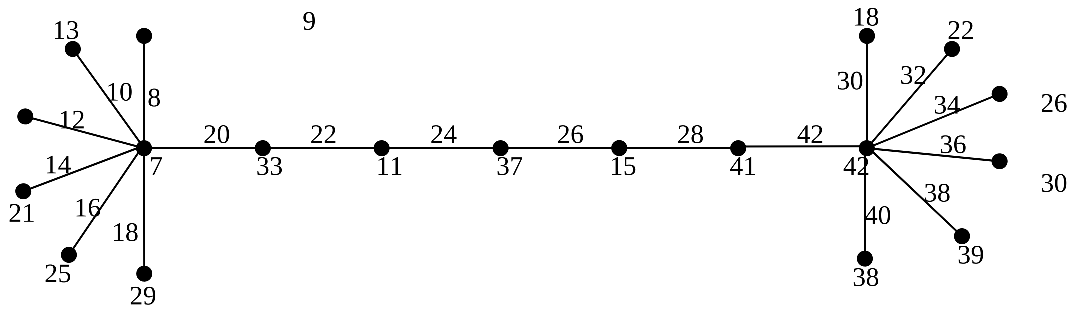

Figure 2.2: 4-EML of $T_{6,\,6,\,7}$

7-EML of $T_{6,\,6,\,6}$ is shown in Figure 2.3.

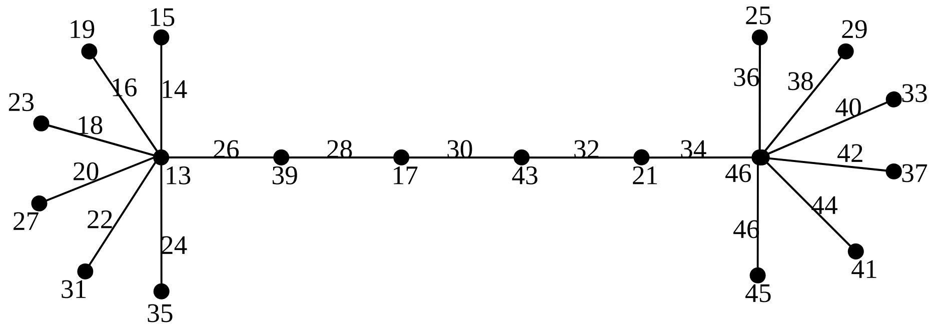

Figure 2.3: 7-EML of $T_{6,\,6,\,6}$

2.3. Theorem

$T_{n,m,t}$ ($n,m \geq 2$ and $t \geq 3$) is a k-even mean graph for any k when t is even and $n + 1 \leq m \leq k + n + t - 2$.

Proof

Let $V(T_{n,m,t}) = \{u_i, 1 \le i \le n\} \cup \{v_i, 1 \le i \le m\} \cup \{p_i, 1 \le i \le t\}$ and
$E(T_{n,m,t}) = \{p_1u_i, 1 \le i \le n\} \cup \{p_i p_{i+1}, 1 \le i \le t-1\} \cup \{p_t v_i, 1 \le i \le m\}$
(see Figure 2.4)

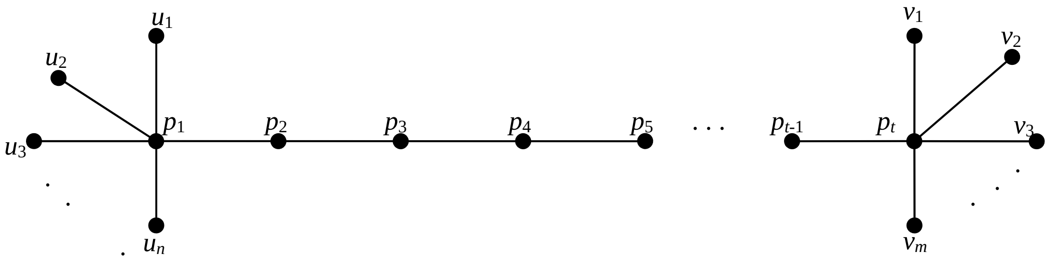

Figure 2.4: Ordinary labeling of $T_{n.m.t}$

First we label the vertices of $T_{n,m,t}$ as follows:

Define $f: V(T_{n,m,t}) \to \{0, 1, 2, \ldots, 2k + 2q - 2\}$ by

Case(i) $\boldsymbol{m = n + 1}$

For $1 \leq i \leq n$, $f(u_i) = 2k + 4i - 3$

For $1 \leq i \leq t - 1$,

$$f(p_i) = \begin{cases} 2k + 2i - 3 & \text{if } i \text{ is odd} \\ 2k + 4(n-1) + 2i + 1 & \text{if } i \text{ is even} \end{cases}$$

$f(p_t) = 2k + 2(n + m + t) - 5$

For $1 \leq i \leq m$,

$f(v_i) = 2k + 2t + 4(i - 1) - 2$

Then the induced edge labels are

For $1 \leq i \leq n$

$f^*(p_1 u_i) = 2k + 2i - 2$

For $1 \leq i \leq t - 1$

$f^*(p_i p_{i+1}) = 2k + 2n + 2i - 2$

For $1 \leq i \leq m$

$f^*(p_t v_i) = 2k + 2(n + t) + 2i - 4$

3-EML of $T_{2,3,6}$ is shown in Figure 2.5.

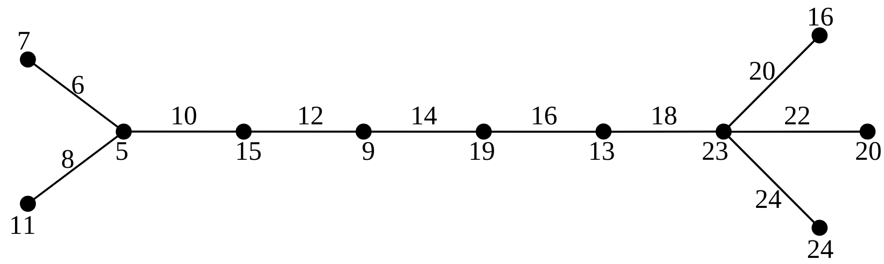

Figure 2.5: 3-EML of $T_{2,3,6}$

Case (ii) $n + 2 \leq m \leq k + n + t - 3$

The vertex labels are

For $1 \leq i \leq n$,

$f(u_i) = 2k + 4i - 3$

For $1 \leq i \leq t - 1$,

$$f(p_i) = \begin{cases} 2k + 2i - 3 & \text{if } i \text{ is odd} \\ 2k + 4(n-1) + 2i + 1 & \text{if } i \text{ is even} \end{cases}$$

Subcase (i) $m - n$ **is even**

$f(p_t) = 2k + 2(n + m + t) - 4$

$$f(v_i) = \begin{cases} 2k + 2(n - m + t) + 4(i-1) - 4, & 1 \leq i \leq \frac{m-n}{2} \\ 2k + 2(n - m + t) + 4(i-1), & \frac{m-n+2}{2} \leq i \leq m-1 \end{cases}$$

$f(v_m) = 2k + 2(n + m + t) - 5$

Subcase (ii) $m - n$ **is odd**

$f(p_t) = 2k + 2(n + m + t) - 5$

$$f(v_i) = \begin{cases} 2k + 2(n - m + t) + 4(i-1) - 4, & 1 \leq i \leq \frac{m-n-1}{2} \\ 2k + 2(n - m + t) + 4(i-1), & \frac{m-n+1}{2} \leq i \leq m-1 \end{cases}$$

$f(v_m) = 2k + 2(n + m + t) - 4$

Then the induced edge labels are

For $1 \leq i \leq n$

$f^*(p_1 u_i) = 2k + 2i - 2$

For $1 \leq i \leq t - 2$,

$f^*(p_i\ p_{i+1}) = 2k + 2n + 2i - 2$

$$f^*(p_{t-1} p_t) = \begin{cases} 2k + 2(n+t) + m - n - 4 & \text{if } m+n \text{ is even} \\ 2k + 2(n+t) + m - n - 5 & \text{if } m+n \text{ is odd} \end{cases}$$

Subcase (i) $m - n$ **is odd**

$$f^*(p_t v_i) = \begin{cases} 2k + 2(n+t+i-1) - 4, & 1 \le i \le \dfrac{m-n-1}{2} \\ 2k + 2(n+t+i-1) - 2, & \dfrac{m-n+1}{2} \le i \le m \end{cases}$$

Subcase (ii) ***m* – *n* is even**

$$f^*(p_t v_i) = \begin{cases} 2k + 2(n+t+i-1) - 4, & 1 \le i \le \dfrac{m-n}{2} \\ 2k + 2(n+t+i-1) - 2, & \dfrac{m-n+2}{2} \le i \le m \end{cases}$$

2-EML of $T_{3,6,4}$ is shown in Figure 2.6.

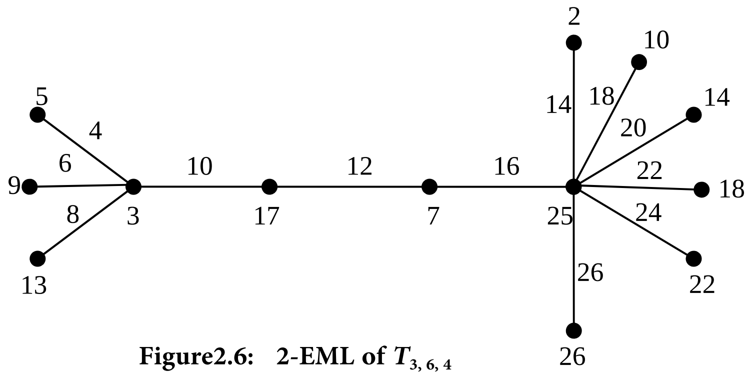

Figure2.6: 2-EML of $T_{3,6,4}$

3-EML of $T_{4,9,6}$ is shown in Figure2.7

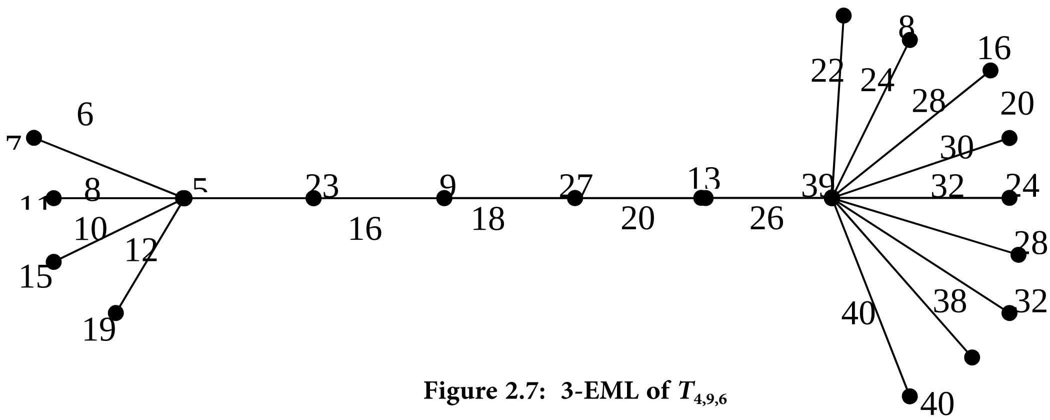

Figure 2.7: 3-EML of $T_{4,9,6}$

Case (iii) ***m* = *k* + *n* + *t* – 2**

For $1 \le i \le n$,

$$f(u_i) = 4k + 2t + 4(i-1) - 5$$

For $1 \le i \le t - 1$,

$$f(p_i) = \begin{cases} 4k + 2t - 2i - 9 & \text{if } i \text{ is odd} \\ 4k + 2t - 2i - 5 & \text{if } i \text{ is even} \end{cases}$$

$$f(p_t) = 2k + 2(m + n + t) - 5$$

For $1 \leq i \leq m$,

$$f(v_i) = 4i$$

Then the induced edge labels are

For $1 \leq i \leq n$,

$$f^*(p_1 u_i) = 2k + 2t + 2(i-1) - 4$$

For $1 \leq i \leq t - 2$

$$f^*(p_i\ p_{i+1}) = 2k + 2t + 2(1 - i) - 6$$

$$f^*(p_{t-1}\ p_t) = 2k + 2(t+n) - 4$$

For $1 \leq i \leq m$; $f^*(p_t v_i) = 2k + 2(t+n) + 2(i-1) - 2$

Therefore, $f^*(E(T_{n,\, m,\, t})) = \{2k,\ 2k+2,\ 2k+4,\ \ldots,\ 2k+2q-2\}$.

So, f is a k -even mean labeling and hence, $T_{n,m,t}$ is a *k*-even mean graph for any *k* when $n+1 \leq m \leq k+n+t-2$.

2-EML of $T_{4,10,6}$ is shown in Figure 2.8.

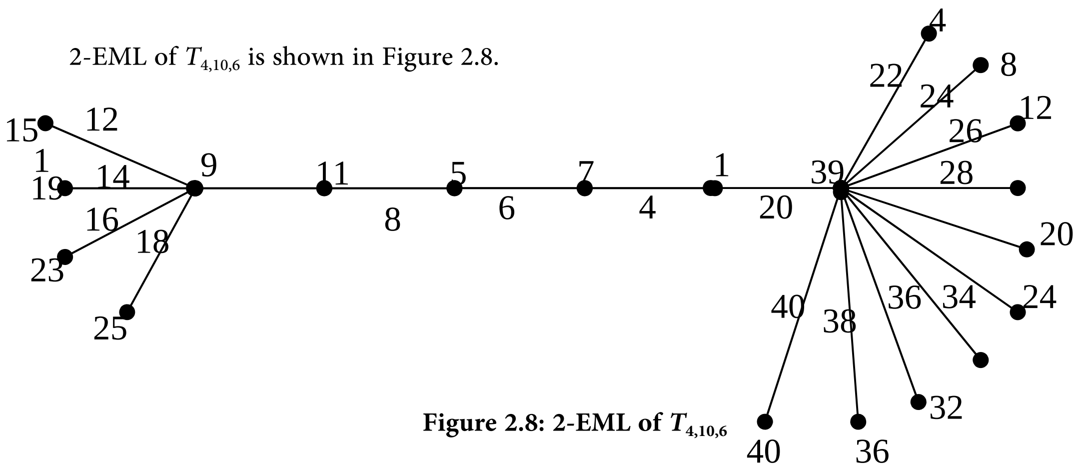

Figure 2.8: 2-EML of $T_{4,10,6}$

2.4. Theorem

$T_{n,m,t}$ (n, $m \geq 2$ and $t \geq 3$) is a *k*-even mean graph for any k when t is odd and $n+1 \leq m \leq k+n+t-2$.

Proof

Let $V(T_{n,m,t}) = \{u_i,\ 1 \leq i \leq n\} \cup \{v_i,\ 1 \leq i \leq m\} \cup \{p_i,\ 1 \leq i \leq t\}$ and

$E(T_{n,m,t}) = \{p_1 u_i,\ 1 \leq i \leq n\} \cup \{p_i p_{i+1},\ 1 \leq i \leq t-1\} \cup \{p_t v_i,\ 1 \leq i \leq m\}$

(see Fig. 2.4)

First we label the vertices of $T_{n,\, m,\, t}$ as follows:

Define $f : V(T_{n,\ m,\ t}) \rightarrow \{0, 1, 2, \ldots, 2k+2q-2\}$ by

Case (i) m = n + 1

For $1 \leq i \leq n$,

$$f(u_i) = 2k + 4i - 3$$

For $1 \leq i \leq t - 1$,

$$f(p_i) = \begin{cases} 2k + 2i - 3 & \text{if } i \text{ is odd} \\ 2k + 4(n-1) + 2i + 1 & \text{if } i \text{ is even} \end{cases}$$

$f(p_t) = 2k + 2(n + m + t) - 5$

For $1 \leq i \leq m - 1$,

$f(v_i) = 2k + 2t + 4(i - 1) - 6$

$f(v_m) = 2k + 2(n + m + t) - 4$

Then the induced edge labels are

For $1 \leq i \leq n$,

$f^*(p_1 u_i) = 2k + 2i - 2$

For $1 \leq i \leq t - 2$,

$f^*(p_i\, p_{i+1}) = 2k + 2n + 2i - 2$

$f^*(p_{t-1}\, p_t) = 2k + 2(n + m + t) - 6$

For $1 \leq i \leq m - 1$

$f^*(p_t\, v_i) = 2k + 2(n + t) + 2i - 6$

$f^*(p_t v_m) = 2k + 2(m + n + t) - 4$

5-EML of $T_{2, 3, 5}$ is shown in Figure 2.9.

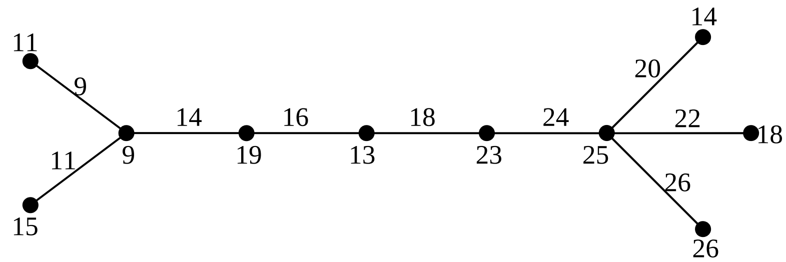

Figure 2.9: 5-EML of $T_{2, 3, 5}$

Case (ii) $n + 2 \leq m \leq k + n + t - 3$

The vertex labels are

For $1 \leq i \leq n$,

$f(u_i) = 2k + 4i - 3$

For $1 \leq i \leq t - 1$,

$$f(p_i) = \begin{cases} 2k + 2i - 3 & \text{if } i \text{ is odd} \\ 2k + 4(n-1) + 2i + 1 & \text{if } i \text{ is even} \end{cases}$$

Subcase (i) $m + n$ **is odd**

$f(p_t) = 2k + 2(n + m + t) - 6$

$$f(v_i) = \begin{cases} 2k + 2(n - m + t) + 4(i - 1) - 4, & 1 \leq i \leq \dfrac{m+n-1}{2} \\ 2k + 2(n - m + t) + 4i - 4, & \dfrac{m+n+1}{2} \leq i \leq m - 1 \end{cases}$$

$f(v_m) = 2k + 2(n + m + t) - 5$

Subcase (ii) ***m* + *n* is even**

$f(p_t) = 2k + 2(n + m + t) - 5$

$$f(v_i) = \begin{cases} 2k + 2(n - m + t) + 4(i-1) - 4, & 1 \le i \le \frac{m+n}{2} \\ 2k + 2(n - m + t) + 4i - 4, & \frac{m+n+2}{2} \le i \le m-1 \end{cases}$$

$f(v_m) = 2k + 2(n + m + t) - 4$

Then the induced edge labels are

For $1 \le i \le n$

$f(p_1 u_i) = 2k + 2i - 2$

For $1 \le i \le t - 2$,

$f^*(p_i\, p_{i+1}) = 2k + 2n + 2i - 2$

$f^*(p_{t-1}\, p_t) = 2k + 3n + 2t + m - 4$

Subcase (i) ***m* + *n* is odd**

$$f^*(p_t v_i) = \begin{cases} 2k + 2(n + t + i - 1) - 4, & 1 \le i \le \frac{m+n-1}{2} \\ 2k + 2(n + t + i - 1) - 2, & \frac{m+n+1}{2} \le i \le m \end{cases}$$

Subcase (ii) ***m* + *n* is even**

$$f^*(p_t v_i) = \begin{cases} 2k + 2(n + t + i - 1) - 4, & 1 \le i \le \frac{m+n}{2} \\ 2k + 2(n + t + i - 1) - 2, & \frac{m+n+2}{2} \le i \le m \end{cases}$$

4-EML of $T_{3,8,5}$ is shown in Figure 2.10.

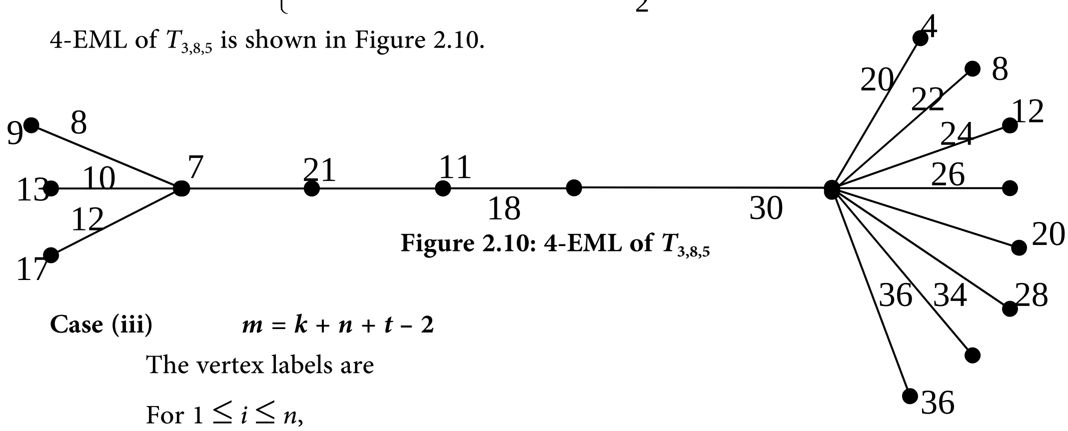

Figure 2.10: 4-EML of $T_{3,8,5}$

Case (iii) ***m* = *k* + *n* + *t* – 2**

The vertex labels are

For $1 \le i \le n$,

$f(u_i) = 2t + 4(i-1) - 1$

For $1 \le i \le t - 1$,

$$f(p_i) = \begin{cases} 4k + 2t - 2i - 5 & \text{if } i \text{ is odd} \\ 2k - 2i - 1 & \text{if } i \text{ is even} \end{cases}$$

$$f(p_t) = 2k + 2(m + n + t) - 5$$

For $1 \leq i \leq m$,

$$f(v_i) = 4i$$

Then the induced edge labels are

For $1 \leq i \leq n$,

$$f^*(p_1 u_i) = 2k + 2t + 2(1 - i) - 4$$

For $1 \leq i \leq t - 2$

$$f^*(p_i\ p_{i+1}) = 2k + 2t + 2(1 - i) - 6$$

$$f^*(p_{t-1}\ p_t) = 2k + 2(t + n) - 4$$

For $1 \leq i \leq m$

$$f^*(p_t\ v_i) = 2k + 2(t + n) + 2(i - 1) - 2$$

Therefore, $f^*(E(T_{n,\,m,\,t})) = \{2k,\ 2k + 2,\ 2k + 4,\ \ldots,\ 2k + 2q - 2\}$

So, f is a k-even mean labeling and hence, $T_{n,m,t}$ is a k-even mean graph for any k when $n + 1 \leq m \leq k + n + t - 2$.

1-EML of $T_{3,9,7}$ is shown in Figure 2.11.

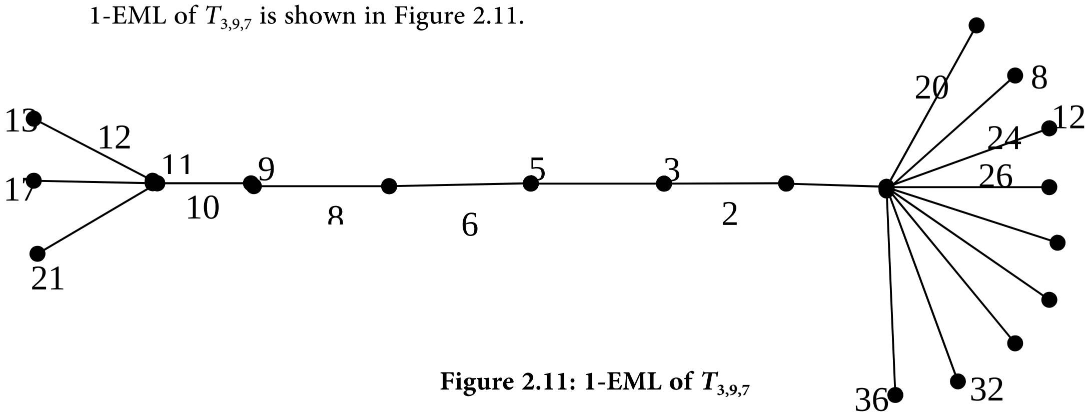

Figure 2.11: 1-EML of $T_{3,9,7}$

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