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Weak Convex Restrained Domination in Graphs

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Abstract: *In a graph G = (V, E), a set D$\subset$V is a weak convex set if $d_{<D>}(u,v) = d_G(u,v)$ for any two vertices u, v in D. A weak convex set D is called as a weak convex dominating (WCD) set if each vertex of V-D is adjacent to at least one vertex in D. A weak convex dominating set D is called weak convex restrained dominating(WCRD) set if every vertex in V(G)-D is adjacent to a vertex in D and another vertex in V(G)-D. The domination number $\gamma_{rc}(G)$ is the smallest order of a weak convex restrained dominating set of G and the codomination number of G, written $\gamma_{rc}(\overline{G})$, is the weak convex restrained domination number of its complement. In this paper we found various bounds for these parameters and characterized the graphs which attain these bounds.*

Keywords: *domination number, distance, eccentricity, radius, diameter, self-centered, neighbourhood, weak convex set, weak convex dominating set, weak convex restrained dominating set.*

1. Introduction

Graphs discussed in this paper are undirected and simple. Unless otherwise stated the graphs which we consider are connected graphs only. For a graph G, let V(G) and E(G) denote its vertex and edge set respectively and p and q denote the cardinality of those sets respectively. The *degree* of a vertex v in a graph G is denoted by $\deg_G(v)$. The minimum and maximum degree in a graph is denoted by δ and Δ respectively. The length of any shortest path between any two vertices u and v of a connected graph G is called the *distance between u and v* and is denoted by $d_G(u, v)$. The distance between two vertices in different components of a disconnected graph is defined to be ∞. For a connected graph G, the *eccentricity* $e_G(v) = \max\{d_G(u, v): u \in V(G)\}$. If there is no confusion, we simply use the notion $\deg(v)$, $d(u, v)$ and $e(v)$ to denote degree, distance and eccentricity respectively for the concerned graph. The minimum and maximum eccentricities are the *radius* and *diameter* of G, denoted r(G) and diam(G) respectively. When these two are equal, the graph is called *self-centered* graph with radius r, equivalently is *r self-centered*. A vertex u is said to be an *eccentric vertex* of v in a graph G, if $d(u, v) = e(v)$. In general, u is called an eccentric vertex, if it is an eccentric vertex of some vertex. For $v \in V(G)$, the *neighbourhood* $N_G(v)$ of v is the set of all vertices adjacent to v in G. The set $N_G(v) = N_G(v) \cup \{v\}$ is called the *closed neighbourhood* of v. A set S of edges in a graph is said to

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be *independent* if no two of the edges in S are adjacent. An edge e = (*u*, *v*) is a *dominating edge* in a graph G if every vertex of G is adjacent to at least one of *u* and *v*.

The concept of domination in graphs was introduced by Ore. A set $D \subseteq V(G)$ is called *dominating set* of G if every vertex in V(G)-D is adjacent to some vertex in D. D is said to be a *minimal* dominating set if D-{*v*} is not a dominating set for any $v \in D$. The *domination number* γ(G) of G is the minimum cardinality of a dominating set. We call a set of vertices a γ-set if it is a dominating set with cardinality γ(G). Different types of dominating sets have been studied by imposing conditions on the dominating sets. A dominating set D is called *connected* (*independent*) dominating set if the induced subgraph <D> is connected (independent). D is called a *total dominating set* if every vertex in V(G) is adjacent to some vertex in D.

A cycle of D of a graph G is called a *dominating cycle* of G, if every vertex in V – D is adjacent to some vertex in D. A dominating set D of a graph G is called a *clique dominating set* of G if <D> is complete. A set D is called an *efficient dominating set* of G if every vertex in V – D is adjacent to exactly one vertex in D. A set $D \subseteq V$ is called a *global dominating set* if D is a dominating set in G and $\overline{G}$. A set D is called a *restrained dominating set* if every vertex in V(G)-D is adjacent to a vertex in D and another vertex in V(G)-D. A set D is a *weak convex dominating set* if each vertex of V-D is adjacent to at least one vertex in D and $d_{<D>}(u,v) = d_G(u,v)$ for any two vertices u, v in D. By $\gamma_c, \gamma_i, \gamma_t, \gamma_o, \gamma_k, \gamma_e, \gamma_g, \gamma_r$ and γ_{wc}, we mean the minimum cardinality of a connected dominating set, independent dominating set, total dominating set, cycle dominating set, clique dominating set, efficient dominating set, global dominating set, restrained dominating set and weak convex dominating set respectively.

In this paper, we introduce a new dominating set called weak convex restrained dominating set of a graph through which we analyse the properties of the graph such as variation in radius and diameter of the graph. We find upper and lower bounds for the new domination number in terms of various already known parameters. Also we obtain several interesting properties like Nordhaus-Gaddum type results relating the graph and its complement.

2. Prior Results

Theorem 2.1:[57]

Let G be any graph and D be any dominating set of G. Then $|V\text{-}D| \leq \sum_{u \in V(D)} deg(u)$ and equality holds in this relation if and only if D has the following properties :

(i) D is independent.

(ii) For every u $\in$ V-D, There exist a unique vertex v $\in$ D such that $N(u) \cap D = \{v\}$.

Theorem 2.2:[8]

Let G be a connected graph. Then $p-q \geq 0$ if and only if G is unicyclic or a tree.

Theorem 2.3:[8]

$[\ p/\Delta+1\ ] \leq \gamma_{wc}$.

3. Main Results

In this section we define a new concept called weak convex restrained dominating set(WCRD) and study some structural properties of graphs related to this WCRD set.

Definition 3.1 :

A weak convex restrained dominating set D is a weak convex dominating set in which every vertex in V(G)-D is adjacent to another vertex in V(G)-D. In short a weak convex restrained dominating set is abbreviated as WCRD set.

The cardinality of the minimum weak convex restrained dominating set is called weak convex restrained domination number and is denoted by $\gamma_{rc}(G)$.

This concept has application in network to find the dominating set D with hereditary property distance, which can cover the remaining set with every node in the V-D, is linked with at least one node as stand by to have many backups.

Observations:

3.1: $\gamma_{rc}(K_n) = 1$ if $n \neq 2$.

3.2: $\gamma_{rc}(T) = n$,for any tree T.

3.3: $\gamma_{rc}(K_{m,n}) = 2$,for $\{m, n\} \geq 2$.

3.4: $\gamma_{rc}(C_n) = n$,for $n \geq 7$.

3.5: If v is a vertex of degree 1, then v belongs to all WCRD sets.

3.6: If $v \in$ V-D, where D is a WCRD set then deg(v) is greater than or equal to 2.

Theorem 3.1:

Let G be a graph. $\gamma_{rc}(G) = 1$ if and only if there exists a vertex of degree n-1 and all others have degree greater than or equal to 2.

Proof:

Let $\gamma_{rc}(G)$ =1. Then there exists a WCRD set D with |D| = 1. Let D = {*v*}, for some *v* ∈ V(G). Then deg(*v*) = n-1. Since D is WCRD set, each vertex in V-{*v*} must be adjacent with some vertex in V-{*v*} and already they are adjacent with *v*, hence degree of any vertex must be greater than or equal to 2.

Conversely, suppose G contains a vertex *v* of degree n-1, then {*v*} will form a convex dominating set and if degree greater than or equal to 2 for other vertices, then every vertex in V-{*v*} must be adjacent with some vertex in V-{*v*}. Hence, $\gamma_{rc}(G) = 1$.

Corollary 3.1:

If G has vertex of degree less than or equal to 1, then $\gamma_{rc}(G) \geq 2$.

Proposition 3.1:

There exists no graph G on n vertices such that $\gamma_{rc}(G) = n-1$.

Proof:

If suppose G has a WCRD set D with |D| = γ_{rc} = n-1, then | V-D | = 1. This implies that, there exists a vertex *v* ∈ V such that V-D = {*v*}. Thus, D cannot be a restrained dominating set.

Proposition 3.2 :

Let D be any weak convex restrained dominating set of G. Then $|V-D| \leq \sum_{u \in V(D)} \deg(u)$, for all u ∈ D.

Proof:

Let D be any weak convex restrained dominating set of G. Then clearly, D is a dominating set of G. Hence from Theorem 2.1, $|V-D| \leq \sum_{u \in V(D)} \deg(u)$.

Theorem 3.2 :

Let D be a weak convex restrained dominating set. Then $|V-D| = \sum_{u \in V(D)} \deg(u)$ if and only if there exists a vertex '*u*' of degree p-1 and deg(*v*) ≥ 2, $\forall v \in V-\{u\}$.

Proof:

Let D be a weak convex restrained dominating set with $|V-D| = \sum_{u \in V(D)} \deg(u)$. Since every weak convex restrained dominating set is a dominating set, then from Theorem 2.1,

|D| =1. Therefore there exists a vertex '*u*' of degree p-1. Since D={*u*} is a restrained dominating set, deg(*v*) ≥ 2, for all $v \in$ V-D. Converse is trivial.

Theorem 3.3 :

If G is a graph on p vertices and q edges such that γ_{rc}(G) = p-q, then G $\cong$ K_1.

Proof :

Let γ_{rc} = p-q > 0. Then by Theorem 2.2, G must be a tree, which implies that p-q =1 and hence γ_{rc} = 1. Hence G $\cong$ $K_{1,\ r}$. But for the existence of weak convex restrained dominating set in star graph G, it would be K_1.

Proposition 3.3 :

$[\ p/\Delta+1\ ] \leq \gamma_{rc}$.

Proof :

Since every weak convex restrained dominating set is a weak convex dominating set . From Theorem 2.3, we have $[\ p/\Delta+1] \leq \gamma_{wc} \leq \gamma_{rc}$.

Theorem 3.4 :

Let G be a graph of order p. Then k = γ_{rc} (G) = [p/Δ+1] if and only if γ_{rc} (G)=1.

Proof :

Assume that k = γ_{rc} (G) = [p/Δ+1]. Let D be a weak convex restrained dominating set such that |D| = γ_{rc}.

Claim: D is independent.

If not, let D be a non-independent set. Since D is itself a dominating set and not independent, $|\ V\text{-}D\ | \neq \sum_{u \in V(D)} \deg(u)$.

$$\text{(i.e.)} \qquad |\ V\text{-}D\ | < \sum_{u \in V(D)} \deg(u) \leq k\Delta$$

$$\Rightarrow \qquad k > p/(\Delta+1) \geq [\ p/\Delta+1\ ]$$

This is a contradiction to k = [p/Δ+1]. Therefore, D is independent. Since D is a weak convex dominating set, |D|= 1, that is γ_{rc} (G) =1. Proof of converse is trivial.

Corollary 3.2:

Let G be a graph of order p. Then k = γ_{rc} (G) = [p/Δ+1] if and only if there exists a vertex '*u*' of degree p-1 and deg(*v*) ≥ 2 $\forall v \in$ V-{*u*}.

Corollary 3.3:

Let G be a graph of order p. Then $k = \gamma_{rc}(G) = [p/\Delta+1]$ if and only if there exists some weak convex restrained dominating set D such that $|V-D| = \sum_{u \in V(D)} deg(u)$

Theorem 3.5 :

Let G be a self-centered graph of diameter 2 with exactly one vertex of degree 2, then $\gamma_{rc}(G) \leq 3$.

Proof:

Let u' be a vertex of degree 2. Then there exists a vertex u such that $u' \in N_1(u)$. Let u'' be the other vertex adjacent to u'.

Case 1: Suppose $u'' \in N_1(u)$.

Since u' is adjacent with only u and u'', u'' is adjacent with all the vertices of $N_2(u)$ (to maintain distance 2 from u' to all the vertices of $N_2(u)$ through u''). Therefore, the set $\{u, u', u''\}$ form a W.C.D set. Surely $N_1(u)$ contains some vertices other than u' and u''(otherwise radius of G will become one). Let w be any vertex in $N_1(u)$ other than u' and u'' which is not adjacent to any vertex of $N_2(u)$. Then to maintain distance 2 between the vertices of $N_2(u)$ and itself, it must be adjacent to u''. Also it is adjacent to u. Therefore, it must be adjacent to some vertex in $N_1(u)$ other than u and u''(since all the vertices other than u' are having degree greater than or equal to 3). Let v be a vertex in $N_2(u)$, which is not adjacent to any vertex of $N_1(u)$. If it is not adjacent to any other vertex of $N_2(u)$, then degree of v will become one. This is not possible, since G is a block(by theorem in zero chapter). Therefore, v must be adjacent to some vertex in $N_2(u)$. Thus $\{u, u', u''\}$ forms a W.C.R.D set for G. Hence $\gamma_{rc}(G) \leq 3$.

Case 2: If $u'' \in N_2(u)$.

In this case also all the vertices of $N_2(u)$ are adjacent to u''. And since all the vertices other than u' are of degree greater than or equal to 3, all the vertices in $V(G)$-$\{u, u', u''\}$ must have degree greater than or equal to one in $\langle V(G)-\{u, u', u''\} \rangle$. Hence the set $\{u, u', u''\}$ forms a WCRD set for G. Thus $\gamma_{rc}(G) \leq 3$.

Theorem 3.6 :

Let G be a self-centered graph of diameter = 2. If for some $u \in V(G)$, $\langle N_1(u) \rangle$ and $\langle N_2(u) \rangle$ are independent then $\gamma_{rc}(G) = 2$.

Proof:

If there exists a vertex $u \in V(G)$ such that $\langle N_1(u) \rangle$ and $N_2(u)$ are independent, then each vertex in $N_2(u)$ must be adjacent to all the vertices of $N_1(u)$. Then clearly, $\{u, u'\}$ will form a dominating set, where $u' \in N_1(u)$. Hence $\gamma_{rc}(G) = 2$.

Theorem 3.7 :

Let G be a self-centered graph of diameter 2 and u be a vertex of degree δ. If every component of $<N_2(u)>$ has cardinality at most δ - 1, then $\gamma_{rc}(G) \leq \delta$.

Proof:

Let u be a vertex of degree δ of a graph G. Assume that $|C| \leq \delta - 1$, where C is a component in $< N_2(u) >$.

Case 1: If $|C| = 1$, then the vertex v in C must be adjacent with all of $N_1(u)$ (since $\deg(u) = \delta$ and $\deg(v) \geq \delta$).

Case 2: If $2 \leq |C| \leq \delta - 1$, then degree of any vertex in C may be a maximum of $\delta - 2$ in $< C >$. Since δ is the minimum degree, to maintain δ, any vertex in C must be adjacent with two of the vertices of $N_1(u)$. Then the set $\{u\} \cup \{N_1(u) - \{u'\}\}$ will form a W.C.R.D set, where $u' \in N_1(u)$ and $N_1(u') \cap N_2(u) \neq \phi$. Hence $\gamma_{rc}(G) \leq \delta$.

Proposition 3.4 :

Let G be a self-centered graph of diameter 2. If for a vertex u, every component of $<N_2(u)>$ is with cardinality at least two, then $\gamma_{rc} \leq \Delta+1$.

Theorem 3.8:

Let G be a self-centered graph of diameter = 2. If there exists a vertex u of degree δ such that each component C of $<N_2(u)>$ is with cardinality at most 2, then $\gamma_{rc}(G) \leq 3$.

Proof :

Any vertex in C, where $|C|=1$, must be adjacent to all of $N_1(u)$. Any vertex in C, where $|C|=2$, must be adjacent to δ-1 number of vertices of $N_1(u)$. That is, it is adjacent to at most one vertex in $N_2(u)$. Hence $\{u, u', u''\}$ will form a W.C.R.D set for G, where $u', u'' \in N_1(u)$. Hence $\gamma_{rc}(G) \leq 3$.

Theorem 3.9:

Let G be a self-centered graph of diameter = 2. Then $\gamma_{rc}(G) = p - 2 \Leftrightarrow G = C_5$ or C_4.

Proof:

Suppose $G = C_5$ or C_4 then clearly $\gamma_{rc}(G) = p - 2$. Conversely, suppose $\gamma_{rc}(G) = p - 2$. Then there exists an edge $uv \in V-D$ for a minimal dominating set D. Clearly $u,v \in N_2(w)$ for some $w \in V(G)$ other than u and v.

Case 1:

If not, then for any $w \in V(G)$ either $u \in N_1(w)$ or $v \in N_1(w)$. This implies uv forms a dominating edge. This implies that $\gamma_{rc} \leq 2$. As G is self-centered graph of diameter 2, $\gamma_{rc} \neq 1$. Thus $\gamma_{rc}(G) = 2$.This implies p = 4. Hence G = C_4 or K_4-e. But diameter(G) = 2 implies that G = C_4.

Case 2:

If $u, v \in N_2(w)$ for some $w \in V(G)$, then clearly no vertex of $N_2(w)$ is adjacent with u or v (if not, then we put that vertex in V-D along with u and v, contradiction to $\gamma_{rc}(G)$ = p-2). Also each of the other components of $N_2(w)$ contains only a single vertex (if not, then those components contain more than 2 vertices, and hence we can put those vertices in V-D along with uv, a contradiction to $\gamma_{rc}(G)$ = p-2). Also we have u(or v) cannot be adjacent to two different vertices(otherwise, we can remove one of those vertices along with all the vertices in $N_2(w)$, which are adjacent with that vertex and uv, the remaining set will form a R.D. set with cardinality less than or equal to p-3, a contradiction to $\gamma_{rc}(G)$ = p-2). Hence u and v are adjacent to only one vertex of $N_1(w)$, this implies that deg(u) = deg(v) = 2. Therefore, $\gamma_{rc}(G)$ = 3. Hence p = 5.

Lemma 3.1:

For any graph G of diameter(G) =2, $\gamma_{rc}(G) \leq p - 2$.

Proof :

Case 1:

Suppose for some u of degree δ, $<N_2(u)>$ is an independent set, then since each vertex of $N_2(u)$ is adjacent with all of $N_1(u)$ (since deg(u) = δ and deg(v) $\geq \delta$, for all $v \in N_2(u)$). Hence u and a vertex in $N_1(u)$ will form a dominating set for the whole of G.

Case 2:

If $N_2(u)$ does not form an independent set then there exist a component $C \in N_2(u)$ with $|C| \geq 2$. Therefore G-C will form a restrained dominating set for G. This implies that $\gamma_{rc}(G) \leq |G\text{-}C| = |G| - |C| \leq p\text{-}2$.

Proposition 3.5:

Let G be a self-centered graph of diameter 2. If u is a vertex of degree δ such that $N_2(u)$ is independent, then $\gamma_{rc}(G) \leq 2$.

Proof:

Proof directly follows from case 1 of the above lemma 3.1.

Theorem 3.10:

Let G and $\overline{G}$ be both self-centered graphs of diameter = 2. If for some vertex u, any vertex in $N_1(u)$ is not adjacent some vertex in $N_1(u)$ and any vertex in $N_2(u)$ is adjacent to some vertex in $N_2(u)$, then $\gamma_{rc}(G) + \gamma_{rc}(\overline{G}) \leq p+1$.

Proof :

Let u be a vertex of G of degree k. If the cardinality of all the components G_i''(i = 1 to k) of $<N_2(u)>$ is greater than or equal to 2, then $\{u\} \cup N_1(u)$ will form a restrained dominating set for G. If for any vertex in $N_1(u)$ is not adjacent some vertex in $N_1(u)$, then $\{u\} \cup N_2(u)$ for a restrained dominating set for $\overline{G}$. Hence $\gamma_{rc}(G) + \gamma_{rc}(\overline{G}) \leq$ k+1+p-k-1+1 = p+1.

Theorem 3.11:

Let G be a graph. If there exist two pair of vertices u_1, u_2 and v_1, v_2 such that $d(u_1, u_2) = d(v_1, v_2) = 3$, then $\gamma_{rc}(G) + \gamma_{rc}(\overline{G}) \leq p+3$.

Proof :

If there exist two pair of vertices u_1, u_2 and v_1, v_2 such that $d(u_1, u_2) = d(v_1, v_2) = 3$. Then clearly u_1u_2 is a dominating edge in $\overline{G}$ and all the vertices are adjacent with v_1 or v_2 in $\overline{G}$. Therefore $\{u_1, u_2\}$ will form a weak convex restrained dominating set in $\overline{G}$. Hence $\gamma_{rc}(\overline{G}) = 2$. Thus $\gamma_{rc}(G) + \gamma_{rc}(\overline{G}) \leq p + 2$.

Theorem 3.12 :

If suppose G is a graph of diameter greater than or equal to 4 , then $\gamma_{rc}(G) + \gamma_{rc}(\overline{G}) \leq p + 2$.

Proof :

Since G is of diameter greater than or equal to 4, there exists a path of length 4 say $uxyzv$, in G. Clearly $d(u, z)= d(x,v)=3$. therefore the $\{u, z)$ will form a dominating set for $\overline{G}$. Also it is a restrained dominating set, since all the vertices in V-$\{u, z\}$ must be adjacent to $\{x, v\}$ in $\overline{G}$. Therefore $\gamma_{rc}(\overline{G}) \leq 2$. Hence $\gamma_{rc}(G) + \gamma_{rc}(\overline{G}) \leq p + 2$.

Theorem 3.13:

Let G be a graph such that diameter(G) = diameter($\overline{G}$) = 3. If there exist three vertices u_1, u_2, u_3 such that $d(u_1, u_2) = d(u_1, u_3) = d(u_2, u_3) = 3$, then $\gamma_{rc}(G)+\gamma_{rc}(\overline{G}) \leq p+3$.

Proof :

If there exist three vertices u_1, u_2, u_3 such that $d(u_1, u_2) = d(u_1, u_3) = d(u_2, u_3) = 3$.

Case 1: Any one is of degree one.

Let u_1 be the vertex of degree one and u_1' be the support of u_1 in G. Then $\{u_1', u_2, u_3\}$ will form a restrained dominating set (since all the vertices are adjacent with u_2 or u_3

and all the vertices are adjacent with u_1 except u_1'). Thus $\gamma_{rc}(G) \leq p + 3$. Hence $\gamma_{rc}(G) + \gamma_{rc}(\overline{G}) \leq p + 3$.

Case 2: Suppose degree of u_1, u_2, u_3 are greater than or equal to 1.

If for every vertex in the set S = V-{u_1, u_2}, there exist a non-adjacent vertex in the set S, then clearly { u_1, u_2} will form a restrained dominating set in $\overline{G}$. Therefore, $\gamma_{rc}(G) + \gamma_{rc}(\overline{G}) \leq p+2$.

If not, then there exist a vertex $v \in S$ such that v is adjacent with all of S. Let T= G-{u_1, u_2, u_3}. Clearly, $v \in T$. Also v is at distance 2 from at least two vertices of u_1, u_2, u_3. Without loss of generality, assume that v is at distance two from u_2.

Case 2.1: If no vertex of $N_1(u_2)$ is adjacent to all of T, then {u_1, v, u_3} form a restrained dominating set for $\overline{G}$ (since u_2 dominates $N_1(u_1)$ and $N_1(u_3)$ and all the vertices of $N_1(u_2)$is adjacent to some vertex of $N_1(u_1)$ and $N_1(u_3)$ in $\overline{G}$).

Case 2.2: If there exists a vertex of $N_1(u_2)$, which is adjacent to all the vertices of T, say $v \neq x \in N_1(u_2)$. Then the set {x, u_1', u_3'} forms a W.C.R.D set in G, where $u_1' \in N_1(u_1)$ and $u_3' \in N_1(u_3)$.Hence from case 2.2.1 and case 2.2.2, we have $\gamma_{rc}(G) + \gamma_{rc}(\overline{G}) \leq p+3$.

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