International Journal of Engineering Sciences, Advanced Computing and Bio-Technology
Vol. 1, No. 4, October –December 2010, pp.152- 157

Graceful Labeling of Generalized Tree with Hanging Stars in Geometric Progression

T.N.Janakiraman[1] and G.Sathiamoorthy[2]

[1]*Department of Mathematics, National Institute of Technology, Tiruchirappalli.*

Email : janaki@nitt.edu

[2]*School of Humanities & Sciences, SASTRA University, Thanjavur.*

Email :sami@maths.sastra.edu

Abstract: *In this paper, it is shown graceful a labeling of generalized tree obtained from a family of n stars having number of branches of those stars, form a geometric progression with common ratio r and one of the branches of each of those stars, merged with one different point of a common path on n vertices successively in increasing order.*

Key words: *geometric progression, growing stars, supporting points, hanging points, free leaves, general ratio.*

1. Introduction

A simple undirected graph G = (V (G), E (G)) with p vertices and q edges. A function f is called graph labeling of graph G if f: V→ {0, 1, 2 … q} is injective and the induced function f*: E→ {1, 2, 3,…, q} defined as f*(e = uv) = $|f(u)-f(v)|$ is bijective. All edge values are unique and distinct.

Gallian, [4] gives extensive survey on graceful labeling. Huang, Kotzig, and Rosa [1] gives a new class of graceful trees, Sethuraman and Jesintha [2] shows a new class of graceful rooted trees, they showed generating new graceful trees [3] and Michelle Edwards and lea Howard, given a survey of graceful trees[6]. In the earlier paper of arithmetic progression by us [5] motivated to find graceful labeling for trees with hanging stars which are in geometric progression.

Let P_n be basic of $T^{(r)}_{(G(n),a)}$ tree. Let s_1, s_2,…, s_n be such vertices, which are, term it as supporting vertices of $T^{(r)}_{(G(n),a)}$ tree. In $T^{(r)}_{(G(n),a)}$ at each s_i, a star S_i with i branches having centre c_i with one of the branch vertex of S_i merged with s_i. Here $|S_1|$, $|S_2|$,…, $|S_n|$ form geometric progression with common ratio r and hence it has been denoted as $T^{(r)}_{(G(n),a)}$ tree, where $|E(S_1)| = a$.

Received: 12 May, 2010; Revised: 18 August, 2010; Accepted: 23 September, 2010

2. Main results

Let the support points of the hanging stars $S_1, S_2, \ldots, S_n$ be $s_1, s_2, s_3, \ldots, s_n$ respectively and denote the free leaves of each of the stars S_i by $f_1^{(i)}, f_2^{(i)}, \ldots, f_{i-1}^{(i)}$ for $i = 1, 2, \ldots, n$.

Let $c_1, c_2, \ldots, c_n$ be the central vertices of the stars $S_1, S_2, S_3, \ldots, S_n$ respectively.

A tree with growing n hanging stars as branches whose cardinality are in geometric progression with common ratio 'r' is denoted by $T^{(r)}_{(G(n),a)}$, where 'a' is number of branches in star S_1 and 'r' is common ratio any two consecutive stars.

Stars of tree $T^{(r)}_{(G(n),a)}$ can be derived by the relation $|V(S_n)| = a\,r^{n-1} + 1$, where n= 1, 2…

It can be verified that the number of vertices of $T^{(r)}_{(G(n),a)}$ can be recursively defined by the relation

$$\left|V(T^{(r)}_{(G(n),a)})\right| = \left|V(T^{(r)}_{(G(n-1),a)})\right| + (1 + r^n) .$$

Also the edges of $T^{(r)}_{(G(n),a)}$ can be defined by the relation $\left|E(T^{(r)}_{(G(n),a)})\right| = \left|E(T^{(r)}_{(G(n-1),a)})\right| + (1 + r^n)$.

Because of the above relation, we define the relation between two successive trees $T^{(r)}_{(G(n),a)}$ and $T^{(r)}_{(G(n),a)}$ as $\left|T^{(r)}_{(G(n),a)}\right| \Theta \left|T^{(r)}_{(G(n-1),a)}\right| = 1 + r^n$,

where Θ denote the difference between the number of vertices (edges) of $T^{(r)}_{(G(n),a)}$ and $T^{(r)}_{(G(n),a)}$.

let us assume that $|E(S_1)| = a = q_1$, $|E(S_2)| = a\,r = q_2$, $|E(S_3)| = ar^2 = q_3, \ldots$, and $|E(Sn)| = ar^{n-1} = q_n$.

For example a general tree $T^{(2)}_{(G(n),2)}$ drawn in Figure 1.

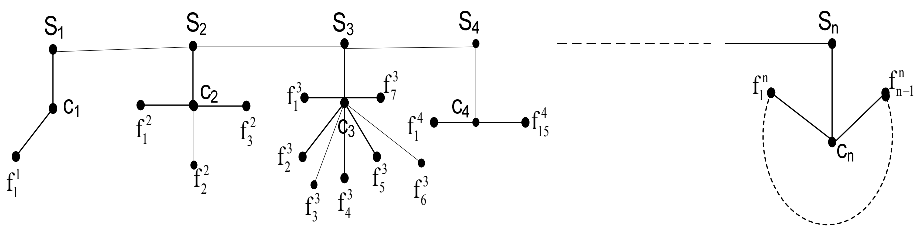


Figure 1

Total number of edges q = $\frac{a(1-r^n)}{1-r}+(n-1)$.

where (n-1) denotes the number of edges in the base path from which the stars are hanging.

We also denote the labeling of node v in the tree as l (v). Here for the tree $T^{(r)}_{(G(n),a)}$, we assign the labeling as follows.

R (1): $l(s_1) = 0$; $l(c_1) = q$; $l(c_2) = a$, $l(s_2) = q\text{-}a$.

R (2): $l(s_{2m+1}) = l(s_{2m-1}) + (a\, r^{2m-1} + 1)$, $m \geq 1$.

R (3): $l(s_{2m+2}) = l(s_{2m}) - (a\, r^{2m} + 1)$, $m \geq 1$.

R (4): $l(c_{2m+1}) = l(c_{2m-1}) - (a\, r^{2m-1} + 1)$, $m \geq 1$.

R (5): $l(c_{2m+2}) = l(c_{2m}) + (a\, r^{2m} + 1)$, $m \geq 1$.

Let the free leaves of growing m^{th} star of $T^{(r)}_{(G(n),a)}$ at s_m be $f_1^{\,m}, f_2^{\,m} \ldots f_k^{\,m}$ where $k = ar^{m-1}\text{-}1$.

Let the free leaves of S_1 are labeled with values 1 to q_1-1.

Then for $m \geq 1$

The labeling of free leaves of odd stars of S_{2m+1} based on its supporting vertex s_{2m+1} as follows.

R(6)a: labeling of (ar^{2m}-1) free leaves of S_{2m+1} are given by the integers starting from $l(c_{2m}) + 1$ to $l(c_{2m}) + ar^{2m}$ except the value of $l(s_{2m+1})$.

The labeling of free leaves of even stars S_{2m} based on its supporting vertex s_{2m} as follows.

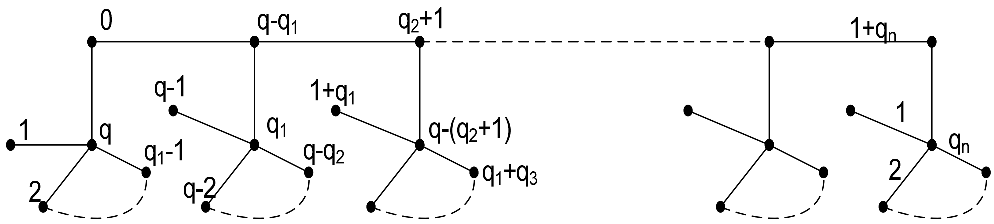
R (6) b: labeling of (ar^{2m-1}-1) free leaves of S_{2m} are given by the integers starting from $l(c_{2m-1}) - ar^{2m-1}$ to $l(c_{2m-1}) - 1$ except the value of $l(s_{2m})$.

The above labeling of vertices (edges) induces a bijective mapping I_E and I_V as follows.

I_E: E ($T^{(r)}_{(G(n),a)}$) $\longrightarrow$ {1, 2, 3, …, $\frac{a(1-r^n)}{1-r}+(n-1)$ }

I_V: V ($T^{(r)}_{(G(n),a)}$) $\longrightarrow$ {0, 1, 2,…, $\frac{a(1-r^n)}{1-r}+(n-1)$ }

The edge assignment as follows.



this could be verified easily that it is a graceful labeling for the given tree $T^{(r)}_{(G(n),a)}$ from the following assignment tables.

The vertex assignment table:

Labeling of T_n	Labeling of vertices
s_1	0
c_1	q
Remaining free leaves of S_1	1 to a-1
s_2	q-a
c_2	a
Remaining free leaves of S_2	{q-1 to ($q-q_2$) except (q-a)}
s_3	$1+q_2$
c_3	$q-(1+q_2)$
Remaining free leaves of S_3	{(a +1) to (a + q_3) except($1+q_2$))}
s_4	$q-(a+q_3+1)$
c_4	$a+ q_3 + 1$
Remaining free leaves of S_4	{($q-q_2-2$) to ($q-(q_2+q_4+1)$ except ($q-(a+q_3+1)$)}
s_{2m}	$q-l(c_{2m})$
c_{2m}	$a+q_3+q_5+\ldots+q_{2m-1}+m-1$
Remaining free leaves of S_{2m}	The relation R (6b) assigns the values.
s_{2m+1}	$q_2+q_4+\ldots+q_{2m}+m$
c_{2m+1}	$q-l(s_{2m+1})$
Remaining free leaves of S_{2m+1}	The relation R (6a) assigns the values.

s_{2n}	q- $l(c_{2n})$
c_{2n}	$l(c_{2n}) = l(c_{2n-2}) + a\, r^{2n-2} + 1$
Remaining free leaves of S_{2n}	The relation R (6b) assigns the values.
s_{2n+1}	$l(s_{2n+1}) = l(s_{2n-1}) + ar^{2n-1} +1)$
c_{2n+1}	q- $l(s_{2n+1})$
Remaining free leaves of S_{2n+1}	The relation R (6a) assigns the values.

Table 1

The edge assignment follows.

Labeling of T_n	Labeling of edge values
edge s_1c_1	q
free leaves of S_1	q-1 to q-(a-1)
edge s_1s_2	q-a
edge s_2c_2	q-2a
free leaves of S_2	$q-(a+1)$ to $q-(a+q_2)$ except (q-2a)
edge s_2s_3	$q-(a+q_2+1)$
edge s_3c_3	$q-(2q_2+2)$
free leaves of S_3	$q-(a+q_2+2)$ to $q-(a+q_2+q_3+1)$ except $q-2(1+q_2)$
edge s_3s_4	$q-(a+q_2+q_3+2)$
edge s_4c_4	$q-2(a+q_3+1)$
free leaves of S_4	$q-(a+q_2+q_3+3)$ to $q-(a+q_2+q_3+q_4+2)$ except $q-2(a+q_3+1)$
edge $s_{2m-1}s_{2m}$	$q-(a+q_2+\ldots+q_{2m-1}+2m-2)$
edge $s_{2m}c_{2m}$	$q-2(a+q_2+\ldots+q_{2m-1}+m-1)$
free leaves of S_{2m}	$q-(a+q_2+\ldots+q_{2m-1}+2m-1)$ to $q-(a+q_2+\ldots+q_{2m}+2m-2)$ except edge $l(s_{2m}c_{2m})$
edge $s_{2m}s_{2m+1}$	$q-(a+q_2+\ldots+q_{2m}+2m-1)$
edge $s_{2m+1}c_{2m+1}$	$q-2(a+q_2+\ldots+q_{2m}+m)$
free leaves of S_{2m+1}	$q-(a+q_2+\ldots+q_{2m}+2m)$ to $q-(a+q_2+\ldots+q_{2m+1}+2m-1)$ except edge $l(s_{2m+1}c_{2m+1})$

edge $s_{n-1}s_n$	1+ | E (S_n)|
edge s_nc_n	|l(s_n)-l(c_n)|
free leaves of S_n	| E (S_n)| to 1 except edge l(s_nc_n)

Table 2

We observe that the labeling of S_i's in which s_{2m+1}, m ≥ 1 are increasing order and s_{2m}, m ≥ 2 are decreasing order in relation with q. l(s_i) + l(c_i) = q. for any i, and hence it can be observed that the reverse property of l(s_i)'s is satisfied for l(c_i)'s.

References

[1] C. Huang, A. Kotzig, and A. Rosa, Further results on tree labellings, Util. Math., 21c (1982) 31-48

[2] G. Sethuraman and J. Jesintha, A new class of graceful rooted trees, J. Disc. Math. Sci. Crypt., 11 (2008) 421-435

[3] G. Sethuraman and J. Jesintha, Generating new graceful trees, Proc. Inter. Conf. Math. Comput. Sci., July (2008) 67-73

[4] Gallian, J A A Dynamic Survey of Graceful Labeling, The Electronic Journal of Combinatories, twelfth edition January 2009.

[5] T.N. Janakiraman and G. Sathiamoorthy, Graceful labeling of a tree with hanging stars in which number of branches in arithmetic progression, Inter. J. Eng. Sci. Comp. and Biotech. Vol (communicated)

[6] Michelle Edwards and lea Howard, A survey of graceful trees, Atlantic journal of Mathematics, vol 1(summer 2006) 5-30.