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Acyclic Distance Closed Domination In Graph

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Abstract: *In a graph G=(V,E), a set $S \subset V(G)$ is a distance closed set of G if for each vertex $u \in S$ and for each $w \in V$-S, there exists at least one vertex $v \in S$ such that $d_{<S>}(u, v) = d_G(u, w)$. Also, a vertex subset D of V(G) is a dominating set of G if every vertex in V-D is adjacent to some vertex in D. Combining the above concepts, a distance closed dominating set of a graph G is defined as follows: A subset $S \subseteq V(G)$ is said to be a distance closed dominating (D.C.D) set, if <S> is distance closed and S is a dominating set. In this paper, we define a new concept of domination called acyclic distance closed domination (A.D.C.D) and analyze some structural properties of graphs and extremal problems relating to the above concepts.*

Keywords: *domination number, distance, eccentricity, radius, diameter, self centered graph, neighborhood, induced sub graph, acyclic, unicyclic graph, geodetic graph, distance closed dominating set, acyclic distance closed dominating set.*

1. Introduction

Graphs discussed in this paper are connected and simple graphs only. For a graph, let V(G) and E(G) denotes its vertex and edge set respectively. The *degree* of a vertex v in a graph G is denoted by $deg_G(v)$. The length of any shortest path between any two vertices u and v of a connected graph G is called the *distance between u and v* and it is denoted by $d_G(u, v)$. The distance between two vertices in different components of a disconnected graph is defined to be ∞. For a connected graph G, the *eccentricity* $e_G(v) = \max \{d_G(u,v): \forall u \in V(G)\}$. If there is no confusion, we simply use the notion
deg(v), d(u, v) and e(v) to denote degree, distance and eccentricity respectively for the connected graph. The minimum and maximum eccentricities are the *radius* and *diameter* of G, denoted by r(G) and diam (G) respectively. If these two are equal in a graph, that graph is called *self-centered* graph with radius r and is called an *r self-centered* graph. Such graphs are 2-connected graphs. A vertex u is said to be an eccentric vertex of v in a graph G, if d (u, v)=e(v) in that graph. In general, u is called an *eccentric vertex*, if it is an eccentric vertex of some vertex. For $v \in V(G)$, the *neighborhood* $N_G(v)$ of v is the set of all vertices adjacent to v in G. The set $N_G[v] = N_G(v) \cup \{v\}$ is called the *closed neighborhood* of v. A set S of edges in a graph is said to be independent if no two of the edges in S are adjacent. An edge e=(u, v) is a *dominating edge* in a graph G if every vertex of G is adjacent to at least one of u and v. For any set S of vertices in G, the induced sub graph

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<S> is the maximal sub graph with vertex set S. Also, a sub graph H of G is a component of G if H is a maximal connected sub graph of G. The concept of distance and related properties are studied in [2], [3], [13] and [14]. Also, the structural properties of some special class of graphs such as self centered graphs, radius critical graphs and eccentricity preserving spanning trees are studied in [4], [7] and [8].

One important aspect of the concept of distance and eccentricity is the existence of polynomial time algorithm to analyze them. The concept of domination in graphs originated from the chess games theory and that paved the way to the development of the study of various domination parameters and its relation to various other graph parameters. The list of survey of domination theory papers are in [16] and [17].

A set $D \subseteq V(G)$ is called dominating set of G if every vertex in V(G)-D is adjacent to some vertex in D and D is said to be a minimal dominating set if D-{v} is not a dominating set for any $v \in D$. The *domination number* $\gamma(G)$ is the minimum cardinality of a dominating set. We call a set of vertices a γ-set if it is a dominating set with cardinality $\gamma(G)$.Different types of dominating sets have been studied by imposing conditions on the dominating sets. A dominating set D is called *connected* (*independent*) dominating set if the induced sub graph <D> is connected (independent). D is called a *total dominating set* if every vertex in V(G) is adjacent to some vertex in D. A set D is called an *efficient dominating set* of G if every vertex in V-D is adjacent to exactly one vertex in D. A set $D \subseteq V(G)$ is called a *global dominating set* if D is a dominating set of G and $\overline{G}$. A set D is called a *restrained dominating set* if every vertex in V-D is adjacent to a vertex in D and another vertex in V-D. A set D is called an *acyclic dominating set* if <D> has no cycle. By γ_c, γ_i, γ_t, γ_e, γ_g, γ_r and γ_a, we mean the minimum cardinality of a connected dominating set, independent dominating set, total dominating set, efficient dominating set, global dominating set, restrained dominating set and acyclic dominating set respectively. The connected and acyclic domination in various graphs and their structural properties are studied in [5], [6], [11], [12] and [15].

The new concepts such as distance closed sets, distance preserving sub graphs, eccentricity preserving sub graphs, Super eccentric graph of a graph, pseudo geodetic graphs are introduced and structural properties of those graphs are studied in [9]. Janakiraman and Alphonse [1] introduced and studied the concept of weak convex dominating sets, which mixes the concept of dominating set and distance preserving set. Using these, structural properties of various dominating parameters are studied. Continuing the above study, the concept of distance closed dominating set was defined and the structural properties of distance closed domination in various graphs are studied in [10].

In this paper, we introduce a new dominating set called acyclic distance closed dominating set of a graph through which we studied the properties of the graph. We find upper and lower bounds of the new domination number for various graphs such as self centered graph, bipartite graph and unicyclic graph. Also, the Nordhaus-Gaddum type results for the acyclic distance closed domination number are established.

2. Prior Results

The concept of distance closed set is defined and studied in the doctoral thesis of Janakiraman [9] and the concept of distance closed sets in graph theory is due to the related concept of ideals in ring theory in algebra. The ideals in a ring are defined with respect to the multiplicative closure property with the elements of that ring. Similarly, the distance closed dominating set is defined with respect to the distance closed property and the dominating set of the graph. Thus, the distance closed dominating set of a graph G is defined as follows:

A subset $S \subseteq V(G)$ is said to be a distance closed dominating (D.C.D) set, if

(i) <S> is distance closed;

(ii) S is a dominating set.

The cardinality of a minimum D.C.D set of G is called the distance closed domination number of G and is denoted by γ_{dcl}.

Clearly, from the definition, $1 \leq \gamma_{dcl} \leq p$ and graph with $\gamma_{dcl}=p$ is called a 0-distance closed dominating graph. Also, if S is a D.C.D set of G then the complement V-S need not be a D.C.D set of G. The definition and the extensive study of the above said distance closed dominating set in Graphs are studied in [10].

Following are some of the results related to the distance closed domination number of a graph presented in [10].

Theorem 2.1 [10]: If T is a tree with number of vertices $p \geq 2$, then $\gamma_{dcl}(T) = p-k+2$, where k is the number of pendant vertices in T.

Theorem 2.2 [10]: Let G be a self centered graph of diameter 2.Then $\gamma_{dcl}(G) \leq \delta+2$.

Theorem 2.3 [10]: Let G be a graph of order p. Then

(i) $\gamma_{dcl}(G)=2$ if and only if G has at least two vertices of degree p-1.

(ii) If G has exactly a vertex of degree p-1, then $\gamma_{dcl}(G)=3$.

Theorem 2.4 [10]: Let G be a graph of order p. If G has exactly a vertex of degree p-1, then $\gamma_{dcl}(G)=3$.

Theorem 2.5 [10]: If a graph G is connected and diam(G) ≥3, then $\gamma_{dcl}(\overline{G})=4$.

Theorem 2.6 [10]: For any connected graph G such that $\overline{G}$ is also connected, $\gamma_{dcl}(G)+\gamma_{dcl}(\overline{G})\leq p+4$, where $\gamma_{dcl}(G)$ and $\gamma_{dcl}(\overline{G})$are the cardinality of minimal distance closed dominating set of G and $\overline{G}$ respectively.

3. Main Results:

In this paper, we define a new domination parameter namely, acyclic distance closed domination as follows.

Acyclic distance closed dominating sets:

A distance closed dominating (D.C.D) set D is said to be an acyclic distance closed dominating (A.D.C.D) set if <D> is acyclic.

The cardinality of the minimum A.D.C.D set is called an acyclic distance closed domination number and it is denoted by γ_{adcl}.

There are many graphs for which γ_{adcl} do not exist. For example, the set of all graphs with radius greater than or equal to 2 with all vertices having first neighborhood of them forming a complete sub graph, will not have an A.D.C.D set. The following theorems give us the bounds of some special class of graphs.

Proposition 3.1: If T is a tree with p vertices, then $\gamma_{adcl}(T)=p-k+2$, where k is the number of pendant vertices of T.

Proof: As any tree is acyclic, we have the proof.

Proposition 3.2: $\gamma_{dcl}\leq\gamma_{adcl}$, for any graph G, if γ_{adcl} exist in G.

Proof: Any acyclic D.C.D set is a D.C.D set.

=> | minimal D.C.D set| ≤ | minimal A.D.C.D set|.

=> $\gamma_{dcl}\leq\gamma_{adcl}$.

Proposition 3.3: If G is a graph (not a path) in which γ_{adcl} exist, then $\gamma_{adcl}(G)\leq p-1$.

Remark 3.1: The bound p-1 exists for the following graphs

(i) Odd cycles;

(ii) Tree with 3 pendant vertices.

Theorem 3.1: If G is a graph with radius 1, then $\gamma_{adcl}(G)\leq 3$.

Proof: Let G be a graph with radius 1 and diameter d ≤ 2.

Case (i): If d=1 then clearly, any two vertices will form an A.D.C.D set of G. Hence, $\gamma_{adcl}(G)$)=2.

Case (ii): If d=2 then we have the following sub cases.

Sub case (a): If G has more than one vertex with eccentricity 1 then also $\gamma_{adcl}(G)$)=2.

Sub case (b): If G has exactly one vertex with eccentricity 1 (say v) then the set {u, v, u'} will form an A.D.C.D set of G (where u is the vertex with eccentricity 2 and u' is its eccentric point in G). Hence $\gamma_{adcl}(G)$)=3.

Proposition 3.4: If G is a 2-self centered graph and for every $v \in V(G)$, $<N_1(v)>$ and $<N_2(v)>$ are independent, then G has no A.D.C.D set.

Proof: Let G be a 2-self centered graph and for every $v \in V(G)$, $<N_1(v)>$ and $<N_2(v)>$ are independent. Then clearly, every vertex in $N_1(v)$ is adjacent to all the vertices of $N_2(v)$. Hence every four vertices {v, u, w, x}, where u, $w \in N_1(v)$ and $x \in N_2(v)$ must have a cycle and hence G has no A.D.C.D set.

Proposition 3.5: If G is a 2-self centered graph and for a vertex $v \in V(G)$, $<N_1(v)>$ is a clique and at least one vertex of $N_1(v)$ has a unique eccentric point, then G must have a A.D.C.D set and $\gamma_{adcl}(G)=4$.

Proof: Let G be a 2-self centered graph and for a vertex $v \in V(G)$, $<N_1(v)>$ is a clique. Then clearly every vertex in $<N_1(v)>$ has its eccentric vertex in $N_2(v)$ only. Let $u \in N_1(v)$ and let x be its unique eccentric point in $N_2(v)$. Then the set {v, u, y, x} forms an A.D.C.D set of G, where $y \in N_2(v)$. Hence $\gamma_{adcl}(G)=4$.

Proposition 3.6: For any geodetic graph G (which is self centered graph of diameter 2 and triangle free), γ_{adcl} exists and $\gamma_{adcl}(G) \leq \delta+2$.

Proof: Let G be a geodetic graph which is self centered graph of diameter 2. Then, G doesn't contain an even cycle (otherwise, there exist two shortest paths between every pair of vertices within the cycle). Since G is 2 self centered, it must contain at least one C_5. That is, G cannot be a triangulated graph. Hence any 4 vertices in C_5 will form an A.D.C.D set of G, if $\gamma_{dcl}(G)=4$. Also, if $\gamma_{dcl}(G)>4$ then , $\gamma_{adcl}(G) \leq \delta+2$.

Theorem 3.2: Let G be a self centered graph of diameter 2. If v is a vertex in G such that $N_1(v)$ has two non adjacent vertices of degree 2 and $|N_2(v)| \geq 2$, then $\gamma_{adcl}(G)=4$.

Proof: Let G be a self centered graph of diameter 2. Let v_1, v_2 be two non adjacent vertices of degree 2. Since the graph is of diameter 2, v_1 and v_2 are adjacent to some vertex, say v. Also v_1 and v_2 are of degree 2, they must be adjacent to the vertices say v_1' and v_2' respectively.

Case 1: $v_1' = v_2'$.

Case 1.1: $v_1' \in N_1(v)$.

Then all the vertices in $N_2(v)$ are adjacent to v_1', to maintain distance 2 between them and $\{v_1, v_2\}$. Therefore, the set $\{v, v_1'\}$ forms a dominating set of G and also the eccentric node of v_1' must be in $N_1(v)$ itself. Let $u \in N_1(v)$ be the eccentric node of v_1'.

Sub case (i): If u is adjacent to all the vertices of $N_2(v)$ then the set $\{v_1, v, u, x\}$, where $x \in N_2(v)$ will form an A.D.C.D set for G. Thus $\gamma_{adcl}(G)=4$.

Sub case (ii): If u is not adjacent to at least one vertex of $N_2(v)$, say x then the set $\{u, v, v_1', x\}$ will form an A.D.C.D set for G. Thus $\gamma_{adcl}(G)=4$.

Case 1.2: $v_1' \in N_2(v)$.

In this case, all the vertices in $N_2(v)$ are adjacent to v_1'. Therefore, $\{v, v_1, v_1', u\}$ where $u \in N_2(v)$ will form an A.D.C.D set of G. Hence $\gamma_{adcl}(G)=4$.

Case 2: $v_1' \neq v_2'$.

Case 2.1: both v_1' and v_2' belongs to $N_1(v)$

Here also to maintain the distance all the vertices of $N_2(v)$ are adjacent to both v_1' and v_2'. Thus, $\{v_2, v, v_1', u\}$ where $u \in N_2(v)$ form an A.D.C.D set for G. Hence $\gamma_{adcl}(G)=4$.

Case 2.2: suppose $v_1' \in N_1(v)$ and $v_2' \in N_2(v)$

Clearly all the vertices of $N_2(v)$ are adjacent to both v_1' and v_2'. Thus, $\{v_2, v, v_1', u\}$ where $u \in N_2(v)$ form an A.D.C.D set for G. Hence $\gamma_{adcl}(G)=4$.

Case 2.3: both $v_1', v_2' \in N_2(v)$

Then all the vertices of $N_2(v)$ are adjacent to both v_1' and v_2'. Therefore, the set of vertices $\{v_2, v, v_1, v_1'\}$, where $u \in N_2(v)$ form an A.D.C.D set for G. Hence $\gamma_{adcl}(G)=4$.

Theorem 3.3: Let G be a self centered graph of diameter 2. If G has any two adjacent vertices of degree 2, then $\gamma_{adcl}(G)=4$.

Proof: Let G be a self centered graph of diameter 2. Let u and v be any two adjacent vertices of degree 2. Let $w \neq v$ be a vertex adjacent to u.

Claim: w is not adjacent to v.

If not, let v and w be adjacent in G. Then all the vertices other than u, v and w must be adjacent to w to maintain the distance 2 from u and v. This implies that radius of G is 1, a contradiction to G is self centered of diameter 2. Thus v and w are non adjacent.

Therefore, we have $u \in N_1(w)$ and $v \in N_2(w)$.

Case1: If v is adjacent to some vertex $x \in N_1(w)$ other than u, then all other vertices in $N_1(w)$ must be adjacent to x to maintain the distance 2 from v and also adjacent to w. Clearly in this case, $N_2(w)=\{v\}$ only. Otherwise the distance from u to any other vertex in

$N_2(w)$ different from v becomes 3. Thus the edge {w, x} forms a dominating set for G. Also, the set {y, w, u, v} where $y \in N_1(w)$ forms an A.D.C.D set for G and hence $\gamma_{adcl}(G)=4$.

Case 2: If v is not adjacent to any vertex of $N_1(w)$. Clearly $|N_2(w)|=2$. Let $x \in N_2(w)$ other than v. Now all the vertices of $N_1(w)$ other than u must be adjacent to x to maintain the distance between them and v. Thus the set {w, x} forms a dominating set for G. Hence {w, u, v, x} forms an A.D.C.D set for G and hence $\gamma_{adcl}(G)=4$.

Proposition 3.7: Let G be a self centered graph of diameter 2. If for a vertex u, there exists some adjacent vertices w, $v \in N_2(u)$ such that $N_1(w) \cap N_1(v)=\Phi$, then $\gamma_{adcl}(\overline{G})=4$.

Proof: Let G be a self centered graph of diameter 2. Suppose, for a vertex u, there exists some adjacent vertices w, $v \in N_2(u)$ such that $N_1(w) \cap N_1(v)=\Phi$. Then clearly the set {w, v} forms a dominating set for $\overline{G}$ and also the set {v, u, w, x}, where $x \in N_1(w)$ or $N_1(v)$ will form an A.D.C.D set in $\overline{G}$. Hence, $\gamma_{adcl}(\overline{G})=4$.

Theorem 3.4: If G is a graph with diameter ≥3, then $\gamma_{adcl}(\overline{G})=4$.

Proof: Let G be a graph of diameter≥3. Then any pair of vertices (u, v) in G of distance equal to 3 will dominate the whole of $\overline{G}$. Also the 4 vertices in the shortest path connecting u to v will form an A.D.C.D set for $\overline{G}$. Hence $\gamma_{adcl}(\overline{G})=4$.

Corollary 3.1: If G is a graph with diameter equal to 3 and $\gamma_{dcl}(G)=4$, then G has a global A.D.C.D set.

Proof: Let G be a graph with diameter 3 and $\gamma_{dcl}(G)=4$. Then, G has at least one A.D.C.D set D such that <D> is a path. Hence by Theorem 3.4, D is also an A.D.C.D set of $\overline{G}$ and $\gamma_{adcl}(\overline{G})=4$.Hence, D becomes a global A.D.C.D set of G.

Theorem 3.5: If G is a graph (not a path) with diameter ≥3, then $\gamma_{adcl}(G)+\gamma_{adcl}(\overline{G}) \leq p+3$.

Proof: For any graph G with diameter ≥3, $\gamma_{adcl}(G) \leq p-1$and $\gamma_{adcl}(\overline{G})=4$(from proposition 3.3 and Theorem 3.4). Hence, $\gamma_{adcl}(G)+\gamma_{adcl}(\overline{G}) \leq p+3$.

Remark 3.2: The bound is attainable for cycles and trees with 3 pendant vertices.

Proposition 3.8: If G is a tree on p vertices, then $\gamma_{adcl}(\overline{G}) \leq 4$.

Proof: Let G be a tree on p vertices with radius r.

Case1: r=1.

In this case clearly, G= $K_{1,n}$. Hence $\gamma_{adcl}(\overline{G})=3$.

Case2: r≥2.

If r≥2, then diameter of G must be greater than or equal to 3. Hence any 4 vertices which form a P_4 in G will form an A.D.C.D set of $\overline{G}$. Hence $\gamma_{adcl}(\overline{G})=4$.

Proposition 3.9: If G is a unicyclic graph with p vertices having a cycle C_n such that at most n-1 vertices of C_n each of which dominates uniquely some vertex in G, then $\gamma_{adcl}(G) \leq$ p-1.

Proof: Let G be a unicyclic graph with p vertices having a cycle C_n such that at most n-1 vertices of C_n each of which dominates uniquely some vertex in G. Let v be a vertex of C_n such that no vertex of G is uniquely dominated by v. Thus, d(v)=2 in G and <G-v> is a tree. Hence V(G)-v will form an A.D.C.D set of G and hence $\gamma_{adcl}(G) \leq$ p-1.

Theorem 3.6: If G is a ciliate, then it doesn't have an A.D.C.D set.

Proof: Let G be a ciliate on p vertices. We know that ciliate is a 0-D.C.D graph. That is $\gamma_{dcl}(G)=p$. If γ_{adcl} exists in G, then $p=\gamma_{dcl} \leq \gamma_{adcl}$. This implies that $\gamma_{adcl}(G)=p$, that is A.D.C.D set of G is the whole of V(G), which is not possible as G is having a cycle. Hence G does not have any A.D.C.D set.

Theorem 3.7: Any prism(G) of a graph G on n vertices, doesn't have an A.D.C.D set.

Proof: The Prism(G) is a graph obtained by taking two isomorphic copies of G and place edges between vertices that correspond under the isomorphism and a n-prism has 2n nodes and 3n edges. Also, prism(G) is (n+1) self centered. Therefore, $\gamma_{dcl}(prism(G)) \geq 2(n+1)$. Hence, prism(G) cannot have an A.D.C.D set.

Theorem 3.8: If G is a complete bipartite graph, then G doesn't have an A.D.C.D set

Proof: Let G be a complete bipartite graph. Then clearly, G is a self centered graph of diameter 2 and every distance closed dominating set contains a 4-cycle. Hence G has no A.D.C.D set.

Theorem 3.9: Let G be 2-self centered graph with $\delta \geq p-3$ and $\Delta=p-2$. If for every pair of non adjacent vertices at least one of them is of degree p-2 in G, then G has no A.D.C.D set.

Proof: Let G be a 2-self centered graph with $\delta \geq p-3$ and $\Delta=p-2$. We have to prove that G has no A.D.C.D set. Suppose that G has an A.D.C.D set D. Let it be D=<u, v, w, . . . x>. Then clearly, |D| ≥ 4. Since D is an A.D.C.D set, the induced sub graph of D, <D> does not contain a cycle. That is <D> is either a path or a tree.

In both the cases, the vertices u and x are pendant vertices of <D>. That is these two vertices are adjacent to exactly one vertex of D. So they are not adjacent at least two vertices of D. Thus the degree of the vertices u and x are at most p-3 in G.

Since the vertices u and x are not adjacent in <D>, they are not adjacent in G also. Hence u and x are any two non adjacent vertices of G with degree at most p-3, which is a contradiction to the fact that, in G for every pair of non adjacent vertices at least one of them is of degree p-2. Hence G has no A.D.C.D set.

Corollary 3.2: If G is a (p-2) regular graph, then G has no A.D.C.D set.
Proof: Any (p-2) regular graph is 2-self centered and hence from theorem 3.9, we have the result.

Open problem:

Find the structure of graphs with a given diameter $d \geq 2$ having $\gamma_{dcl} < \gamma_{adcl}$.

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