International Journal of Engineering Science, Advanced Computing and Bio-Technology
Vol. 1, No. 1, January –March 2010, pp. 1-13

Weak Convex Domination in Graphs

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Abstract: *In a graph G = (V, E), a set* $D \subset V$ *is a weak convex set if* $d_{<D>}(u,v) = d_G(u,v)$ *for any two vertices u, v in D. A weak convex set is called as a weak convex dominating (WCD) set if each vertex of V-D is adjacent to at least one vertex in D. The weak convex domination number* $\gamma_{wc}(G)$ *is the smallest order of a weak convex dominating set of G and the codomination number of G, denoted by* $\gamma_{wc}(\bar{G})$*, is the weak convex domination number of its complement. In this paper, we found various bounds of these parameters and characterized the graphs, for which bounds are attained.*

Keywords: *domination number, distance, eccentricity, radius, diameter, self-centered, neighbourhood, weak convex set, weak convex dominating set.*

1. Introduction

Graphs discussed in this paper are undirected and simple. Unless otherwise stated the graphs which we consider are connected graphs only. For a graph G, let V(G) and E(G) denote its vertex and edge set respectively and p and q denote the cardinality of those sets respectively. The *degree* of a vertex *v* in a graph G is denoted by $deg_G(v)$. The minimum and maximum degree in a graph is denoted by δ and Δ respectively. The length of any shortest path between any two vertices *u* and *v* of a connected graph G is called the *distance between u and v* and is denoted by $d_G(u, v)$. The distance between two vertices in different components of a disconnected graph is defined to be ∞. For a connected graph G, the *eccentricity* $e_G(v) = \max\{d_G(u, v): u \in V(G)\}$. If there is no confusion, we simply use the notion deg(*v*), d(*u*, *v*) and e(*v*) to denote degree, distance and eccentricity respectively for the concerned graph. The minimum and maximum eccentricities are the *radius* and *diameter* of G, denoted r(G) and diam(G) respectively. When these two are equal, the graph is called *self-centered* graph with radius r, equivalently is *r self-centered*. A vertex *u* is said to be an *eccentric vertex* of *v* in a graph G, if d(*u*, *v*) = e(*v*). In general, *u* is called an eccentric vertex, if it is an eccentric vertex of some vertex. For $v \in V(G)$, the *neighbourhood* $N_G(v)$ of *v* is the set of all vertices adjacent to *v* in G. The set

Received: 02 February, 2009; Revised: 18 February, 2009; Accepted: 24 February, 2009

$N_G[v] = N_G(v) \cup \{v\}$ is called the *closed neighbourhood* of v. A set S of edges in a graph is said to be *independent* if no two of the edges in S are adjacent. An edge e = (u, v) is a *dominating edge* in a graph G if every vertex of G is adjacent to at least one of u and v.

The concept of domination in graphs was introduced by Ore. A set $D \subseteq V(G)$ is called *dominating set* of G if every vertex in V(G)-D is adjacent to some vertex in D. D is said to be a *minimal* dominating set if D-$\{v\}$ is not a dominating set for any $v \in D$. The *domination number* γ(G) of G is the minimum cardinality of a dominating set. We call a set of vertices a γ-set if it is a dominating set with cardinality γ(G). Different types of dominating sets have been studied by imposing conditions on the dominating sets. A dominating set D is called *connected* (*independent*) dominating set if the induced subgraph <D> is connected (independent). D is called a *total dominating set* if every vertex in V(G) is adjacent to some vertex in D.

A cycle of D of a graph G is called a *dominating cycle* of G, if every vertex in V–D is adjacent to some vertex in D. A dominating set D of a graph G is called a *clique dominating set* of G if <D> is complete. A set D is called an *efficient dominating set* of G if every vertex in V – D is adjacent to exactly one vertex in D. A set $D \subseteq V$ is called a *global dominating set* if D is a dominating set in G and $\overline{G}$. A set D is called a *restrained dominating set* if every vertex in V(G)-D is adjacent to a vertex in D and another vertex in V(G)-D. By $\gamma_c, \gamma_i, \gamma_t, \gamma_o, \gamma_k, \gamma_e, \gamma_g$ and γ_r, we mean the minimum cardinality of a connected dominating set, independent dominating set, total dominating set, cycle dominating set, clique dominating set, efficient dominating set, global dominating set and restrained dominating set respectively.

In this paper, we introduce a new dominating set called weak convex dominating set of a graph through which we analyse the properties of the graph such as variation in radius and diameter of the graph. We find upper and lower bounds for the new domination number in terms of various already known parameters. Also we obtain several interesting properties like Nordhaus-Gaddum type results relating the graph and its complement.

2. Prior Results

Theorem 2.1:[14]

Let G be any graph and D be any dominating set of G. Then $|V\text{-}D| \leq \sum_{u \in V(D)} \deg(u)$ and equality holds in this relation if and only if D has the following properties :

(i) D is independent.

(ii) For every $u \in$ V-D, There exist a unique vertex $v \in$ D such that $N(u) \cap D = \{v\}$.

Theorem 2.2:[3]

For any tree T of order p $\geq$ 3, γ_c(S(T))= 2p-e-1, where e denotes the number of pendent vertices of T.

Theorem 2.3:[10]

Let be a geodetic graph, which is neither K_2 nor $K_1 \cup K_1$ such that $\overline{G}$ is also geodetic. Then G satisfies one of the following:

(i) G has diameter 3 and radius 2.

(ii) G is self-centered implies $G \cong C_5$.

3. Main Results

3.1. Weak convex dominating sets in Graphs.

Mulder[11] defined interval of a graph G as a subgraph S suchthat <S> includes all shortest paths of G connecting every pair of vertices in S.

Instead of all shortest paths joining every pair of vertices in S, if we consider the inclusion property of at least one shortest of every pair then that will induce a weaker set S of earlier definition of Mulder and hence we define the new concept of weak convex set with domination property as follows.

Definition 3.1 :

A dominating set D with $d_{<D>}(u,v) = d_G(u,v)$ for any two vertices *u*, *v* in D is called as a Weak Convex Dominating (W.C.D) set.

The cardinality of a minimum weak convex dominating set of G is called as a weak convex domination number of G and is denoted by γ_{wc}.

Most of the parameters so far defined on domination in graphs is a subclass of this weak convex domination, because the weak convex dominating set is a dominating and distance preserving set of a graph in which it is defined.

Observations :

3.1: Clearly from the definition, $1 \leq \gamma_{wc} \leq p$.

3.2: If G is geodetic, then for any spanning sub graph H, $\gamma_{wc}(G) \leq \gamma_{wc}(H)$.

3.3: For any tree T, $\gamma_{wc}(T)=\gamma_c(T)$= p-e, where e is the number of pendant vertices of T.

3.4: For any graph G $\gamma_{wc}(G^n) \leq \gamma_{wc}(G)$, where G^n is the n^{th} power of a graph G.

3.5: If the diameter of G is n, then $\gamma_{wc}(G^n) = 1$

3.6: Every weak convex dominating set contains a minimal dominating set.

3.7: Every weak convex dominating set of a connected graph contains a minimal connected dominating set.

Clearly, from observations 3.6 and 3.7 we have the relation that $\gamma \leq \gamma_t \leq \gamma_c \leq \gamma_{wc}$. The following Lemma is trivial from the definition of W.C.D set.

Lemma 3.1 :

A weak convex dominating set D is a minimal weak convex dominating set if and only if for each $d \in$ D, one of the following conditions hold:

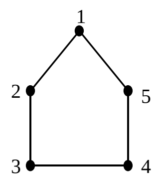
(i) there exists a vertex $c \in$ V-D such that N(c) $\cap$ D = {d}.

(ii) d must be in the geodesic of two vertices of D satisfying property (i).

(iii) d lies in the geodesic of any two vertices of D, which may satisfy any of the above two properties.

Can V-D be a weak convex dominating set if D is a weak convex dominating set? The answer is that it need not be.

Example 3.1:



Here, D={1, 2, 3} form a W.C.D set, but V-D={4, 5} is not a W.C.D set.

Proposition 3.1 :

Let D be any weak convex dominating set of G. Then $|\text{V-D}| \leq \sum_{u \in V(D)} \deg(u)$, for all u $\in$ D.

Proof:

Let D be any weak convex dominating set of G. Then clearly, D is a dominating set of G. Hence from Theorem 2.1, $|\text{V-D}| \leq \sum_{u \in V(D)} \deg(u)$.

Remark 3.2 :

Equality holds for any complete graph.

Proposition 3.2 :

$$|\text{V-D}| = \sum_{u \in V(D)} \deg(u) \Leftrightarrow |D| = 1.$$

Proof:

Since every weak convex dominating set is a dominating set, then the result follows from Theorem 2.1.

Corollary 3.1 :

If $|V\text{-}D| = \sum_{u \in V(D)} \deg(u) \Leftrightarrow$ there exists a vertex of degree p-1.

Lemma 3.2 :

Let G be a connected graph. Then $p\text{-}q \geq 0 \Leftrightarrow$ G is unicyclic or a tree.

Proof :

If G is unicyclic then p-q = 0. If G is a tree then p-q =1. Hence $p\text{-}q \geq 0$.

Assume that G is neither unicyclic nor a tree. That is G has at least two cycles. Let H be a spanning tree of G. Then q(H) = p-1. Since G has at least two cycles, $q(G) \geq q(H) + 2$ $\Rightarrow p\text{-}q(G) \leq p\text{-}q(H)\text{-}2 \Rightarrow p\text{-}q \leq -1$. This implies that G is either unicyclic or a tree.

Theorem 3.1 :

Let G be a connected graph. Then $\gamma_{wc}(G) = p\text{-}q \Leftrightarrow$ G is isomorphic to $k_{1,\,r}$.

Proof :

Let $\gamma_{wc} = p\text{-}q>0$. Then by lemma 3.2, G must be a tree, which implies that p-q =1 and hence $\gamma_{wc} = 1$. Therefore radius(G) = 1. Hence $G \cong K_{1,\,r}$.

Proposition 3.3 :

$[\, p/\Delta+1 \,] \leq \gamma_{wc}$.

Proof :

Let D be a minimum weak convex dominating set. From proposition 3.1, we have $|\, V\text{-}D \,| \leq \sum_{u \in V(D)} \deg(u) \leq |D|\, \Delta \Rightarrow |V| \leq |D|(\Delta+1) \Rightarrow [\, p/\Delta+1] \leq |D| = \gamma_{wc}$.

Proposition 3.4 :

Let G be a graph of order p. Then $k = \gamma_{wc}(G) = [p/\Delta+1]$ if and only if $\gamma_{wc}(G)=1$.

Proof :

Assume that $k = \gamma_{wc}(G) = [\, p/\Delta+1 \,]$. Let D be a weak convex dominating set such that $|D| = \gamma_{wc}$.

Claim: D is independent.

If not, let D be a non-independent set. Since D is itself a dominating set and not independent, $|\, V\text{-}D \,| \neq \sum_{u \in V(D)} \deg(u)$.

$$\text{(i.e.)} \quad |\, V\text{-}D \,| < \sum_{u \in V(D)} \deg(u) \leq k\Delta$$

$$\Rightarrow \quad k > p/(\Delta+1) \geq [\, p/\Delta+1 \,]$$

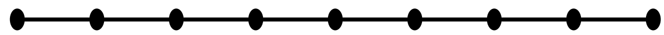
This is a contradiction to k = [p/Δ+1]. Therefore, D is independent. Since D is a weak convex dominating set, |D|= 1, that is $\gamma_{wc}(G)$ =1. Proof of converse is trivial.

Corollary 3.2 :

Let G be a graph of order p such that $\gamma_{wc}(G) = [p/(\Delta+1)]$. Then Δ+1 divides p.

The converse of the above corollary is not true, that is if Δ+1 divides p, then γ_{wc} need not be equal to $[p/(\Delta+1)]$.

Example 3.2:

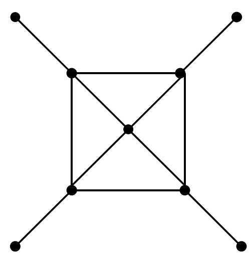


Here, γ_{wc} = 7, but $[p/(\Delta+1)] = 3$.

Remark 3.3:

One can construct a family of infinite number of graphs, which has weak convex dominating sets that have no vertex of G with eccentricity r (radius of G).

Example 3.3:



Theorem 3.2 :

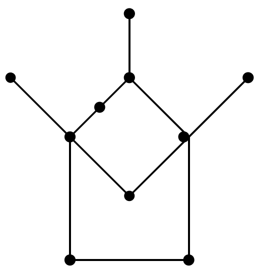
Let G be any distance hereditary graph. Then for any spanning sub graph H of G, $\gamma_{wc}(G) \leq \gamma_{wc}(H)$.

Remark 3.4 :

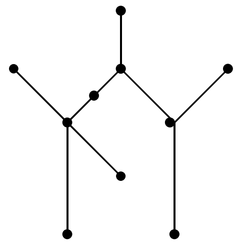
Usually this inequality is not true for any graph G. There are so many graphs, which have the other inequality.

Example 3.4:

G



H



Here $\gamma_{wc}(G) = 5$ and $\gamma_{wc}(H) = 4$

Following question generally arise when one tries to analyse the structural property of W.C.D. set of a graph.

Question:

If d and r are the diameter and radius of the graph G then what about the diameter and radius of the induced graph induced by the convex dominating set of G? The following two theorems give the answer for this question.

Theorem 3.3 :

Let G be a graph and D be a weak convex dominating set. Then the radius of the induced graph <D>, induced by D, is at least r-1, where r is the radius of G.

Proof :

Let *u* be a vertex in the weak convex dominating set D, whose eccentricity is reduced to r-2 in the induced graph <D>. Let *v* be an eccentric point of *u* in G. Then surely *v* does not belong to D (This is because $v \in G$ implies that the shortest path between *u* and *v* must occur in the induced graph and it is greater than are equal to r. This contradicts our assumption). Then v must be dominated by some vertex say, *w* in D. Clearly, $d(u, w) \leq$ r-2 in the induced graph <D>.

This implies that $d(u, w) + d(w, v) \leq$ r-1, that is $d(u, v) \leq$ r-1 in G. This contradicts the fact that the distance between *u* and *v* is greater than or equal to r. Hence, there is no point in the weak convex dominating set, which has eccentricity r-2 in the induced graph <D>.

Remark 3.5:

It is easy to verify that the radius of the induced graph <D> cannot be increased more than the diameter of the original graph.

Corollary 3.3:

Let d and r be the diameter and radius of the given graph G and let r_c be the radius of the induced graph induced by the weak convex dominating set of G. Then $r-1 \leq r_c \leq d$.

The following two theorems gives the picture of diameter of W.C.D. set of a graph.

Theorem 3.4:

There exists no graph, which has a weak convex dominating set that induces a sub graph of diameter less than are equal to d-3, where d is the diameter of the original graph.

Proof :

Let d be the diameter of the given graph. To get an induced sub graph of diameter d-3, we have to eliminate at least three edges from all diameteral paths. We can eliminate three edges from the diameteral path only by the following two ways :

(i) Two edges from one end and one edge from the other end.

or (ii) three consecutive edges from one end.

But in both the cases domination property is lost. Hence there exists no graph, which has a weak convex dominating set that induces a sub graph of diameter less than or equal to d-3.

Theorem 3.5 :

Let d, d_c be the diameters of the given graph G and the induced sub graph of the weak convex dominating set of G respectively. Then $d-2 \leq d_c \leq d$.

Proof :

Proof follows from the fact of Theorem 3.4.

Corollary 3.3:

Let G be a connected graph with diameter d. Then $d-1 \leq \gamma_{wc}(G)$.

Proof :

Proof follows from Theorem 3.5.

Following propositions are direct and hence we state without details of proofs.

Proposition 3.5 :

Let cut(G) denote the number of cut vertices of a connected graph G. Then $cut(G) \leq \gamma_{wc}(G)$.

Proposition 3.6 :

Let G be a self-centered graph of diameter 2. Then $\gamma_{wc}(G) \leq \delta+1$.

Proof:

Any δ degree vertex with its first neighborhood forms a W.C.D. set and hence the proposition.

Proposition 3.7 :

Let G be a graph with radius 2. Then $\gamma_{wc}(G) \leq \Delta+1$.

Proposition 3.8 :

If $\gamma_{wc}(\bar{G}) \geq 3$, then diam(G) ≤ 3.

Theorem 3.6 :

If a graph G is connected and diam(G) ≥ 3, then $\gamma_{wc}(\bar{G}) = 2$.

Theorem 3.7 :

If a graph G has $\delta \geq 2$ and girth g(G) ≥ 7, then $\gamma_{wc}(G) = p$.

Proof :

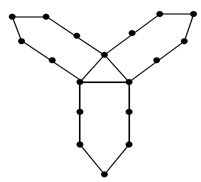
Let D be a γ_{wc} set of G. We know that $\gamma_{wc}(G) \leq p$. If $\gamma_{wc}(G) \neq p$, then $\gamma_{wc}(G) < p$. This implies that $|V\text{-}D| \geq 1$. Let $u \in V\text{-}D$. If two vertices of D dominate u, then as D is convex there must exists a C_3 or C_4 in G. Therefore, only one vertex of D dominates u. Since $\delta(G) \geq 2$, there must exists another vertex $v \in V\text{-}D$ such that u and v are adjacent. And also u and v are not be dominated by the same vertex of D (if possible, then C_3 arises). Therefore, u and v are dominated by two different vertices say some u' and v' of D respectively. This implies that $d(u', v') \leq 3$ (since length of the path u'uvv' = 3). This implies that there must be a C_4 or C_5 or C_6 exist in G, which is a contradiction to g(G) $\geq$ 7. Hence $\gamma_{wc}(G) = p$.

Remark 3.6:

Is converse of the above true?

Need not be. Consider the following :

Example 3.5:



For the above graph g(G) = 3 and $\gamma_{wc}(G) = p$.

Theorem 3.8:

For any connected graph G such that $\bar{G}$ also connected , $\gamma_{wc}(G) + \gamma_{wc}(\bar{G}) \leq p+2$, where $\gamma_{wc}(G)$ and $\gamma_{wc}(\bar{G})$ are the cardinality of minimum weak convex sets of G and $\bar{G}$ respectively.

Proof :

Case 1: For r = 1, d = 1 and r = 1, d = 2.

Clearly, there is no graph G such that G and $\bar{G}$ are connected.

Case 2 : For r = 2 , d = 2.

Consider a vertex *v*. Clearly, $\{v\} \cup N_1(v)$ forms a weak convex dominating set for G . We have the property that every vertex in $N_1(v)$ has at least one eccentric point in $N_2(v)$. That is every point in $N_1(v)$ is not adjacent to at least one point of $N_2(v)$. Also every point of $N_2(v)$ is not adjacent to v. Hence $\{v\} \cup N_2(v)$ forms a weak convex dominating set for $\bar{G}$. Hence, $\gamma_{wc}(G)+\gamma_{wc}(\bar{G}) \leq p+1$.

Case 3 : For $r \geq 2$, $d \geq 3$.

In G, there must exist at least two vertices with distance between them is greater than or equal to 3. Then that two vertices form a weak convex dominating set for $\bar{G}$. Hence, $\gamma_{wc}(G) + \gamma_{wc}(\bar{G}) \leq p+2$.

Remark 3.7:

The cycles of order greater than or equal to 7 attain this bound p+2.

Theorem 3.9:

For a given graph G,

$$\gamma_{wc}(G) \cdot \gamma_{wc}(\bar{G}) \leq p \quad ; \text{ if } r=1 \text{ or } \bar{r}=1$$
$$\leq (\delta+1)^2 \; ; \text{ if } d=\bar{d}=2$$
$$\leq 2p \quad ; \text{ if } d \text{ or } \bar{d} \geq 3,$$

where (r, d) and $(\bar{r}, \bar{d})$ denote the radius and diameter of G and $\bar{G}$ respectively.

Definition 3.2:

A graph is said to be k-weak convex dominating special graph, if it has exactly k-disjoint weak convex dominating sets.

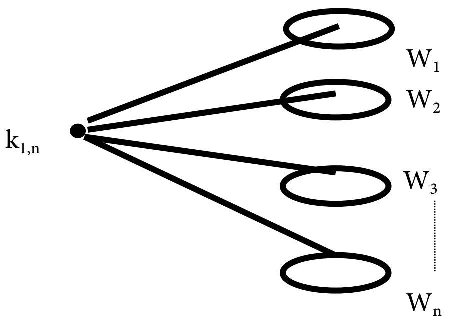
If the whole vertex set of G is the only W.C.D set of G, then G is said to be a 0-weak convex dominating graph.

Remark 3.8:

Whether all 0-weak convex dominating graphs are 2-connected ?

The answer is no, we could construct a family of graphs by attaching several 0-weak convex dominating graphs at pendent vertices of $K_{1,\,n}$ for all positive integer n.

Example 3.6 :



Separable 0-weak convex graphs.

Here W_1, W_2, …, W_n are 0-weak convex graphs.

Proposition 3.9 :

A graph G is 0-weak convex dominating graph, then the diameter d of G is greater than or equal to 3. The smallest graph is C_7.

Proof :

Follows from the fact that if G has diameter less than or equal to 2 then $\gamma_{wc} \leq \Delta+1$ and hence a contradiction to G is 0-weak convex dominating graph.

Proposition 3.10:

A graph G is 0-weak convex dominating graph, then $\bar{G}$ is not 0-weak convex dominating graph.

Proof:

As the graph has diameter greater than or equal to 3, $\bar{G}$ has a dominating edge and hence $\gamma_{wc}(\bar{G})=2$. Hence the proposition.

Following corollary is immediate from the previous two propositions.

Corollary 3.4 :

There is no graph G such that both G and $\bar{G}$ are 0-weak convex dominating graphs.

Theorem 3.10:

For any graph G , $\gamma_{wc}(G) + \gamma_{wc}(\bar{G}) = p+2 \Leftrightarrow$ one of the graphs G or $\bar{G}$ is a 0-weak convex dominating graph.

Proof :

Let G be any graph. Assume that $\gamma_{wc}(G) + \gamma_{wc}(\bar{G}) = p+2$. Then clearly, $\gamma_{wc}(G)$ and $\gamma_{wc}(\bar{G})$ are not less than are equal to 2 (from the fact of proof of theorem 3.8). Thus, either diam(G) or diam($\bar{G}$) is greater than or equal to 3. Without loss of generality assume that diam(G) is greater than or equal to 3. Then clearly, $\gamma_{wc}(\bar{G}) = 2$ and hence, $\gamma_{wc}(G)+\gamma_{wc}(\bar{G}) = p+2$ implies that $\gamma_{wc}(G) = p$, that is G is 0-weak convex dominating graph.

Conversely, assume that G or $\bar{G}$ is a 0-weak convex dominating graph. Without loss of generality, let G be a 0-weak convex dominating graph, that is $\gamma_{wc}(G) = p$. Then from the proposition 3.9, diam(G) $\geq$ 3. This implies that $\gamma_{wc}(\bar{G}) = 2$. Hence, $\gamma_{wc}(G) + \gamma_{wc}(\bar{G}) = p+2$.

The following proposition gives the relation between p, q and γ_{wc}.

Proposition3.11:

If G is a (p, q) - graph , then $q \geq \frac{1}{2}(p + \gamma_{wc})$.

Proof:

Clearly from the previous proposition, every pendent vertex of G belongs to V – D. This implies that deg(u) $\geq$ 2 for all u $\in$D.

Therefore, $2q = \sum_{u\in V(G)} \deg(u) = 2\,|\,D\,|+|\,V - D\,| = 2\gamma_{wc} + p - \gamma_{wc} = \gamma_{wc} + p$.

Hence, $q \geq \frac{1}{2}(p + \gamma_{wc})$.

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