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A New Form of Generalized Yield Criteria of Porous Sintered Powder Metallurgy Metals

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Abstract: *This note is intended to present a new and corrected version of generalized yield criteria of porous sintered powder metallurgy metals, which was proposed in our earlier paper (Narayanasamy et.al.,2001). It is noticed that equations (14)-(17) are dimensionally and mathematically incorrect. This is a slip in our part. In the case of anisotropic non-porous sheet metals with the parameters m=2, n=2 and p=1, mathematical expression for yield equation obtained from generalized yield equation (see equation (15)) in our paper (Narayanasamy et.al.,2001) has not been reduced to the corresponding cited equation (20) which was taken from the book written by Johnson and Mellor(1973). It is wrongly mentioned in our paper (Narayanasamy et.al., 2001) that when substituting m=2, p=1 and n=2 in Eq.(17), Eq.(24) is obtained, which is impossible. It is further observed that Eqs.(14) and (22) are not the same when m=2 and p=1. At this juncture, it is pertinent to pin-point out that the theoretical prediction of forming limit strain, wrinkling tendency, localized necking behaviour etc. of sheet materials using improper generalized yield criteria leads to wrong conclusions which, in turn, could not be helpful for us to have a crystal clear understanding the actual characteristics of plastic deformation of sheet metals and materials and to the development of designing tools for manufacturing process. Based on the forgoing views, it is pivotal to propose a new form of genreralized yield equation and obtain the corresponding particular cases that form the content of the present paper.*

Key words: *New and corrected version of Generalized yield equation; porous-anisotropic metals; P/M perform*

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Case 1: Generalized Yield Theory

As reported by Lee and Kim (1992), we write

$$(2+R^n)J_2^{'}+\frac{1}{3}(1-R^n)J_1^2=Y_R^2 \qquad (1)$$

where J_1, $J_2^{'}$ are called linear stress invariant and the quadratic stress deviator invariant respectively. It is observed from the work done by Lee and Kim (1992) that the value of n has been taken as 2 where n is a constant, which takes values like 1.80,1.9,2.0 and 2.1, depending upon the level of anisotropic nature. The yield stress of porous metal Y_R may be related to the yield stress of the non-porous metal Y_0 through a geometrical hardening parameter (η) as follows.

$$Y_R^2=\eta Y_0^2 \qquad (2)$$

where $\eta = \{(R-R_c) / (1-R_c)\}^2$ with $0 < R_c \leq R \leq 1$,

which is given in Lee and Kim (1992). Here R and R_c are relative density and critical relative density of P/M perform respectively. Yield equation for anisotropic metals in terms of the principal stresses can obtained as follows.

$$Y_0^m(1+r^p)=|\sigma_2-\sigma_3|^m+|\sigma_3-\sigma_1|^m+r^p|\sigma_1-\sigma_2|^m \qquad (3)$$

where p is a constant r^p varies from say, 0.8 to 1.2. The volumetric strain energy expression can be written as:

$$J_1^m=\left(\frac{3}{2+r^p}\right)|\sigma_1+\sigma_2+\sigma_3|^m \qquad (4)$$

The modified distortion strain energy term is

$$J_2' = \left\{ \frac{1}{2+r^p} \right\} \frac{\{|\sigma_2-\sigma_3|^m + |\sigma_3-\sigma_1|^m + r^p|\sigma_1-\sigma_2|^m\}^{\frac{2}{m}}}{(2+(p-1)^2 r^p)} \tag{5}$$

From Eqs.(1)-(5), the new and corrected version of yield criterion with five parameter constants for porous anisotropic metals becomes;

$$\frac{(2+R^n)}{(2+r^p)(2+(p-1)^2 r^p)}\{|\sigma_2-\sigma_3|^m + |\sigma_3-\sigma_1|^m + r^p|\sigma_1-\sigma_2|^m\}^{\frac{2}{m}} + \left(\frac{3}{2+r^p}\right)^{\frac{2}{m}} \frac{(1-R^n)}{3}|\sigma_1+\sigma_2+\sigma_3|^2 = Y_0^2 \frac{(R-R_c^2)}{(1-R_c)^2} \tag{6}$$

For Uniaxial compression or porous –anisotropic metals, Eq.(6) becomes:

$$\frac{(1+r^p)^{\frac{2}{m}}}{(2+(p-1)^2 r^p)} = \frac{(2+R^n)}{2+r^p}\sigma_1^2 + \left(\frac{3}{2+r^p}\right)^{\frac{2}{m}} \frac{(1-R^n)}{3}\sigma_1^2 = Y_R^2$$

(7)

Eq.(7) can be rewritten as follows:

$$\frac{\sigma_1^2}{Y_0^2}\left[\frac{(1+r^p)^{\frac{2}{m}}}{(2+(p-1)^2 r^p)} \frac{(2+R^n)}{(2+r^p)} + \frac{\left(\frac{3}{2+r^p}\right)^{\frac{2}{m}}(1-R^n)}{3}\right] = \frac{(R-R_c^2)}{(1-R_c)^2} \tag{8}$$

Case 2: Porous Anisotropic Metals

The yield stress of a porous metal Y_R may be related to the yield stress of a non-porous metal Y_0 through a geometrical hardening parameter (η) as follows, as given in case 1.

$$Y_R^2 = \eta Y_0^2 \tag{9}$$

where $\eta = \{(R-R_c) / (1-R_c)\}^2$ with $0 < R_c \le R \le 1$,

which is reported by Lee and Kim(1992). Here R and R_c are relative density and critical relative density of P/M perform respectively. Yield equation for anisotropic metals in terms of the principal stresses can obtained as follows(Johnson and Mellor,1973):

$$\bar{\sigma} = \sqrt{\frac{3}{2}}\left[\frac{(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2+r(\sigma_1-\sigma_2)^2}{(2+r)}\right]^{\frac{1}{2}} \tag{10}$$

Substituting the values of n = m = 2 and p =1 in Eq.(6), the yield criterion for porous-anisotropic metals becomes:

$$(2+R^2)\frac{1}{2}\left[\frac{(\sigma_1-\sigma_2)^2 r+(\sigma_2-\sigma_3)^2+(\sigma_2-\sigma_3)^2}{(2+r)}\right] + \left(\frac{1-R^2}{3}\right)\left[\frac{3}{(2+r)}(\sigma_1+\sigma_2+\sigma_3) = Y_0^2\frac{(R-R_c^2)}{(1-R_c)^2} \tag{11}$$

For Uni-axial compression of porous-anisotropic metals, Eq.(11) becomes:

$$\frac{\sigma_1^2}{Y_0^2} = \frac{(R-R_c^2)(4+2r)}{[(2+R^2)(1+r)+2(1-R^2)](1-R_c)^2} \tag{12}$$

When substituting m = n = 2 and p = 1 in Eq.(8) , Eq.(12) is obtained.

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