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Domination Numbers on the Complement of the Boolean Function Graph $B(\overline{K_p}, NINC, L(G))$ of a Graph

T. N. Janakiraman[1], S. Muthammai[2] and M. Bhanumathi[2]

[1]*Department of Mathematics and Computer Applications,*
National Institute of Technology, Triuchirapalli, 620015, TamilNadu, India
E-Mail: janaki@nitt.edu, tnjraman2000@yahoo.com
[2]*Government Arts College for Women, Pudukkottai.622001, TamilNadu, India.*
E-Mail: bhanu_ksp@yahoo.com, muthammai_s@yahoo.com

Abstract: *For any graph G, let V(G) and E(G) denote the vertex set and edge set of G respectively. The Boolean function graph* $B(\overline{K_p}, NINC, L(G))$ *of G is a graph with vertex set* $V(G)\cup E(G)$ *and two vertices in* $B(\overline{K_p}, NINC, L(G))$ *are adjacent if and only if they correspond to two adjacent edges of G or to a vertex and an edge not incident to it in G. For brevity, this graph is denoted by* $B_2(G)$*. In this paper, domination number, independent, connected, total, cycle, point set, restrained, split and non split domination numbers in the complement* $\overline{B_2}(G)$ *of* $B_2(G)$ *are determined. Also the bounds for the above numbers are obtained.*

1. Introduction

Graphs discussed in this paper are undirected and simple graphs. For a graph G, let V(G) and E(G) denote its vertex set and edge set respectively. For a connected graph G, the *eccentricity* $e_G(v)$ of a vertex v in G is the distance to a vertex farthest from v. Thus, $e_G(v) = \{d_G(u, v) : u\in V(G)\}$, where $d_G(u, v)$ is the distance between u and v in G. If there is no confusion, then we simply denote the eccentricity of vertex v in G as e(v) and d(u, v) to denote the distance between two vertices u, v in G respectively. The minimum and maximum eccentricities are the *radius and diameter* of G, denoted r(G) and diam(G) respectively. The neighborhood $N_G(v)$ of a vertex v is the set of all vertices adjacent to v in G. $N_G[v] = N_G(v)\cup\{v\}$ is called the closed neighborhood of v. A set S of edges in a graph G is said to be independent, if no two of the edges in S are adjacent.

The concept of domination in graphs was introduced by Ore [12]. A set $D \subseteq V(G)$ is said to be a *dominating set* of G, if every vertex in $V(G)-D$ is adjacent to some vertex in D. D is said to be a minimal dominating set if $D-\{u\}$ is not a dominating set for any $u\in D$. The *domination number* $\gamma(G)$ of G is the minimum cardinality of a dominating set. We call a set of vertices a γ-set, if it is a dominating set with cardinality $\gamma(G)$. Different types of dominating sets have been studied by imposing conditions on the dominating

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sets. A dominating set D is called a *connected* (*independent*) *dominating* set, if the induced sub graph <D> is connected (independent) [14]. D is called a total dominating set, if every vertex in V(G) is adjacent to some vertex in D[1]. A dominating set D is called a *cycle dominating set*, if the sub graph <D> has a Hamiltonian cycle and is called a *perfect dominating set*, if every vertex in V(G)−D is adjacent to exactly one vertex in D [2]. D is called a *restrained dominating set*, if every vertex in V(G)−D is adjacent to another vertex in V(G)−D[3]. By γ_c, γ_i, γ_t, γ_0, γ_p and γ_r, we mean the minimum cardinality of a connected dominating set, independent dominating set, total dominating set, cycle dominating set, perfect dominating set and restrained dominating set respectively.

Sampathkumar and Pushpalatha[13] introduced the concept of point set domination number of a graph. A set $D \subseteq V(G)$ is called a point set dominating set (psd-set), if for every set $T \subseteq V(G)-D$, there exists a vertex $v \in D$ such that the sub graph $\langle T \cup \{v\} \rangle$ induced by $T \cup \{v\}$ is connected. The point set domination number $\gamma_{ps}(G)$ is the minimum cardinality of a psd-set of G. Kulli and Janakiram[11] introduced the concept of split and non split domination in graphs. A dominating set D of a connected graph G is a split (non split) dominating set, if the induced sub graph <V(G)−D> is disconnected (connected). The split (non split) domination number $\gamma_s(G)$ ($\gamma_{ns}(G)$) of G is the minimum cardinality of a split(non split) dominating set.

The *Boolean function graph* $B(\overline{K_p}, NINC, L(G))$ of G is a graph with vertex set $V(G) \cup E(G)$ and two vertices in $B(\overline{K_p}, NINC, L(G))$ are adjacent if and only if they correspond to two adjacent vertices of G, two adjacent edges of G or to a vertex and an edge not incident to it in G. For brevity, this graph is denoted by $B_2(G)$. In other words, $V(B_2(G)) = V(G) \cup V(L(G))$; and $E(B_2(G)) = [E(T(\overline{G})) - (E(\overline{G}) \cup E(\overline{L(G)}))] \cup E(L(G))$, where $\overline{G}$, L(G) and T(G) denote the complement, the line graph and the total graph of G respectively. The vertices of G and L(G) are referred as point and line vertices respectively.

The mixed relations of incident, non-incident, adjacent and non-adjacent can be used to analyze nature of clustering of elements of communication networks. The concept of domination set can be visualized in each cluster as that cluster representatives and the domination set of whole network can be taken as representatives of entire network. If any clustering or a partition of vertices network such that each cluster having at least one representative or at least one element of dominating set of the given network

In this paper, we determine the domination numbers mentioned above for the complement $\overline{B_2}(G)$ of the graph $B_2(G)$. The definitions and details not furnished in this paper may found in [4].

2. Prior Results

In this section, we list some results with indicated references, which will be used in the subsequent main results.

Theorem 2.1[13]: Let G = (V, E) be a graph. A set $S \subseteq V$ is a point set dominating set of G if and only if for every independent set W in V−S, there exists a vertex u in S such that $W \subseteq N_G(u) \cap (V-S)$.

Observation [10]:

2.2: Degree of a point vertex v in $\overline{B_2}(G)$ is p −1 + $deg_G(v)$ and the degree of line vertex e' is q + 1 - $deg_{L(G)}(e')$.

2.3: $\overline{B_2}(G)$ is connected, for any graph G.

2.4: No vertex of $\overline{B_2}(G)$ is a cut-vertex.

Theorem 2.5[10]: $\overline{B_2}(G)$ is bi-eccentric with radius 1 if and only if $G \cong K_{1,n} \cup mK_1$, $K_2 \cup tK_1$, for $n \geq 2$, $m \geq 0$ and $t \geq 1$.

Theorem 2.6[10]: Let G be a graph with at least three vertices and not totally disconnected. $\overline{B_2}(G)$ is complete if and only if $G \cong nK_1$ or K_2, for $n \geq 2$.

3. Main Results

In the following, we prove that the domination number of $\overline{B_2}(G)$ is at most 3.

Theorem 3.1: $\gamma(\overline{B_2}(G)) = 1$ if and only if G is one of the graphs, nK_1, $K_{1,n} \cup mK_1$, $K_2 \cup tK_1$, where $n \geq 2$ and m, $t \geq 0$.

Proof: By Theorem 2.5. and 2.6., radius of $\overline{B_2}(G))$ is 1 if and only if $G \cong nK_1$ or $K_{1,n} \cup mK_1$, $K_2 \cup tK_1$, where $n \geq 2$ and m, $t \geq 0$. Hence, the theorem follows.

Remark 3.1: For the above graphs, the restrained domination $\gamma_{\geq}$ of $\overline{B_2}(G)$ is 1.

Theorem 3.2: Let G be any graph not totally disconnected and $G \neq K_{1,n} \cup mK_1$, where $n \geq 1$ and $m \geq 0$. Then, $\gamma(\overline{B_2}(G)) = 2$ if and only if one of the following is true.

(i). There exists a point cover for G containing two vertices.

(ii). G contains pendant vertices.

(iii). G contains $K_2 \cup K_1$, P_3 or C_3 as one of its components.

Proof: Assume $\gamma(\overline{B_2}(G)) = 2$. Then there exists a dominating set D of $\overline{B_2}(G)$ containing two vertices.

Case(i): D contains two point vertices.

Then D must contain a pair of vertices v_1 and v_2 such that each edge in G is incident with at least one of v_1 and v_2. Hence, there exists a point cover for G with cardinality 2.

Case(ii): D contains one point vertex v and one line vertex e'.

Let e be the edge in G corresponding to e' and e be incident with v. Let e = (u, v), where $u \in V(G)$. Since D is a dominating set of $\overline{B_2}(G)$, there exists no edge in G incident with u. Hence, u is a pendant vertex in G. If e is not incident with v, then the edges adjacent to e must incident with v or there exist vertices and edges in G not adjacent to v and e respectively. Thus, G contains $K_2 \cup K_1$, P_3 or C_3 as one of its components.

Case(iii): D contains two line vertices.

Then either $G \cong P_3$ or $G \cong 2K_2$.

Conversely, if one of the conditions (i), (ii) and (iii) is true, then there exists a dominating set of $\overline{B_2}(G)$, containing two vertices. Hence, $\gamma(\overline{B_2}(G)) \leq 2$. Since G is not totally disconnected and $G \neq K_{1,n} \cup mK_1$, where $n \geq 1$ and $m \geq 0$, $\gamma(\overline{B_2}(G)) \geq 2$. Therefore, $\gamma(\overline{B_2}(G)) = 2$.

Theorem 3.3: Let G be any graph not totally disconnected. Then $\gamma(\overline{B_2}(G)) \leq 3$.

Proof: Since G is not totally disconnected, there exists an edge, say e = (u, v) $\in E(G)$, where u, $v \in V(G)$. Let e' be the line vertex in $\overline{B_2}(G)$ corresponding to e. Then $D = \{u, v, e'\} \subseteq V(\overline{B_2}(G))$ is a dominating set of $\overline{B_2}(G)$. Hence, $\gamma(\overline{B_2}(G)) \leq 3$. This bound is attained, if $G \cong C_n$, for $n \geq 5$.

Remark 3.2: Since the sub graph of $\overline{B_2}(G)$ induced by D is a cycle on 3 vertices, D is also a connected (total, cycle) dominating set of $\overline{B_2}(G)$. Thus, $\gamma_c(\overline{B_2}(G)) = \gamma_t(\overline{B_2}(G)) \leq 3$ and $\gamma_0(\overline{B_2}(G)) = 3$. Also, $V(\overline{B_2}(G))$–D contains no isolated vertices, D is also a restrained dominating set of $\overline{B_2}(G)$. Hence, $\gamma_r(\overline{B_2}(G)) \leq 3$, when $p \geq 4$.

In the following, upper bounds for independent domination number γ_i of $\overline{B_2}(G)$ are obtained.

Theorem 3.4: $\gamma_i(\overline{B_2}(G)) = 2$ if and only if G contains $K_2 \cup K_1$, P_3 or C_3 as one of its components.

Proof: Follows from Theorem 3.2.

Theorem 3.5: $\gamma_i(\overline{B_2}(G)) \leq 3$ if and only if either G contains $P_3 \cup K_1$ as a sub graph such that the end vertices of P_3 are not adjacent in G or $G \cong C_3$ or $K_{1,3}$.

Proof: Assume $\gamma_i(\overline{B_2}(G)) = 3$. Then there exists an independent dominating set D of $\overline{B_2}(G)$ containing three vertices. Since any two point vertices in $\overline{B_2}(G)$ are adjacent, D contains at most one point vertex.

Case(i): D contains one point vertex v and two line vertices e_1' and e_2'(say).

Then $v \in V(G)$. Let e_1 and e_2 be the edges in G corresponding to e_1' and e_2' respectively. Since D is independent, e_1 and e_2 must be adjacent edges in G and v is not incident with both e_1 and e_2. Since D dominates line vertices of $\overline{B_2}(G)$, edges of G are nonadjacent to at least one of e_1 and e_2 or edges adjacent to both e_1 and e_2 must be incident with v. Hence, G contains $P_3 \cup K_1$ as a sub graph. If the end vertices of P_3 are adjacent in G, then D does not dominate the corresponding line vertex.

Case(ii): D contains 3 line vertices, say e_1', e_2', e_3'.

Let e_1, e_2, e_3 be the corresponding edges in G. Since D is independent, e_1, e_2 and e_3 must be mutually adjacent in G. Since D dominates point vertices of $\overline{B_2}(G)$, all the vertices of G are incident with at least one of the edges e_1, e_2 and e_3. Hence, $G \cong C_3$ or $K_{1,3}$. Converse can be verified easily.

Similarly, the following theorem can be proved.

Theorem3.6: $\gamma_i(\overline{B_2}(G)) \leq 4$ if and only if either G contains $C_3 \cup K_1$ or $K_{1,3} \cup K_1$ as a sub graph or $G \cong K_{1,4}$.

Remark 3.3: In general, $\gamma_i(\overline{B_2}(G)) \leq n$, $n \geq 5$ if and only if either G contains $K_{1,n-1} \cup K_1$ as a sub graph or $G \cong K_{1,n}$.

In the following, the perfect domination number γ_p of $\overline{B_2}(G)$ is determined.

Theorem 3.7: Let G be any graph not totally disconnected having at least three vertices. If there exists a perfect dominating set D with $|D| \geq 2$ in $\overline{B_2}(G)$, then D must contain line vertices only such that the corresponding edges in G are independent.

Proof: Let D be a perfect dominating set in $\overline{B_2}(G)$ such that $|D| \geq 2$. If D contains at least two point vertices, then a point vertex (if any) in $V(\overline{B_2}(G))-D$ is adjacent to all the point vertices in D and hence D cannot be a perfect dominating set of $\overline{B_2}(G)$. Assume $V(\overline{B_2}(G))-D$ contains no point vertices. Let v_1 and v_2 be any two point vertices in D. If v_1 and v_2 are adjacent in G, then the line vertex in $\overline{B_2}(G)$ corresponding to the edge (v_1, v_2) must also be in D, since it is adjacent to both v_1 and v_2 in D. Hence, $G \cong K_2$. If v_1 and v_2 are nonadjacent vertices in G, then $G \cong 2K_1$. But, the number of vertices in G is at least three. Therefore, D must contain exactly one point vertex. Let D contain a point vertex v and a line vertex e' and e be the corresponding edge in G. If e = (v, w), where $w \in V(G)$, then $w \in V(\overline{B_2}(G))-D$ is adjacent to both v and e'. Let e = (u, w), where $u, w \in V(G)$. Then, both $u, w \in V(\overline{B_2}(G))-D$ and are adjacent to both v and e', which is a contradiction. Hence, all the vertices of D must be the line vertices. Let e_1', e_2' be the two line vertices in $\overline{B_2}(G)$ and e_1, e_2 be the corresponding edges in G. If e_1 and e_2 are adjacent edges in G, then the common vertex, say $v_1 \in V(G)$ and hence $v_1 \in V(\overline{B_2}(G))-D$ is adjacent to both e_1' and e_2'. Thus, D contains only line vertices such that the corresponding edges in G are independent.

Remark 3.4:

$$\gamma_p(\overline{B_2}(G)) = 1 \text{ if and only if } G \cong nK_1,\ n \geq 2 \text{ or } K_{1,n} \cup mK_1,\ n \geq 1 \text{ and } m \geq 0.$$

Theorem 3.8: Let G be any graph not totally disconnected and $G \neq K_{1,n} \cup mK_1$, $n \geq 1$ and $m \geq 0$. Then $\overline{B_2}(G)$ has a perfect dominating set if and only if $G \cong nK_2$, $n \geq 2$.

Proof: Let D be a perfect dominating set of $\overline{B_2}(G)$. Then by Theorem 3.7.,D must contain line vertices only and the corresponding edges in G are independent. Since D is a dominating set of $\overline{B_2}(G)$, the set of edges in G corresponding to the line vertices in D is a line cover for G. Let $e_1', e_2' \in D$ and e_1, e_2 be the corresponding edges in G and are independent. If there exists an edge in G adjacent to both e_1 and e_2, then the corresponding line vertex is not adjacent to both e_1' and e_2'. Hence, there exists no edge in G joining the end vertices of e_1 and e_2. Let there exist an edge say, e = (u, v), where $u, v \in V(G)$, adjacent to exactly one of e_1 and e_2, say e_1, then one of the point vertices u and v is not adjacent to both e_1 and e_2. Hence, $G \cong nK_2$, $n \geq 2$.

Conversely, if $G \cong nK_2$, $n \geq 2$, then V(L(G)) is a perfect dominating set of $\overline{B_2}(G)$.

In the following, the graphs G for which the point set domination number γ_{ps} of $\overline{B_2(G)}$ is 1 or 2 are obtained by using Theorem 2.1.

Theorem 3.9: $\gamma_{ps}(\overline{B_2(G)}) = 1$ if and only if $G \cong nK_1, K_{1,m} \cup tK_1$, for $n \geq 2$, $m \geq 1$ and $t \geq 0$.

Theorem 3.10: Let G be not totally disconnected and $G \neq K_{1,n} \cup mK_1$, where $n \geq 1$ and $m \geq 0$. Then, $\gamma_{ps}(\overline{B_2(G)}) = 2$ if and only if one of the following holds.

(i). There exists a point cover $\{v_1, v_2\}$ for G such that $N(v_1) \cap N(v_2) = \Phi$.

(ii). $G \cong C_3 \cup K_2 \cup mK_1, P_3 \cup mK_1, m \geq 0$ or $2K_2 \cup nK_1, n \geq 1$.

Proof: Assume $\gamma_{ps}(\overline{B_2(G)}) = 2$. By Theorem 3.2.,$\gamma(\overline{B_2(G)}) = 2$ if and only if one of the following is true.

(a). There exists a point cover for G containing two vertices.

(b). G contains pendant vertices.

(c). G contains $K_2 \cup K_1$, P_3 or C_3 as one of its components.

Let $D = \{v_1, v_2\}$ be a dominating set of $\overline{B_2(G)}$, where $v_1, v_2 \in V(G)$. Then D is a point cover for G. If $N(v_1) \cap N(v_2) \neq \Phi$, then there exists a vertex, say $v \in V(G)$, adjacent to both v_1 and v_2. Let e_1', e_2' be the line vertices in $\overline{B_2(G)}$ corresponding to the edges (v, v_1) and (v, v_2) respectively. Then $W = \{e_1', e_2'\}$ is an independent set in $V(\overline{B_2(G)}) - D$ and there exists no vertex in D adjacent to both e_1' and e_2'. Hence, $N(v_1) \cap N(v_2) = \Phi$. Assume G contains pendant vertices. Let u be a pendant vertex in G and $e = (v, u) \in E(G)$, where $v \in V(G)$. Then $D = \{v, e'\} \subseteq V(\overline{B_2(G)})$ is a point set dominating set (psd-set) of $\overline{B_2(G)}$ if and only if all the edges of G are incident with v. Thus, $G \cong K_{1,m}$, for $m \geq 2$. But $\gamma(\overline{B_2(K_{1,m})}) = 1$, $m \geq 2$. Assume G contains $K_2 \cup K_1$, P_3 or C_3 as one of its components. $D = \{v, e'\}$, where e' is the line vertex in $\overline{B_2(G)}$ corresponding to an edge e, incident with the vertex v, where $v \in V(G)$. Then D is a psd-set of $\overline{B_2(G)}$, if $G \cong C_3 \cup K_2 \cup mK_1, P_3 \cup K_2 \cup mK_1, m \geq 0$ or $G \cong 2K_2 \cup nK_1, n \geq 1$. Converse follows easily.

The next theorem relates point set domination number γ_{ps} of $\overline{B_2(G)}$ with the point covering number of G.

Theorem 3.11: Let G be any graph not totally disconnected. Then, $\gamma_{ps}(\overline{B_2(G)}) \leq \alpha_0(G)$ if and only if there exists a point cover D of G with $|D| = \alpha_0(G)$ satisfying

(i). <D> has no triangles; and

(ii). For any two vertices v_1 and v_2 in D, $[N_G(v_1) \cap N_G(v_2)] \cap (V(G)-D) = \Phi$.

Proof: Assume conditions (i) and (ii). Since D is a point cover for G, V(G)–D is independent. Also since any two point vertices in $\overline{B_2}(G)$ are adjacent, any independent set W in <V($\overline{B_2}$(G))–D> contains at most one point vertex.

(i). Let W contain one point vertex and the other vertices be the line vertices. Then the edges in G corresponding to the line vertices in W are adjacent in G and the vertex in G corresponding to the point vertex in W is not incident with these edges. Since V(G)–D is independent, there exists a vertex in D adjacent to all the vertices in W.

(ii). Let W contains line vertices only. By the assumption, the edges corresponding to the line vertices are the edges of a star, the point vertex in $\overline{B_2}(G)$ corresponding to the center vertex of the star is in D. Hence, there exists a point vertex v in D such that all the vertices of W are adjacent to v. Thus, D is a psd-set of $\overline{B_2}(G)$. $\gamma_{ps}(\overline{B_2}(G)) \leq \alpha_0(G)$. Conversely, assume a point cover D of G is a psd-set of $\overline{B_2}(G)$. If <D> has triangles, then the set of line vertices in $\overline{B_2}(G)$ corresponding to the edges of a triangles is an independent set of V($\overline{B_2}$(G))–D and there exists no vertex in D adjacent to these line vertices, which is a contradiction. Let v_1 and v_2 be any two vertices in D, both are adjacent to the vertex v in V(G)–D. Then the set of line vertices in $\overline{B_2}(G)$ corresponding to the edges (v_1, u) and (v_2, u) is an independent set in V($\overline{B_2}$(G))–D and there exists no vertex in D is adjacent to these line vertices, which is a contradiction. Hence, conditions (i) and (ii) are true.

Remark 3.5: The set of all point vertices in $\overline{B_2}(G)$ is a psd-set of $\overline{B_2}(G)$ if and only if G is triangle-free.

Example 3.1:

(i). $\gamma_{ps}(\overline{B_2}(P_n))$ = 2, if n = 4; and
= 3, if $n \geq 5$.

(ii). $\gamma_{ps}(\overline{B_2}(C_n))$ = 2, if n = 4; and
= 3, if $n \geq 5$

(iii). $\gamma_{ps}(\overline{B_2}(K_n))$ = n, if $n \geq 4$.

In the following, the bounds for the split domination number γ_s of $\overline{B_2}(G)$ are determined.

Theorem 3.12: $\gamma_s(\overline{B_2}(G)) \leq p + \delta(G) - 1$, where $\delta(G)$ is the minimum degree of G.

Proof: Let v be a vertex of minimum degree in G and let D′ be the set of line vertices in $\overline{B_2}(G)$ corresponding to the edges incident with v. Then D = (V(G)−v)∪D′ ⊆ V($\overline{B_2}(G)$) is a dominating set of $\overline{B_2}(G)$. Also <V($\overline{B_2}(G)$)–D> is disconnected, since v is an isolated vertex in <V($\overline{B_2}(G)$)-D>. Hence, D is a split domination set of $\overline{B_2}(G)$. Thus, $\gamma_s(\overline{B_2}(G)) \leq p + \delta(G) - 1$.

This bound is attained, if $G \cong K_2$.

Theorem 3.13: For any graph G, $\gamma_s(\overline{B_2}(G)) \leq p + \delta_e(G) - 1$, where $\delta_e(G)$ is the minimum degree of L(G).

Proof: Let $e' \in V(L(G))$ be such that $deg_{L(G)}(e') = \delta_e(G)$ and e = (u, v) be the corresponding edge in G, where u, v∈V(G). Then, D = {V(G)–{u, v}, $N_{L(G)}[e']$} ⊆ V($\overline{B_2}(G)$) is a dominating set of $\overline{B_2}(G)$. Also <V($\overline{B_2}(G)$)–D> contains K_2 as one of its components and hence disconnected. Hence, D is a split dominating set of $\overline{B_2}(G)$ and $\gamma_s(\overline{B_2}(G)) \leq p - 2 + \delta_e(G) + 1$. Thus, $\gamma_s(\overline{B_2}(G)) \leq p + \delta_e(G) - 1$.

This bound is attained, if $G \cong 2K_2$.

Theorem 3.14: Let G be any graph with $\Delta(G) \geq 2$. Then, $\gamma_s(\overline{B_2}(G)) \leq p - 1$.

Proof: Let v∈V(G) be such that $deg_G(v) = \Delta(G) \geq 2$ and D′ be the set of line vertices in $\overline{B_2}(G)$ corresponding to the edges in G incident with v. Then D = D′∪{V(G)–N[v]} ⊆ V($\overline{B_2}(G)$) is a dominating set of $\overline{B_2}(G)$ and v is isolated in <V($\overline{B_2}(G)$)–D> and is disconnected. Hence, D is a split dominating set of $\overline{B_2}(G)$. Thus, $\gamma_s(\overline{B_2}(G)) \leq \Delta(G)+p-(\Delta(G)+1)$ and $\gamma_s(\overline{B_2}(G)) \leq p-1$.

Remark 3.6: If v is a vertex of maximum degree in G, then D = {V(G)–N(v), line vertices in $\overline{B_2}(G)$ corresponding to the edges incident with vertices in N[v]} is a split dominating set of $\overline{B_2}(G)$.

Next, an upper bound of γ_s of $\overline{B_2}(G)$ is given in terms of the number of edges of G.

Theorem 3.15: For any graph G with at least one edge, $\gamma_s(\overline{B_2}(G)) \leq q + 1$.

Proof: Let v be a vertex of maximum degree in G. Let D′ be the set of line vertices in $\overline{B_2}(G)$ corresponding to the edges not incident with v in G. Then D = N[v]∪D′ ⊆ V($\overline{B_2}(G)$) is a dominating set of $\overline{B_2}(G)$ and <V($\overline{B_2}(G)$)–D> contains isolated vertices. Hence, D is a split dominating set of $\overline{B_2}(G)$ and $\gamma_s(\overline{B_2}(G) \leq q + 1$.

This bound is attained, if $G \cong 3K_2$.

Remark 3.7: If L(G) is disconnected, then $\gamma_s(\overline{B_2}(G)) \leq p$.

In the following, we characterize the graphs G for which γ_s of $\overline{B_2}(G)$ is 2.

Theorem 3.16: Let G be any graph with at least three vertices. Then $\gamma_s(\overline{B_2}(G)) = 2$ if and only if there exists a point cover D for G with $|D| = 2$ and $\langle D \rangle \cong K_2$.

Proof: Assume $\gamma_s(\overline{B_2}(G)) = 2$. Then there exists a split dominating set D of $\overline{B_2}(G)$ containing two vertices. If D contains either two line vertices or one point and one line vertex, then $\langle V(\overline{B_2}(G)) - D \rangle$ is connected. Therefore, D contains two point vertices say, v_1 and v_2. Since D is a dominating set of $\overline{B_2}(G)$, each edge in G is incident with at least one of v_1 and v_2. If v_1 and v_2 are nonadjacent in G, then $\langle V(\overline{B_2}(G)) - D \rangle$ is connected and hence v_1 and v_2 are adjacent in G. Hence, there exists a point cover $D = \{v_1, v_2\}$ for G such that v_1 and v_2 are adjacent. Conversely, if there exists a point cover D for G with $|D| = 2$ and $\langle D \rangle \cong K_2$, then $\gamma_s(\overline{B_2}(G)) \leq 2$. From Theorem 3.1., $\gamma(\overline{B_2}(G)) = 1$ if and only if $G \cong nK_1, K_{1,n} \cup mK_1$, where $n \geq 1$ and $m \geq 0$. But G is a graph with at least three vertices. Hence, $\gamma_s(\overline{B_2}(G)) \geq 2$. Thus, $\gamma_s(\overline{B_2}(G)) = 2$.

Example 3.2:

(i). $\gamma_s(\overline{B_2}(P_n)) = 2$, if $n = 3$; and

$= n - 2$, if $n \geq 4$.

(ii). $\gamma_s(\overline{B_2}(C_n)) = n - 1$, if $n \geq 3$.

(iii). $\gamma_s(\overline{B_2}(K_{1,n})) = 2$, if $n \geq 1$.

In the following, we prove the non split domination number γ_{ns} of $\overline{B_2}(G)$ is at most 3.

Theorem 3.17: Let G be any graph with at least three vertices. Then, $\gamma_{ns}(\overline{B_2}(G)) = 1$ if and only if $G \cong nK_1$, $n \geq 3$ or $G \cong K_{1,n} \cup mK_1$, $m, n \geq 1$.

Theorem 3.18: Let G be any graph not totally disconnected with at least three vertices and $G \neq K_{1,n} \cup mK_1$, where $m, n \geq 1$. Then $\gamma_{ns}(\overline{B_2}(G)) = 2$ if and only if one of the following holds.

(i). There exists an independent point cover for G with cardinality 2.

(ii). δ(G) = 1; and

(iii). G contains $K_2 \cup K_1$, P_3 or C_3 as one of its components.

Theorem 3.19: Let G be any graph with at least 4 vertices. If none of the conditions (i), (ii) and (iii) is not true, then $\gamma_{ns}(\overline{B_2}(G)) = 3$.

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