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K – Even Mean Labeling of $D_{m,n}$ @ C_n

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Abstract: *Mean labeling of graphs was discussed in [10] and the concept of odd mean labeling was introduced in [9]. k-odd mean labeling and (k, d)-odd mean labeling are introduced and discussed in [5], [6], [7]. In this paper, we introduce the concept of k-even mean labeling and investigate k-even mean labeling of* $D_{m,n}$ @ C_n.

Keywords: *k-even mean labeling, k-even mean graph.*

AMS (MOS) Subject Classification: 05C78

1. Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [8]. The symbols $V(G)$ and $E(G)$ will denote the vertex set and edge set of a graph. Labeled graphs serve as useful models for a broad range of applications [1-3].

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling).

Graph labeling was first introduced in the late 1960's. Many studies in graph labeling refer to Rosa's research in 1967 [11].

Labeled graphs serve as useful models for a broad range of applications such as X-ray crystallography, radar, coding theory, astronomy, circuit design and communication network addressing. Particularly interesting applications of graph labeling can be found in [4].

Mean labeling of graphs was discussed in [10] and the concept of odd mean labeling was introduced in [9]. k-odd mean labeling and (k, d) – odd mean labeling are introduced and discussed in [5], [6], [7]. In this paper, we introduce the concept of k-even mean labeling and here we investigate the k-even mean labeling of $D_{m,n}$ @ C_n.

Throughout this paper, k denotes any positive integer ≥ 1. For brevity, we use k-EML for k-even mean labeling.

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2. Main Results

2.1. Definition: *k*-even mean labeling

A (p, q) graph G is said to have a k-even mean labeling if there exists a injection $f: V \longrightarrow \{0, 1, 2, ..., 2k + 2(q-1)\}$ such that the induced map $f^*: E(G) \longrightarrow \{2k, 2k+2, 2k+4, ..., 2k+2(q-1)\}$ defined by

$$f^*(uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases} \quad \text{is a bijection.}$$

A graph that admits a k-even mean labeling is called a k-even mean graph.

2.2. Definition :

A dragon is formed by joining the end point of a path to a cycle. In fact, it is the one-point union of the end point of a path to a vertex of a cycle. Koh et al. call these tadpoles. Kim and Park call them kites [4].

Let $D_{m,n}$ denote the one-point union of the end point of the path P_m to a vertex of a cycle C_n and $D_{m,n}$ @ C_n denote the graph obtained by the one-point union of the end point of the dragon $D_{m,n}$ to a vertex of a cycle C_n.

2.3. Theorem

$D_{m,n}$ @ C_n, $n \equiv 0 \pmod 4$ is a k-even mean graph for any k and $m > 2$.

Proof

Let $V(D_{m,n}$ @ $C_n) = \{v_i, 1 \leq i \leq n+m-2\} \cup \{v_i', 1 \leq i \leq n\}$ and

$E(D_{m,n}$ @ $C_n) = \{e_i, 1 \leq i \leq n+m-1\} \cup \{e_i', 1 \leq i \leq n\}$ (see Fig. 2.1)

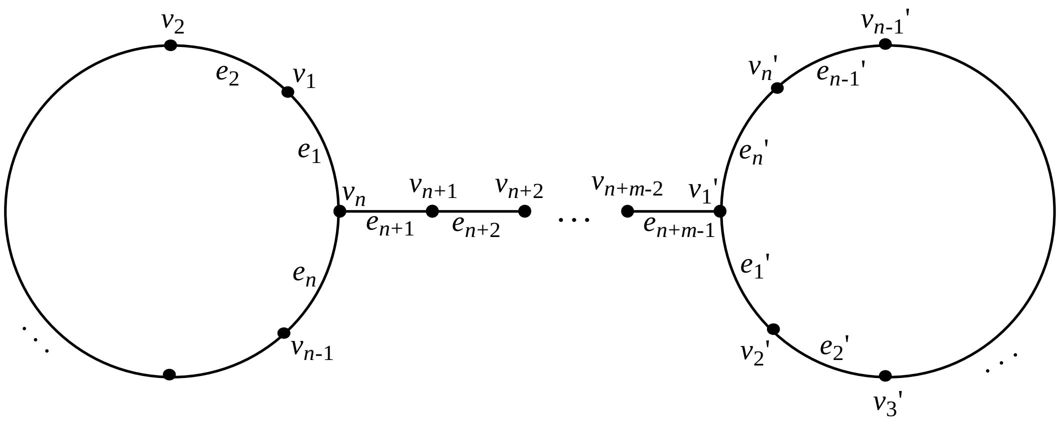

Fig. 2.1: Ordinary labeling of $D_{m,n}$ @ C_n

First we label the vertices of $D_{m,n}$ @ C_n as follows:

Define $f: V(D_{m,n}$ @ $C_n) \longrightarrow \{0, 1, 2, ..., 2k + 2q - 2\}$ by

For $1 \leq i \leq \frac{n-2}{2}$,

$$f(v_i) = \begin{cases} 2k+2n-4(i+1)+1 & \text{if } i \text{ is odd} \\ 2k+2n-4i-2 & \text{if } i \text{ is even} \end{cases}$$

For $\frac{n}{2} \leq i \leq n$,

$$f(v_i) = \begin{cases} 2k+4i-2n+1 & \text{if } i \text{ is odd} \\ 2k-2(n+1)+4i+1 & \text{if } i \text{ is even} \end{cases}$$

For $n + 1 \leq i \leq n + m - 2$,

$$f(v_i) = 2k + 2(i - 1) + 1$$

For $1 \leq i \leq \frac{n}{2}$,

$$f(v_i') = \begin{cases} 2k+2(n+m)+4(i-2)+1 & \text{if } i \text{ is odd} \\ 2k+2(n+m)-1+4(i-2) & \text{if } i \text{ is even} \end{cases}$$

For $\frac{n+2}{2} \leq i \leq n$,

$$f(v_i') = \begin{cases} 2k+2(3n+m)-4i & \text{if } i \text{ is odd} \\ 2k+2(3n+m)+3-4i & \text{if } i \text{ is even} \end{cases}$$

Then the induced edge labels are

$$f^*(e_i) = \begin{cases} 2k+2n-4i, & 1 \leq i \leq \frac{n}{2} \\ 2k+4i-2n-2, & \frac{n+2}{2} \leq i \leq n \end{cases}$$

For $n + 1 \leq i \leq n + m - 1$,

$$f^*(e_i) = 2k + 2i - 2$$

$$f^*(e_i') = \begin{cases} 2k+2(n+m)+4(i-1)-2, & 1 \leq i \leq \frac{n}{2} \\ 2k+2(3n+m)-4i, & \frac{n+2}{2} \leq i \leq n \end{cases}$$

Therefore, $f^*(E(D_{m,n} @ C_n) = \{2k, 2k + 2, 2k + 4, ..., 2k + 2q - 2\}$. So, f is a k-even mean labeling and hence, $D_{m,n}$ @ C_n, $n \equiv 0 \pmod 4$ is a k-even mean graph for any k.

7-*EML* of $D_{4,8}$ @ C_8 is shown in Fig. 2.2.

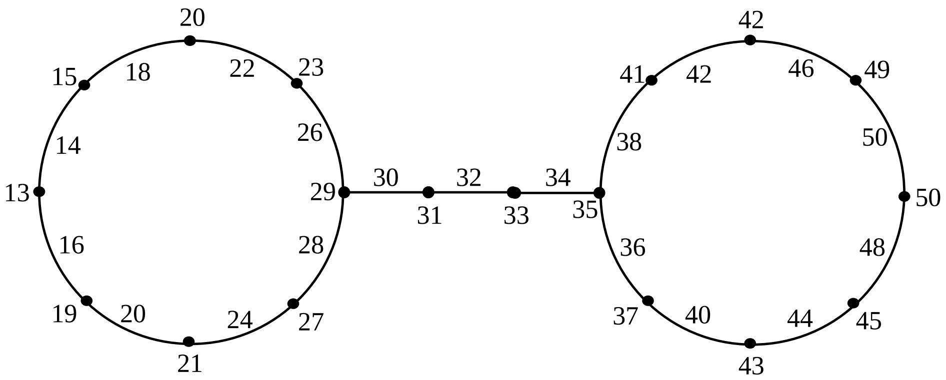

Fig. 2.2: 7-*EML* of $D_{4,8}$ @ C_8

2.4. Theorem

$D_{m,n}$ @ C_n, $n \equiv 1 \pmod 4$ is a k-even mean graph for any k and $m > 2$.

Proof

Let $V(D_{m,n} @ C_n) = \{v_i, 1 \leq i \leq n + m - 2\} \cup \{v_i', 1 \leq i \leq n\}$ and

$E(D_{m,n} @ C_n) = \{e_i, 1 \leq i \leq n + m - 1\} \cup \{e_i', 1 \leq i \leq n\}$ (see Fig. 2.1)

First we label the vertices of $D_{m,n}$ @ C_n as follows:

Define $f: V(D_{m,n} @ C_n) \longrightarrow \{0, 1, 2, ..., 2k + 2q - 2\}$ by

For $1 \leq i \leq \frac{n-3}{2}$,

$$f(v_i) = 2k + 2n - 4i - 2$$

For $\frac{n-1}{2} \leq i \leq n - 1$,

$$f(v_i) = \begin{cases} 2k + 4i - 2n - 1 & \text{if } i \text{ is odd} \\ 2k + 4i - 2n + 1 & \text{if } i \text{ is even} \end{cases}$$

$$f(v_n) = 2k + 2n - 2$$

For $n + 1 \leq i \leq n + m - 2$,

$$f(v_i) = 2k + 2i - 1$$

For $1 \leq i \leq \frac{n+1}{2}$,

$$f(v_i') = \begin{cases} 2k + 2(n+m) + 4(i-2) + 1 & \text{if } i \text{ is odd} \\ 2k + 2(n+m) + 4(i-2) - 1 & \text{if } i \text{ is even} \end{cases}$$

For $\frac{n+3}{2} \leq i \leq n$,

$$f(v_i') = 2k + 2(3n + m) - 4i + 2$$

Then the induced edge labels are

$$f^*(e_i) = \begin{cases} 2k+2n-4i, & 1 \le i \le \dfrac{n-1}{2} \\ 2k+2(2i-n)-2, & \dfrac{n+1}{2} \le i \le n \end{cases}$$

For $n + 1 \le i \le n + m - 1$,

$$f^*(e_i) = 2k + 2i - 2$$

$$f^*(e_i') = \begin{cases} 2k+2(n+m)+4(i-1)-2, & 1 \le i \le \dfrac{n+1}{2} \\ 2k+2(3n+m)-4i, & \dfrac{n+3}{2} \le i \le n \end{cases}$$

Therefore, $f^*(E(D_{m,n} @ C_n)) = \{2k, 2k+2, 2k+4, ..., 2k+2q-2\}$. So, f is a k-even mean labeling and hence, $D_{m,n}$ @ C_n, $n \equiv 1 \pmod 4$ is a k-even mean graph for any k.

7-*EML* of $D_{5,5}$ @ C_5 is shown in Fig. 2.3.

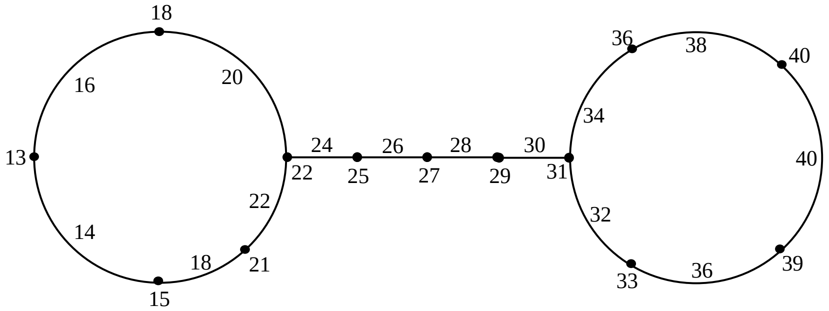

Fig. 2.3: 7-*EML* of $D_{5,5}$ @ C_5

3-*EML* of $D_{3,9}$ @ C_9 is shown in Fig. 2.4.

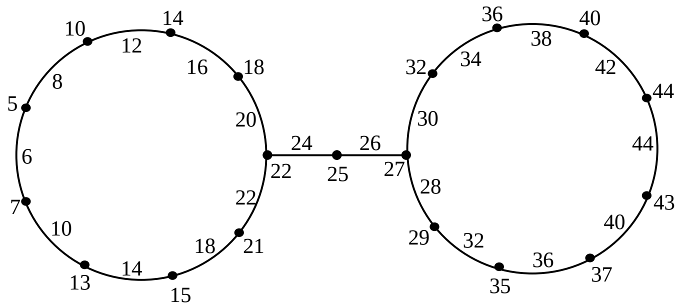

Fig. 2.4: 3-*EML* of $D_{3,9}$ @ C_9

2.5. Theorem

$D_{m,n}$ @ C_n, $n \equiv 2 \pmod 4$ is a k-even mean graph for any k when $n > 6$ and $m > 2$.

Proof

Let $V(D_{m,n} @ C_n) = \{v_i, 1 \le i \le n + m - 2\} \cup \{v_i', 1 \le i \le n\}$ and

$E(D_{m,n} @ C_n) = \{e_i, 1 \leq i \leq n + m - 1\} \cup \{e_i', 1 \leq i \leq n\}$ (see Fig. 2.1)

First we label the vertices of $D_{m,n} @ C_n$ as follows:

Define $f : V(D_{m,n} @ C_n) \longrightarrow \{0, 1, 2, ..., 2k + 2q - 2\}$ by

$$f(v_1) = 2k + 2n - 3$$

For $2 \leq i \leq \frac{n-2}{2}$,

$$f(v_i) = 2k + 2(i - 3) + 1$$

$$f\left(v_{\frac{n}{2}}\right) = 2k + n - 6$$

$$f\left(v_{\frac{n+2}{2}}\right) = 2k + n - 2$$

For $\frac{n+4}{2} \leq i \leq n - 2$,

$$f(v_i) = 2k + 2(i - 2) + 1$$

$$f(v_{n-1}) = 2k + 2n - 6$$

$$f(v_n) = 2k + 2n - 2$$

For $n + 1 \leq i \leq n + m - 2$,

$$f(v_i) = 2k + 2i - 1$$

For $1 \leq i \leq \frac{n-4}{2}$,

$$f(v_i') = 2k + 2(n + m) + 2i - 5$$

$$f\left(v_{\frac{n-2}{2}}'\right) = 2k + 3n + 2m - 8$$

$$f\left(v_{\frac{n}{2}}'\right) = 2k + 3n + 2m - 4$$

For $\frac{n+2}{2} \leq i \leq n - 3$,

$$f(v_i') = 2k + 2(n + m) + 2(i - 2) + 1$$

$$f(v_{n-2}') = 2k + 2(2n + m) - 8$$

$$f(v_{n-1}') = 2k + 2(2n + m) - 4$$

$$f(v_n') = 2k + 2(2n + m) - 5$$

Then the induced edge labels are

$$f^*(e_1) = 2k + 2n - 2$$

$f^*(e_2) = 2k + n - 2$

For $3 \leq i \leq \frac{n+2}{2}$,

$f^*(e_i) = 2k + 2i - 6$

For $\frac{n+4}{2} \leq i \leq n$,

$f^*(e_i) = 2k + 2i - 4$

For $n + 1 \leq i \leq n + m - 1$,

$f^*(e_i) = 2k + 2i - 2$

For $1 \leq i \leq \frac{n-2}{2}$,

$f^*(e_i') = 2k + 2(n + m) + 2(i - 1) - 2$

For $\frac{n}{2} \leq i \leq n - 1$,

$f^*(e_i') = 2k + 2(n + m) + 2i - 2$

$f^*(e_n') = 2k + 3n + 2m - 4$

Therefore, $f^*(E(D_{m,n} @ C_n) = \{2k, 2k + 2, 2k + 4, ..., 2k + 2q - 2\}$. So, f is a k-even mean labeling and hence, $D_{m,n}$ @ C_n, $n \equiv 2 \pmod 4$ is a k-even mean graph for any k when $n > 6$.

6-*EML* of $D_{4,10}$ @ C_{10} is shown in Fig. 2.5.

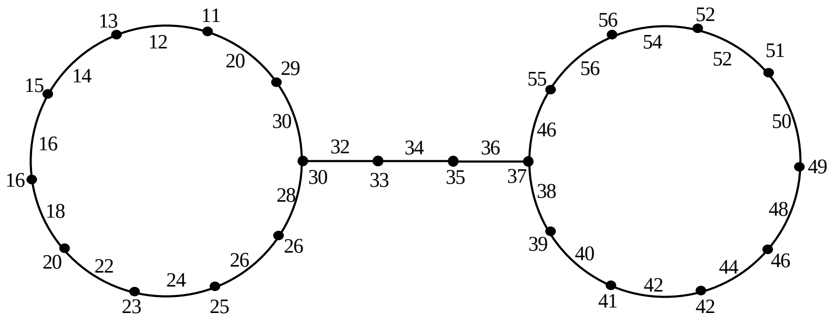

Fig. 2.5: 6-*EML* of $D_{4,10}$ @ C_{10}

2.6. Theorem

$D_{m,n}$ @ C_n, $n \equiv 3 \pmod 4$ is a k-even mean graph for any k and $m > 2$.

Proof

Let $V(D_{m,n} @ C_n) = \{v_i, 1 \leq i \leq n + m - 2\} \cup \{v_i', 1 \leq i \leq n\}$ and

$E(D_{m,n} @ C_n) = \{e_i, 1 \leq i \leq n + m - 1\} \cup \{e_i', 1 \leq i \leq n\}$ (see Fig. 2.1)

First we label the vertices of $D_{m,n} @ C_n$ as follows:

Define $f : V(D_{m,n} @ C_n) \longrightarrow \{0, 1, 2, ..., 2k + 2q - 2\}$ by

For $1 \leq i \leq \frac{n-3}{2}$,

$$f(v_i) = 2k + 2i - 3$$

$$f\left(v_{\frac{n-1}{2}}\right) = 2k + n - 5$$

$$f\left(v_{\frac{n+1}{2}}\right) = 2k + n - 1$$

For $\frac{n+3}{2} \leq i \leq n + m - 2$,

$$f(v_i) = 2k + 2i - 1$$

For $1 \leq i \leq \frac{n-3}{2}$,

$$f(v_i') = 2k + 2(n + m) + 2i - 5$$

$$f\left(v_{\frac{n-1}{2}}'\right) = 2k + 3n + 2m - 7$$

$$f\left(v_{\frac{n+1}{2}}'\right) = 2k + 3n + 2m - 3$$

For $\frac{n+3}{2} \leq i \leq n - 1$,

$$f(v_i') = 2k + 2(n + m) + 2(i - 2) + 1$$

$$f(v_n') = 2k + 2(2n + m) - 4$$

Then the induced edge labels are

$$f^*(e_1) = 2k + n - 1$$

$$f^*(e_i) = \begin{cases} 2k + 2i - 4, & 2 \leq i \leq \frac{n+1}{2} \\ 2k + 2i - 2, & \frac{n+3}{2} \leq i \leq n + m - 1 \end{cases}$$

$$f^*(e_i') = \begin{cases} 2k + 2(n+m) + 2i - 4, & 1 \leq i \leq \frac{n-1}{2} \\ 2k + 2(n+m) + 2i - 2, & \frac{n+1}{2} \leq i \leq n - 1 \end{cases}$$

$$f^*(e_n') = 2k + 3n + 2m - 2$$

Therefore, $f^*(E(D_{m,n} @ C_n)) = \{2k, 2k + 2, 2k + 4, ..., 2k + 2q - 2\}$. So, f is a k-even mean labeling and hence, $D_{m,n}$ @ C_n, $n \equiv 3 \pmod 4$ is a k-even mean graph for any k. 2-*EML* of $D_{3,11}$ @ C_{11} is shown in Fig. 2.6.

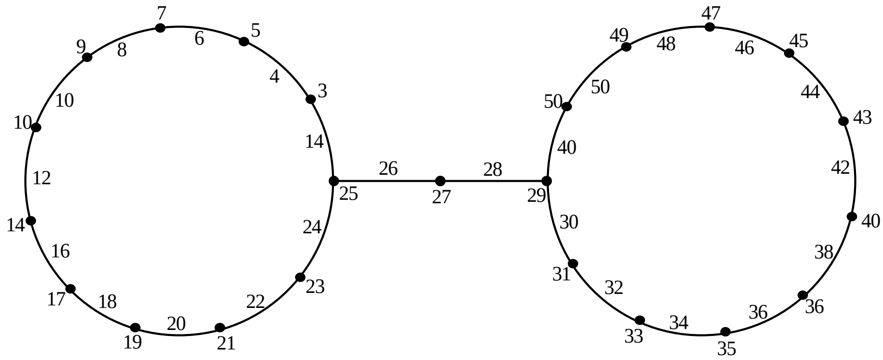

Fig. 2.6: 2-*EML* of $D_{3,11}$ @ C_{11}

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