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Graceful Labeling of Generalized Tree of Hanging Stars in Arithmetic Progression

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Abstract: *In this paper, it is shown graceful a labeling of generalized tree obtained from a family of n stars having number of branches of those stars form an arithmetic progression with common difference j and one of the branches of each of those stars merged with one different point of a common path on n vertices successively in increasing order.*

Key words: *arithmetic progression, growing stars, supporting points, hanging points, free leaves, general difference.*

Mathematical classification 2010: 05C78

1. Introduction

A simple undirected graph G = (V (G), E (G)) with p vertices and q edges. A function f is called graph labeling of graph G if f: V→ {0, 1, 2 … q} is injective and the induced function f*: E→ {1, 2, 3,…,q} defined as f*(e = uv) = $|f(u)-f(v)|$ is bijective. All edge values are unique and distinct.

Gallian, [4] gives extensive survey on graceful labeling. Huang, Kotzig, and Rosa [1] gives a new class of graceful trees, Sethuraman and Jesintha [2] shows a new class of graceful rooted trees, they showed generating new graceful trees[3] and Michelle Edwards and Lea Howard, given a survey of graceful trees[5].

Let P_n be basic path of $T^{(j)}_{(A(n),a)}$ tree. Let s_1, s_2,…, s_n be such vertices, which are termed as supporting vertices of $T^{(j)}_{(A(n),a)}$ tree. In $T^{(j)}_{(A(n),a)}$ at each s_i, a star S_i with i branches having centre c_i with one of the branch vertex of S_i merged with s_i. Here {$|S_1|$, $|S_2|$,…,$|S_n|$} forms an arithmetic progression with common difference j and hence it has been denoted as $T^{(j)}_{(A(n),a)}$ tree, where $|E(S_1)| = a$.

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2. Main results

Let the support points of the hanging stars $S_1, S_2, \ldots, S_n$ be $s_1, s_2, s_3, \ldots, s_n$ respectively and denote the free leaves of each of the stars S_i by $f_1^{(i)}, f_2^{(i)}, \ldots, f_{i-1}^{(i)}$ for i = 1, 2, …, n.

Let $c_1, c_2, \ldots, c_n$ be the central vertices of the stars $S_1, S_2, S_3, \ldots, S_n$ respectively.

A tree with growing n hanging stars as branches whose cardinality are in generalized arithmetic progression with common difference j is denoted by $T^{(j)}_{(A(n),a)}$, where 'a' is number of branches in star S_1 and 'j' is common difference between number of branches of any two consecutive stars.

Stars of tree $T^{(j)}_{(A(n),a)}$ can be derived by the relation $|V(S_i)| = a + (i-1)j+1$, where i = 1, 2, 3,…, n.

It can be verified that the number of vertices of $T^{(j)}_{(A(n),a)}$ can be recursively defined by the relation

$$\left|V(T^{(j)}_{(A(n),a)})\right| = \left|V(T^{(j)}_{(A(n-1),a)})\right| + (1+j).$$

Also the edges of $T^{(j)}_{(A(n),a)}$ can be defined by the relation

$$\left|E(T^{(j)}_{(A(n),a)})\right| = \left|E(T^{(j)}_{(A(n-1),a)})\right| + (1+j).$$

Because of the above relation, we define the relation between two successive trees $T^{(j)}_{(A(n-1),a)}$ and $T^{(j)}_{(A(n),a)}$ as

$$\left|T^{(j)}_{(A(n),a)}\right| \Theta \left|T^{(j)}_{(A(n-1),a)}\right| = 1+j,$$

where Θ denote the difference between the number of vertices (edges) of $T^{(j)}_{(A(n-1),a)}$ and $T^{(j)}_{(A(n),a)}$.

Let us assume that $\left|E(S_1)\right| = a = q_1$, $\left|E(S_2)\right| = a + j = q_2$, $\left|E(S_3)\right| = a + 2j = q_3, \ldots$, and $\left|E(S_n)\right| = a + (n-1)j = q_n$.

For an example a general tree $T^{(1)}_{(A(n),3)}$ is as shown in figure 1.

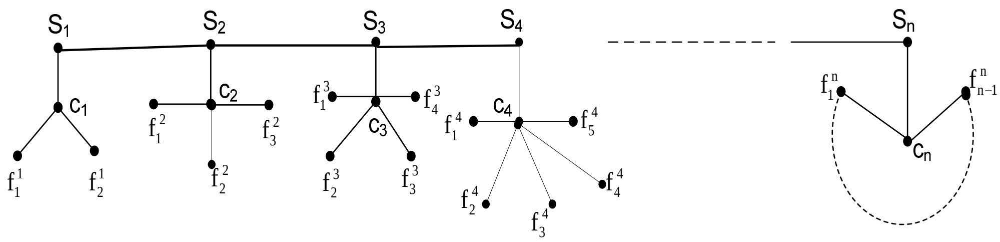


Figure 1

Total number of edges in $T^{(j)}_{(A(n),a)}$ is $q = \frac{n(2a+(n-1)j)}{2} + (n-1)$, n ≥ 1,

where (n-1) denotes the number of edges in the base path from which the stars are hanging.

We also denote the labeling of node v in the tree as l (v). Here for the tree $T^{(j)}_{(A(n),a)}$ we assign the labeling as follows.

R (1): $l(s_1) = 0$; $l(c_1) = q$; $l(c_2) = a$, $l(s_2) = q-a$.

R (2): $l(s_{2m+1}) = l(s_{2m-1}) + (j(2m-1) + 1 + a)$, m ≥ 1.

R (3): $l(s_{2m+2}) = l(s_{2m}) - (2mj + 1 + a)$, m ≥ 1.

R (4): $l(c_{2m+1}) = l(c_{2m-1}) - (j(2m-1) + 1 + a)$, m ≥ 1.

R (5): $l(c_{2m+2}) = l(c_{2m}) + (2mj + 1 + a)$, m ≥ 1.

Let the free leaves of growing mth star of $T^{(j)}_{(A(n),a)}$ at s_m be f_1^m, f_2^m… f_k^m, where k = a+ (m-1) j-1.

Let the free leaves of S_1 are labeled with values 1 to q_1-1.

Then for m ≥ 1

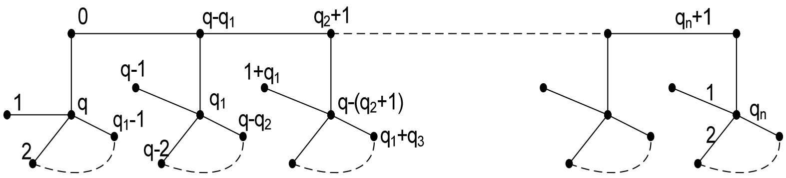
The labeling of free leaves of odd stars of S_{2m+1} based on its supporting vertex s_{2m+1} as follows.

R(6)a: labeling of (a+ 2mj-1) free leaves of S_{2m+1} are given by the integers starting from $l(c_{2m}) + 1$ to $(l(c_{2m}) + a+2mj)$ except the value of $l(s_{2m+1})$.

The labeling of free leaves of even stars S_{2m} based on its supporting vertex s_{2m} as follows.

R(6)b: labeling of (a+(2m-1)j-1)free leaves of S_{2m} are given by the integers starting from $(l(c_{2m-1}) - (a+(2m-1)j))$ to $(l(c_{2m-1}) - 1)$ except the value of $l(s_{2m})$.

The edge assignment follows.



The above labeling of edges induces a bijective mapping I_E and I_V as follows.

I_E: $E(T^{(j)}_{(A(n),a)}) \rightarrow \{1, 2, 3, \ldots, \frac{n(|V(S_1)| + |V(S_n)|)}{2} + n - 1\}$

I_V: $V(T^{(j)}_{(A(n),a)}) \rightarrow \{0, 1, 2, \ldots, \frac{n(|V(S_1)| + |V(S_n)|)}{2} + n - 1\}$

and it can be verified easily that it is a graceful labeling for the given tree $T^{(j)}_{(A(n),a)}$ from the following vertex and edge assignments tables.

The vertex assignment table:

Labeling of T_n	Labeling of vertices
s_1	0
c_1	q
Remaining free leaves of S_1	1 to a-1
s_2	q-a
c_2	a
Remaining free leaves of S_2	{q-1 to (q-q_2) except (q-a)}
s_3	1+q_2
c_3	q-(1+q_2)
Remaining free leaves of S_3	{(a +1) to (a + q_3)except(1+q_2))}
s_4	q-(a+q_3+1)
c_4	a+ q_3 + 1
Remaining free leaves of S_4	{(q-q_2-2) to (q-(q_2+q_4+1)except (q-(a+q_3+1)}
s_{2m}	q-l(c_{2m})
c_{2m}	a+q_3+q_5+…+q_{2m-1}+m-1
Remaining free leaves of S_{2m}	The relation R (6b) assigns the values.
s_{2m+1}	q_2+q_4+…+q_{2m}+m
c_{2m+1}	q-l(s_{2m+1})
Remaining free leaves of S_{2m+1}	The relation R (6a) assigns the values.
s_{2n}	q- l(c_{2n})
c_{2n}	l(c_{2n-2}) + 1+a+(2n-2)j
Remaining free leaves of S_{2n}	The relation R (6b) assigns the values.
s_{2n+1}	l(s_{2n-1}) +1+a+ (2n-1)j
c_{2n+1}	q- l(s_{2n+1})
Remaining free leaves of S_{2n+1}	The relation R (6a) assigns the values.

Table 1

The edge assignment table:

Labeling of T_n	Labeling of edge values
edge s_1c_1	q
free leaves of S_1	q-1 to q-(a-1)
edge s_1s_2	q-a
edge s_2c_2	q-2a
free leaves of S_2	q-(a+1) to q-(a+q_2) except (q-2a)
edge s_2s_3	q-(a+q_2+1)
edge s_3c_3	q-(2q_2+2)
free leaves of S_3	q-(a+q_2+2) to q-(a+q_2+q_3+1) except q-2(1+q_2)
edge s_3s_4	q-(a+q_2+q_3+2)
edge s_4c_4	q-2(a+q_3+1)
free leaves of S_4	q-(a+q_2+q_3+3) to q-(a+q_2+q_3+q_4+2) except q-2(a+q_3+1)
edge $s_{2m-1}s_{2m}$	q-(a+q_2+…+q_{2m-1}+2m-2)
edge $s_{2m}c_{2m}$	q-2(a+q_2+…+q_{2m-1}+m-1)
free leaves of S_{2m}	q-(a+q_2+…+q_{2m-1}+2m-1) to q-(a+q_2+…+q_{2m}+2m-2)except edge l($s_{2m}c_{2m}$)
edge $s_{2m}s_{2m+1}$	q-(a+q_2+…+q_{2m}+2m-1)
edge $s_{2m+1}c_{2m+1}$	q-2(a+q_2+…+q_{2m}+m)
free leaves of S_{2m+1}	q-(a+q_2+…+q_{2m}+2m) to q-(a+q_2+…+q_{2m+1}+2m-1) except edge l($s_{2m+1}c_{2m+1}$)
edge $s_{n-1}s_n$	1+|E(S_n)|
edge s_nc_n	|l(s_n)-l(c_n)|
free leaves of S_n	|E (S_n)| to 1 except edge l(s_nc_n)

Table 2

We observe that the labeling of S_i's in which s_{2m+1}, m ≥ 1 are increasing order and s_{2m}, m ≥ 2 are decreasing order in relation with q. l(s_i) + l(c_i) = q. for any i, and hence it can be observed that the reverse property of l(s_i)'s is satisfied for l(c_i)'s.

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