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Acyclic Weak Convex Domination in Graphs

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Abstract: *In a graph G = (V, E), a set $D \subset V$ is a weak convex dominating(WCD) set if each vertex of V-D is adjacent to at least one vertex in D and $d_{<D>}(u,v) = d_G(u,v)$ for any two vertices u, v in D. A weak convex dominating set D, whose induced graph <D> has no cycle is called acyclic weak convex dominating (AWCD) set. The domination number γ_{ac} (G) is the smallest order of a acyclic weak convex dominating set of G and the codomination number of G, written γ_{ac} ($\overline{G}$), is the acyclic weak convex domination number of its complement. In this paper we found various bounds for these parameters and characterized the graphs which attain these bounds.*

Keywords: *domination number, distance, eccentricity, radius, diameter, self-centered, neighbourhood, weak convex dominating set, acyclic weak convex dominating set.*

1. Introduction

Finding a delay preserving sub network that can communicate with all the nodes of a communication network is a pioneer problem in communication network models. This delay preserving substructure performs with good tolerant index. This structure is studied in detail with the use of *Weak Convex Dominating* (W.C.D) set by considering the underlying graph of the communication network in [4]. Also the distance preserving dominating set concept will be useful in identifying the delay preserving sub network which can cover entire network as connected centre location of the given network. These concepts can be applied also to management and social network for similar applications. In this paper we concentrate on these sub structure with minimum number of links by defining a set called *Acyclic Weak Convex Dominating* (A.W.C.D) set.

Graphs discussed in this paper are undirected and simple. Unless otherwise stated the graphs which we consider are connected graphs only. For a graph G, let V(G) and E(G) denote its vertex and edge set respectively. The *degree* of a vertex *v* in a graph G is denoted by $\deg_G(v)$. The length of any shortest path between any two vertices *u* and *v* of a connected graph G is called the *distance between u and v* and is denoted by $d_G(u, v)$. The distance between two vertices in different components of a disconnected graph is defined to be ∞. For a connected graph G, the *eccentricity* $e_G(v) = \max\{d_G(u, v): u \in V(G)\}$. If there

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is no confusion, we simply use the notion deg(v), d(u, v) and e(v) to denote degree, distance and eccentricity respectively for the concerned graph. The minimum and maximum eccentricities are the *radius* and *diameter* of G, denoted r(G) and diam(G) respectively. When these two are equal, the graph is called *self-centered* graph with radius r, equivalently is *r self-centered*. A vertex u is said to be an *eccentric vertex* of v in a graph G, if d(u, v) = e(v). In general, u is called an eccentric vertex, if it is an eccentric vertex of some vertex. For $v \in$ V(G), the *neighbourhood* $N_G(v)$ of v is the set of all vertices adjacent to v in G. The set $N_G(v) = N_G(v) \cup \{v\}$ is called the *closed neighbourhood* of v. A set S of edges in a graph is said to be *independent* if no two of the edges in S are adjacent. An edge e = (u, v) is a *dominating edge* in a graph G if every vertex of G is adjacent to at least one of u and v.

The concept of domination in graphs was introduced by Ore[13]. A set D $\subseteq$ V(G) is called *dominating set* of G if every vertex in V(G)-D is adjacent to some vertex in D. D is said to be a *minimal* dominating set if D-{v} is not a dominating set for any $v \in$ D. The *domination number* γ(G) of G is the minimum cardinality of a dominating set. We call a set of vertices a γ-set if it is a dominating set with cardinality γ(G). Different types of dominating sets have been studied by imposing conditions on the dominating sets. A dominating set D is called *connected* (*independent*) dominating set if the induced subgraph <D> is connected (independent). D is called a *total dominating set* if every vertex in V(G) is adjacent to some vertex in D.

A cycle D of a graph G is called a *dominating cycle* of G, if every vertex in V – D is adjacent to some vertex in D. A dominating set D of a graph G is called a *clique dominating set* of G if <D> is complete. A set D is called an *efficient dominating set* of G if every vertex in V – D is adjacent to exactly one vertex in D. A set D $\subseteq$ V is called a *global dominating set* if D is a dominating set in G and $\overline{G}$. A set D is called a *restrained dominating set* if every vertex in V(G)-D is adjacent to a vertex in D and another vertex in V(G)-D. A set D is a *weak convex dominating set* if each vertex of V-D is adjacent to at least one vertex in D and the distance between any two vertices u and v in the induced graph <D> is equal to that of those vertices u and v in G. By $\gamma_c, \gamma_i, \gamma_t, \gamma_o, \gamma_k, \gamma_e, \gamma_g, \gamma_r$ and γ_{wc}, we mean the minimum cardinality of a connected dominating set, independent dominating set, total dominating set, cycle dominating set, clique dominating set, efficient dominating set, global dominating set and restrained dominating set respectively.

In this paper we introduce a new dominating set called acyclic weak convex dominating set of a graph through which we studied the properties of the graph such as variation in radius and diameter of the graph. We find upper and lower bounds for the new domination number in terms of various already known parameters. Also we studied several interesting properties like Nordhaus-Gaddum type results relating the graph and its complement.

2. Prior Results

Theorem 2.1:[14]

Let G be any graph and D be any dominating set of G. Then | V-D | $\leq \sum_{u \in V(D)} \deg(u)$ and equality holds in this relation if and only if D has the following properties :

(i) D is independent.

(ii) For every $u \in$ V-D, There exist a unique vertex $v \in$ D such that N(u) $\cap$ D = {v}.

Theorem 2.2:[3]

For any tree T of order p $\geq$ 3, γ_c(S(T))= 2p-e-1, where 'e' denotes the number of pendent vertices of T.

3. Main Results

3.1 Existence of Acyclic Weak Convex Dominating sets:

Definition3.1:

A Weak convex dominating (W.C.D) set is said to be an *Acyclic weak convex dominating* (A.W.C.D) set, if D is acyclic. The cardinality of the minimum A.W.C.D set is denoted by γ_{ac}.

Proposition 3.1:

Let G be a unicyclic graph having a cycle C_n of length less than or equal to 6. Then G does not have any A.W.C.D set, if one of the following hold:

(i) there exists a cycle C_n, where n < 6 such that at least $\lfloor n/2 \rfloor$ + 2 number of vertices of C_n, each of which dominates uniquely some vertex in G

(ii) there exists a C_6 such that n/3 number of vertices of C_n each of which dominate uniquely some vertex in G.

Theorem 3.1:

Let G be any graph having girth $\geq$ 7, then it doesn't have A.W.C.D set.

Proof:

Let G be a graph having girth $\geq$ 7. Let D be a γ_{wc} set of G. We know that γ_{wc} (G) $\leq$ p. If γ_{wc}(G) = p, then G doesn't have A.W.C.D set. If γ_{wc}(G) $\neq$ p, then γ_{wc} (G) < p. This implies that | V-D | $\geq$ 1. Let $u \in$ V-D. If two vertices of D dominate u, then as D is convex there must exists a C_3 or C_4 in G. Therefore, only one vertex of D dominates u. If deg(u) $\neq$ 1,then there exists another vertex $v \in$ V-D such that u and v are adjacent. And also u and v are not dominated by the same vertex of D (if possible, then C_3 arises). Therefore, u and v are dominated by two different vertices say some u' and v' of D respectively. This implies that d(u', v') $\leq$ 3 (since length of the path $u'uvv'$ = 3). This implies that there must be a C_4 or C_5 or C_6 exist in G, which is a contradiction to g(G) $\geq$ 7. Hence V-D contains only the pendent vertices. That is all the vertices other than pendent vertices are in D. Hence, G does not have any A.W.C.D set.

Proposition 3.2:

Let G be a graph having a geodesic cycle C_n, where n$\geq$ 7 does not have any A.W.C.D set.

Proof:

Let G be a graph on n vertices having C_p, where p$\geq$ 7 as a geodesic cycle. For n = 7, G = C_7 which doesn't have any A.W.C.D set. Assume that for n $\geq$ 8, the graph G on n vertices having C_p, where p $\geq$ 7, has no A.W.C.D set.

To prove for a graph on (n + 1) vertices, let G be a graph on n+1 vertices such that it has an acyclic dominating set D in G. Let $u \in$V – D and G' = G – u. Without loss of generality, let us assume that u lies on a cycle of length at most 6. Clearly, by induction hypothesis, G' has no acyclic dominating set, which is a contradiction to the assumption that D is an acyclic dominating set for both G and G'. Thus the proposition is true.

Proposition 3.3:

Let G be a self-centred graph of diameter 2. Then γ_{ac} exists if and only if there exists a dominating set isomorphic to $K_{1,n}$.

Proof :

Let G be a self-centred graph of diameter 2. Suppose γ_{ac} exists in G. Let D be an A.W.C.D set in G. Then the induced sub graph <D> induced by D must be a tree. Clearly, |D| is not equal to one, since G is a self-centred graph of diameter 2. If |D|=2, then the claim is true. Suppose |D| $\geq$ 3. Let u be a vertex of degree greater than or equal to 2 in <D>. Then all the vertices in D must be adjacent to u.

If not, some vertex $v \in$ D is not adjacent to u. That is v is in the second neighbourhood of u. Since <D> is a connected graph, v must be connected to u through some vertex w in $N_1(u)$. Since u is the internal vertex of <D>, there exists some vertex w', which is adjacent to u in D. Now, to maintain the distance two between v and w', D must

contain some vertex of $N_1(u)$ or $N_2(u)$. In both the cases <D> must have a cycle, which is a contradiction to D is an A.W.C.D set. Hence, all the vertices in D must be adjacent to u. Also, any other vertices, which are different from u, cannot be adjacent. Otherwise, that induces a three cycle in <D>. Hence, <D> $\cong K_{1,n}$.
Converse part is a trivial one.

3.2 Bounds on γ_{ac}

Proposition 3.4:

If T is a tree then $\gamma_{ac}(T)$ = p-e, where e is the number of pendant vertices of T.

Proposition 3.5:

$\gamma_{ac}(K_{m,n}) = 2$, for m, n $\geq$ 2.

Proposition 3.6:

$\gamma_{ac}(K_p) = 1$.

Proposition 3.7:

$\gamma_{wc} \leq \gamma_{ac}$, for any graph G, if γ_{ac} exists in G.

Proof:

Any acyclic W.C.D set is a W.C.D set. Therefore, cardinality of minimal W.C.D set is always less than or equal to cardinality of minimal A.W.C.D set. Thus, $\gamma_{wc} \leq \gamma_{ac}$.

Proposition 3.8:

If T is any tree, then $\gamma_{ac} \leq$ p-2.

Proof:

Proof follows from proposition 3.4 and the fact that the number of pendant vertices in a tree is greater than or equal to 2.

Proposition 3.9:

If G is a graph in which γ_{ac} exists, then $\gamma_{ac} \leq$ p-2.

Proof :

Proof follows from the previous proposition 3.8.

Proposition 3.10:

For any isolate-free disconnected graph G, $\gamma_{ac}(\overline{G})=2$.

Proposition 3.11:

Let G be a self-centred graph of diameter 2. If for a vertex u, there exists some adjacent vertices $w, v \in N_2(u)$ such that $N_1(w) \cap N_1(v) = \phi$, then $\gamma_{ac}(\overline{G}) \leq 3$.

Proof :

Let G be a self-centred graph of diameter 2. Suppose for a vertex u, there exists an adjacent vertices $w, v \in N_2(u)$ such that $N_1(w) \cap N_1(v) = \phi$. Then clearly the set $\{u, v, w\}$ will form an A.W.C.D set for $\overline{G}$ and hence $\gamma_{ac}(\overline{G}) \leq 3$.

Proposition 3.12:

For any self-centred graph G of diameter 2, $\gamma_{ac}(G) \leq \Delta+1$.

Proof :

Proof follows directly from the previous proposition 3.3.

Proposition 3.13:

Let G be a self-centred graph of diameter 2 with one vertex of degree 2, then $\gamma_{ac}(G) \leq 3$.

Proof:

Let G be a self-centred graph of diameter 2 with a vertex u' of degree 2. Consider a vertex u, which is adjacent to u'. Consider the other vertex u'', which is adjacent to u'.

Case 1:

If u'' is adjacent to u, that is $u' \in N_1(u)$. Then clearly, all the vertices of $N_2(u)$ is adjacent to u'' (to maintain the distance 2 between them and u'). Also, all of $N_1(u)$ is adjacent to u. Thus, the set $\{u, u''\}$ forms an A.W.C.D. set for G. Hence, $\gamma_{ac} = 2$.

Case 2:

If u'' is not adjacent to u, that is $u'' \in N_2(u)$. To maintain the distance between u' and the vertices of $N_2(u)$, all the vertices of $N_2(u)$ must be adjacent to u''. Also, all the vertices of $N_1(u)$ are adjacent to u. Thus, the set $\{u, u', u''\}$ forms an A.W.C.D set for G. Hence, $\gamma_{ac}(G) \leq 3$.

Therefore, by combining both the cases, we have $\gamma_{ac} \leq 3$.

Proposition 3.14:

Let G be self-centered graph of diameter 2. If G has two non-adjacent vertices of degree 2, then $\gamma_{ac}(G) \leq 3$.

Proof:

Let G be a self-centered graph of diameter 2. Let v_1, v_2 be two non-adjacent vertices of degree 2. Since the graph is of diameter 2, v_1 and v_2 are adjacent to some vertex, say v. Also v_1 and v_2 are of degree 2, they must be adjacent to the vertices say v_1^1 and v_2^1 respectively.

Case 1: If $v_1^1 = v_2^1$

Case 1.1: If v_1^1 lies in $N_1(v)$.

Then all the vertices in $N_2(v)$ are adjacent to v_1^1 to maintain the distance 2 between them and v_1^1. Therefore, v and v_1^1 will form an acyclic W.C.D set for G. Thus $\gamma_{ac}(G) = 2$.

Case 1.2: If $v_1^1 \in N_2(v)$.

In this case, all the vertices of $N_2(v)$ are adjacent to v_1^1. Therefore $\{v, v_1, v_1^1\}$ will form an acyclic W.C.D set for G. Hence $\gamma_{ac}(G) \leq 3$.

Case 2: If $v_1^1 \neq v_2^1$.

Case 2.1: If both v_1^1 and v_2^1 belongs to $N_1(v)$.

Here also to maintain the distance all the vertices of $N_2(v)$ are adjacent to both v_1^1 and v_2^1. Thus $\{v, v_1^1\}$ from and A.W.C.D set for G. Hence $\gamma_{ac}(G) = 2$.

Case 2.2: If suppose $v_1^1 \in N_1(v)$ and $v_2^1 \in N_2(v)$.

Clearly all the vertices of $N_2(v)$ are adjacent to both v_1^1 and v_2^1. Hence $\{v, v_1^1\}$ from A.W.C.D set. Thus $\gamma_{ac}(G) = 2$.

Case 2.3: If both v_1^1 , $v_2^1 \in N_2(v)$.

Then all vertices of $N_2(v)$ are adjacent to both v_1^1 and v_2^1. Therefore, $\{v, v_1, v_1^1\}$ will form an A.W.C.D set. Hence $\gamma_{ac}(G) \leq 3$.

Therefore, combining all the above cases, we get $\gamma_{ac}(G) \leq 3$. Hence the Proof.

Theorem 3.2:

Let G be a self-centered graph of diameter 2. If G has any two adjacent vertices of degree 2, then $\gamma_{ac}(G) \leq 3$.

Proof:

Let G be a self-centered graph of diameter 2. Let u and v be any two adjacent vertices of degree 2. Let $w \neq v$ be a vertex adjacent to u.

Claim 1 : w is not adjacent to v.

If not, let v and w be adjacent in G. Then all the vertices other than u, v and w must be adjacent to w to maintain the distance 2 from u and v. This implies that radius of G is 1. This is a contradiction to G is self-centered of diameter 2. Thus, v and w are non-adjacent.

Therefore, we have, $u \in N_1(w)$ and $v \in N_2(w)$.

Case 1 :

If v is adjacent to some vertex $x \in N_1(w)$ other than u. Then all other vertices in $N_1(w)$ must be adjacent to x to maintain distance 2 from v and also adjacent to w. Clearly in this case, $N_2(w) = \{v\}$ only. Otherwise the distance from u to any other vertex in $N_2(w)$ different from v becomes 3. Thus the edge $\{w, x\}$ forms an A.W.C.D. set for G. Hence $\gamma_{ac}(G) = 2$.

Case 2 :

If v is not adjacent to any of the vertex $N_1(w)$. Clearly $|N_2(w)| = 2$. Let $x \in N_2(w)$ other than v. Now all the vertices of $N_1(w)$ other than u must be adjacent to x to maintain the distance between them and v. Thus the set $\{w, y, x\}$ forms an A.W.C.D set for G. Hence $\gamma_{ac}(G) = 3$.

3.3 Nordhaus-Gaddum Type Results

Next we prove a result relating Nordhaus-Gaddum type result.

Theorem 3.3:

Let G be a graph with diameter greater than or equal to 3, then $\gamma_{ac}(G)+\gamma_{ac}(\overline{G}) \leq p$.

Proof :

Let G be a graph of diameter greater than or equal to 3. Then any pair of vertices in G of distance greater than or equal to 3 will dominate the whole of $\overline{G}$. Hence, $\gamma_{ac}(\overline{G}) \leq 2$. Thus, from the proposition 3.9, we have $\gamma_{ac}(G) + \gamma_{ac}(\overline{G}) \leq p-2+2 = p$.

Theorem 3.4:

If both G and $\overline{G}$ are self-centred graph of diameter 2, then $\gamma_{ac}(G) + \gamma_{ac}(\overline{G}) \leq p+\Delta-\delta+1$.

Proof :

As $\Delta(\overline{G}) = p-\delta-1$, $\gamma_{ac}(G) + \gamma_{ac}(\overline{G}) \leq \Delta(G)+1+\Delta(\overline{G})+1 = p+\Delta-\delta+1$.

Theorem 3.5:

Let G and $\overline{G}$ be self-centred graph of diameter 2 and let v be a vertex in G. If $N_2(v)$ is a clique, then $\gamma_{ac}(G) + \gamma_{ac}(\overline{G}) \leq \Delta+4$.

Proof :

If $N_2(v)$ is a clique, then take a vertex w in $N_2(v)$ and some $u \in N_1(v)$, which is adjacent to both v and w. Then $\{u, v, w\}$ forms a A.W.C.D set for G. In the complement of G $\gamma_{ac}(\overline{G}) \leq \Delta+1$. Therefore, $\gamma_{ac}(G) + \gamma_{ac}(\overline{G}) \leq \Delta+4$.

Theorem 3.6:

Let v be a vertex in self-centred graph G of diameter 2. If $N_1(v)$ forms an independent set, then $\gamma_{ac}(G) + \gamma_{ac}(\overline{G}) \leq \Delta+4$.

Proof :

The proof follows from the above proof by replacing the role of G and $\overline{G}$.

Theorem 3.7:

Let G and $\overline{G}$ be self-centred graphs of diameter 2. If there exists a vertex v with radius one in γ_{ac} sets for both G and $\overline{G}$, then $\gamma_{ac}(G) + \gamma_{ac}(\overline{G}) \leq p+1$.

Proof:

$\gamma_{ac}(G) + \gamma_{ac}(\overline{G}) \leq \Delta(G) + 1 + \delta(\overline{G}) + 1 = (p-1) + 2 = p+1$.

Theorem 3.8:

For any isolate-free disconnected graph G with k components, $\gamma_{ac}(G) + \gamma_{ac}(\overline{G}) \leq p-2(k-1)$.

Proof:

Let $C_1, C_2, \ldots, C_k$ be the k components of G. Then from the proposition 3.5 we get,

$$\begin{aligned}\gamma_{ac}(G) &\leq |C_1|-2 + |C_2|-2 + \ldots + |C_k|-2 \\ &= |C_1|+|C_2|+\ldots+|C_k|-2k \\ &= p-2k\end{aligned}$$

Also, any two vertices one from each of the two different components will form a A.W.C.D set for $\overline{G}$. Thus, $\gamma_{ac}(G) + \gamma_{ac}(\overline{G}) \leq$ p-2k+2 = p-2(k-1).
Hence the proof.

Remark 3.1:
Let r, d be the radius and diameter of G. Let r' and d' be the radius and diameter of <D>, where D is an A.W.C.D set of G.
Then following are the various cases.

As <D> is a tree, we have two major cases (1) d' = 2r' and (2) d' = 2r'-1.

As we have the properties that d-2 $\leq$ d' $\leq$ d and r-1 $\leq$ r' $\leq$ r, let us derive the relation between r and d of G for the various relations between r' and d' in <D>.

Case 1: d' = d and r' = r.

Case 1.1: d' = 2r'

As d = d' = 2r' = 2r, we have d= 2r in this case.

Case 1.2: d' = 2r'-1.

As d = d' = 2r'-1 = 2r-1, we have d = 2r-1 in this case.

Case 2: d = d' and r' = r-1.

Case 2.1: d' = 2r'.

Here, d = d' = 2r' = 2(r-1) = 2r-2. Therefore, d = 2r-2.

Case 2.2: d' = 2r'-1.

Here, d = d' = 2r'-1 = 2(r-1)-1 = 2r-3. Therefore d = 2r-3.

Case 3: d' = d-1 and r' = r.

Case 3.1: d' = 2r'.

Here, d= d'+1 = 2r'+1 = 2r+1, which is not possible in G. Therefore, <D> cannot have this structure.

Case 3.2: d' = 2r'-1.

Here, d = d'+1 = 2r'-1+1=2r. Therefore, in this case d = 2r.

Case 4: d' = d-2 and r' =r.

Case 4.1: d' = 2r'.

Here, d = d'+2 = 2r'+2 = 2r+2, which is not possible in G.

Case 4.2: d' = 2r'-1.

Here, d = d' +2 = 2r'-1+2 = 2r'+1 = 2r+1, which is also not possible in G.

Thus, case 4 is not possible in G.

Case 5: d' = d-2 and r' = r-1.

Case 5.1: d' = 2r'.

Here, d = d' + 2 = 2r'+2 = 2(r-1)+2 =2r. Thus, in this case d = 2r.

Case 5.2: d' = 2r'-1.

Here, $d = d'+2 = (2r'-1)+2 = 2r' = 2(r-1) = 2r-2$. Thus, $d = 2r-2$.

Case 6: $d' = d-1$ and $r' = r-1$.

Case 6.1: $d' = 2r'$.

Here, $d = d' +1 = 2r'+1 = 2(r-1)+1 = 2r-1$. Thus, $d = 2r-1$.

Case 6.2: $d' = 2r'-1$.

Here, $d = d'+1 = (2r'-1)+1 = 2r' = 2(r-1) = 2r-2$. Thus, $d = 2r-2$.

From the above all cases, we have

$2r-3 \leq d \leq 2r$ for a graph G having acyclic dominating set.

Thus we have the following theorem,

Theorem 3.9:

Let G be a connected graph with diameter d and radius r having a γ_{ac} set then $2r-3 \leq d \leq 2r$.

From these above cases it is clear that

1. If G is self-centered with diameter equal to 3 then A.W.C.D set D has $d' = 3$ and $r' = 2$.

2. If G has diameter 3 and radius 2, then A.W.C.D set D has pair (d', r') with solutions (3, 2) and (1, 1).

3. If G has diameter 4 and radius 2, then (d', r') will be (3, 2), (4, 2), (2, 1).

4. If G has diameter 4 and radius 3, then (d', r') will be (4, 2).

We mention, if G has no A.W.C.D set, then $\gamma_{ac} = 0$. Then the following results can be easily verified from the solution set (d', r') from (d, r) satisfying $d-2 \leq d' \leq d$ and $r-1 \leq r' \leq r$.

5. If G is self-centered with diameter $d \geq 4$, then G has no A.W.C.D set , that is $\gamma_{ac} = 0$.

6. If G has diameter $d \geq 2r-4$, then $\gamma_{ac} = 0$.

7. If G is self-centered with diameter greater than or equal to 4, then $d_{ac}(G)+d_{ac}(\overline{G}) \leq p/2$.

8. If G is self-centered with diameter 3, then $d_{ac}(G)+d_{ac}(\overline{G}) \leq \frac{p}{4}+\frac{p}{2}=\frac{3p}{4}$, as each dominating set is with $|V(P_4)|=4$.

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