

# On Multiplicative Sum connectivity index, Multiplicative Randic index and Multiplicative Harmonic index of some Nanostar Dendrimers

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**Abstract:** Topological indices are numbers associated with molecular graphs for the purpose of allowing quantitative structure- activity/property/toxicity relationships. These topological indices correlate certain Physico-Chemical properties like boiling point, stability, strain energy etc of chemical compounds. In this paper, we determine Multiplicative Sum connectivity, Multiplicative Randic and Multiplicative Harmonic index for some Nanostar Dendrimers

**Keywords:** Multiplicative Sum connectivity, Multiplicative Randic , Multiplicative Harmonic indices and Nanostar Dendrimers.

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## 1. Introduction

All graphs considered in this paper are finite, undirected and simple. For terminology and notation not defined here we follow those in Bondy and Murty [1]. In [2], Zhou and Trinajstić first introduced the sum-connectivity index in 2008, and it is found that the sum-connectivity index and the product connectivity index correlate well among themselves and with the  $\pi$ -electronic energy of Benzenoid hydro carbons [3]. It is defined as

$$X(G) = \sum_{e=uv \in E(G)} \frac{1}{\sqrt{d_u + d_v}} \quad (1)$$

One of the best known and widely used Topological index is the product connectivity index or Randić index introduced by Randić in [4]. The product connectivity index of a graph G is defined as

$$R(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}} \quad (2)$$

The Harmonic index gives somewhat better correlations with Physical and Chemical properties comparing with the well known Randić index. The harmonic index H(G) of a graph G was first appeared in [5] and it is defined as

$$H(G) = \sum_{uv \in E(G)} \frac{2}{d_u + d_v} \quad (3)$$

where  $d_u$  denotes the degree of the vertex u and the summation is taken over all pairs of adjacent vertices of the graph G.

Motivated by the definition of the Product connectivity index, the multiplicative Sum connectivity index and multiplicative Product connectivity index was proposed by Kulli in [6]. They are defined as follows:

The multiplicative sum connectivity index of a graph  $G$  is defined as

$$X\pi(G) = \prod_{e=uv \in E(G)} \frac{1}{\sqrt{d_u+d_v}} \quad (4)$$

The multiplicative product connectivity index of a graph  $G$  is defined as

$$R\pi(G) = \prod_{e=uv \in E(G)} \frac{1}{\sqrt{d_u d_v}} \quad (5)$$

Now we define, the multiplicative Harmonic index  $H\pi(G)$  of a graph  $G$  and is defined as

$$H\pi(G) = \prod_{e=uv \in E(G)} \frac{2}{d_u+d_v} \quad (6)$$

## 2. Nanostar Dendrimers

Nano biotechnology is a rapidly advancing area of scientific and technological opportunity that applies the tools and processes of nano fabrication build devices for studying biosystems. Dendrimers are one of the main objects of this new area of science. A dendrimer is an artificially manufactured or synthesized molecule built up from branched units called monomers using a nano scale fabrication process. Dendrimers are recognized as one of the major commercially available nano scale building blocks, large and complex molecules with very well defined chemical structure. From a polymer chemistry point of view, dendrimers are nearly perfect mono disperse macromolecules with are regular and highly branched three dimensional architecture. They consist of three major architectural components: core, branches and end groups. The Nanostar dendrimer is a part of a new group of macro particles that appear to be photon funnels just like artificial antennas. These macro molecules and more precisely those containing phosphorus are used in the formation of nano tubes, micro and macro capsules, nanola-tex, colored glasses, chemical sensors, modified electrodes and soon[8,9]. A  $k$ -polyomino system is a finite 2-connected plane graph such that each interior face (also called cell) is surrounded by a regular  $4k$ -cycle of length one. In other words, it is an edge-connected union of cells[10].

Dendrimer is a synthetic 3-dimensional macromolecule that is prepared in a step-wise fashion from simple branched monomer units see[11]. The Nanostar dendrimer is a part of a new group of macro molecules that appear to photon funnels just like artificial antennas.

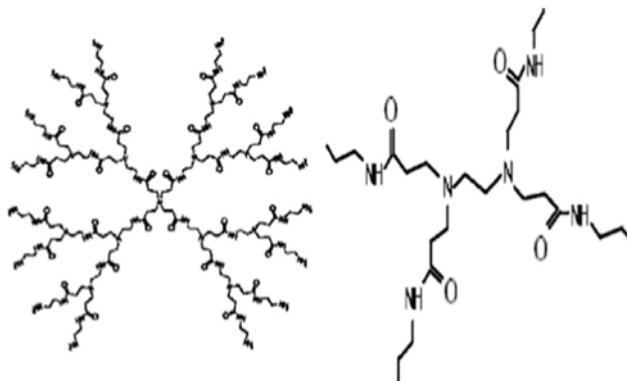


Figure.1  $NS_1[1]$  and  $NS_1[n]$  poly propyleneimine octaamin dendrimer.

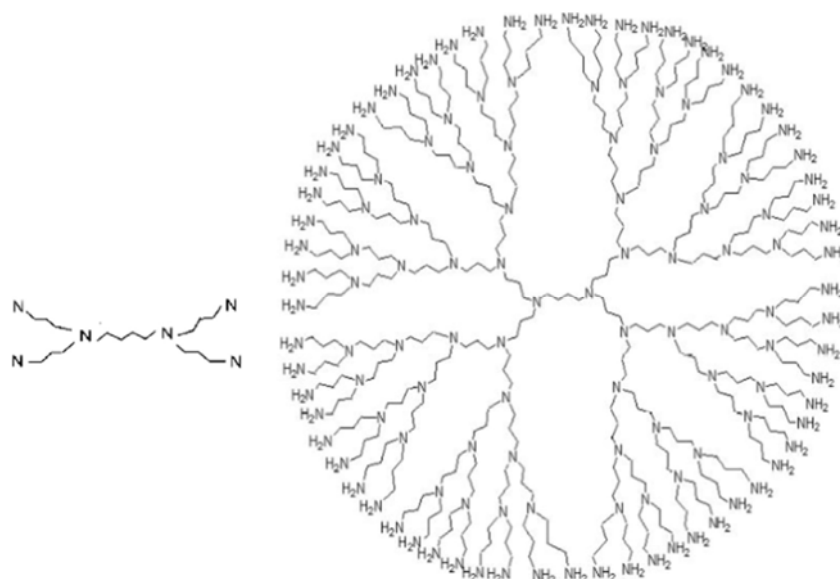


Figure.2  $NS_2[1]$  and  $NS_2[n]$  Polypropyleneimine octaamin dendrimer

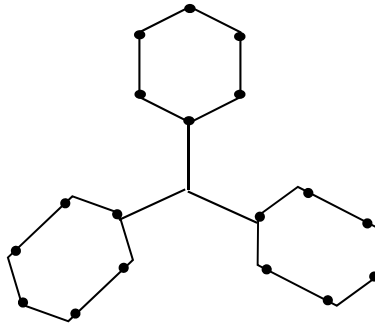


Figure.3 The nanostar dendrimer  $D_n$  for  $n = 1$ .

### 3. Main results and discussion

The Multiplicative Sum connectivity ( $X\pi(G)$ ) index, Multiplicative Randić ( $R\pi(G)$ ) index and Multiplicative Harmonic index ( $H\pi(G)$ ) of some special graphs:

**Lemma 3.1:** Consider the complete graph  $K_n$  of order  $n$ .

- (i) The multiplicative Sum connectivity Index

$$X\pi(K_n) = \left[ \frac{1}{\sqrt{2(n-1)}} \right]^{\frac{n(n-1)}{2}} \quad (7)$$

- (ii) The multiplicative Randić Index

$$R\pi(K_n) = \left[ \frac{1}{(n-1)} \right]^{\frac{n(n-1)}{2}} \quad (8)$$

- (iii) The multiplicative Harmonic Index of

$$H\pi(K_n) = \left[ \frac{1}{n-1} \right]^{\frac{n(n-1)}{2}} \quad (9)$$

**Proof.** The degree of all the vertices of a complete graph  $K_n$  of order  $n$  is  $n - 1$  and the number of edges for  $K_n$  is equal to  $\frac{1}{2} n(n - 1)$  i.e.,  $|E(K_n)| = \frac{1}{2} n(n - 1)$ . Then

$$\begin{aligned} (i) X\pi(K_n) &= \prod_{uv \in E(K_n)} \frac{1}{\sqrt{d_u + d_v}} \\ &= \left[ \frac{1}{\sqrt{2(n-1)}} \right]^{\frac{n(n-1)}{2}} = \left[ \frac{1}{\sqrt{2(n-1)}} \right]^{\frac{n(n-1)}{2}} \\ (ii) R\pi(K_n) &= \prod_{uv \in E(K_n)} \frac{1}{\sqrt{d_u d_v}} \\ &= \left[ \frac{1}{\sqrt{(n-1)^2}} \right]^{\frac{n(n-1)}{2}} = \left[ \frac{1}{(n-1)} \right]^{\frac{n(n-1)}{2}} \\ (iii) H\pi(K_n) &= \prod_{uv \in E(K_n)} \frac{2}{d_u + d_v} \\ &= \left[ \frac{2}{2(n-1)} \right]^{\frac{n(n-1)}{2}} = \left[ \frac{1}{n-1} \right]^{\frac{n(n-1)}{2}} \end{aligned}$$

**Lemma 3.2:**

Suppose  $C_n$  is a cycle of length  $n$  labeled by  $1, 2, \dots, n$ . Then

- (i) The multiplicative Sum connectivity Index of this cycle is

$$X\pi(C_n) = \left[\frac{1}{2}\right]^n \quad (10)$$

- (ii) The multiplicative Randić Index

$$R\pi(C_n) = \left[\frac{1}{2}\right]^n \quad (11)$$

- (iii) The multiplicative Harmonic Index of

$$H\pi(C_n) = \left[\frac{1}{2}\right]^n \quad (12)$$

**Proof.** Here  $|V(C_n)| = n = |E(C_n)|$  and all the vertices have the degrees 2. Hence

$$\begin{aligned} (i) X\pi(C_n) &= \prod_{uv \in E(C_n)} \frac{1}{\sqrt{d_u + d_v}} \\ &= \left[\frac{1}{\sqrt{2+2}}\right]^n = \left[\frac{1}{2}\right]^n \end{aligned}$$

$$\begin{aligned} (ii) R\pi(C_n) &= \prod_{uv \in E(C_n)} \frac{1}{\sqrt{d_u d_v}} \\ &= \left[\frac{1}{\sqrt{2 \cdot 2}}\right]^n = \left[\frac{1}{2}\right]^n \end{aligned}$$

$$\begin{aligned} (iii) H\pi(C_n) &= \prod_{uv \in E(C_n)} \frac{2}{d_u + d_v} \\ &= \left[\frac{2}{2+2}\right]^n = \left[\frac{1}{2}\right]^n \end{aligned}$$

**Lemma 3.1.3:**

Suppose  $S_n$  is the Star graph on  $n$  vertices,

- (i) The multiplicative Sum connectivity Index of Star graph is

$$X\pi(S_n) = \left[\frac{1}{\sqrt{n}}\right]^{n-1} \quad (13)$$

- (ii) The multiplicative Randić Index

$$R\pi(S_n) = \left[\frac{1}{\sqrt{(n-1)}}\right]^{n-1} \quad (14)$$

- (iii) The multiplicative Harmonic Index of

$$H\pi(S_n) = \left[\frac{2}{n}\right]^{n-1} \quad (15)$$

**Proof :**

In a star  $S_n$  all the leaves are of degree 1 and the degree of internal node is  $n - 1$ . Also the number of edges for  $S_n$  is  $n - 1$  i.e.,  $|E(S_n)| = (n - 1)$ . Then

$$\begin{aligned} (i) X\pi(S_n) &= \prod_{uv \in E(S_n)} \frac{1}{\sqrt{d_u + d_v}} \\ &= \left[\frac{1}{\sqrt{1+(n-1)}}\right]^{n-1} = \left[\frac{1}{\sqrt{n}}\right]^{n-1} \end{aligned}$$

$$\begin{aligned} (ii) R\pi(S_n) &= \prod_{uv \in E(S_n)} \frac{1}{\sqrt{d_u d_v}} \\ &= \left[\frac{1}{\sqrt{1 \times (n-1)}}\right]^{n-1} = \left[\frac{1}{\sqrt{(n-1)}}\right]^{n-1} \end{aligned}$$

$$\begin{aligned} (iii) H\pi(S_n) &= \prod_{uv \in E(S_n)} \frac{2}{d_u + d_v} \\ &= \left[ \frac{2}{1+(n-1)} \right]^{n-1} = \left[ \frac{2}{n} \right]^{n-1} \end{aligned}$$

**Lemma 3.1.4:**

If  $W_n, n \geq 5$  is the wheel graph on  $n$  vertices, then

- (i) The multiplicative Sum connectivity Index is

$$X\pi(W_n) = \left[ \frac{1}{\sqrt{6(n+2)}} \right]^{(n-1)} \quad (16)$$

- (ii) The multiplicative Randić Index

$$R\pi(W_n) = \left[ \frac{1}{3\sqrt{3(n-1)}} \right]^{(n-1)} \quad (17)$$

- (iii) The multiplicative Harmonic Index of

$$H\pi(W_n) = \left[ \frac{2}{3(n+2)} \right]^{(n-1)} \quad (18)$$

**Proof:** The wheel graph  $W_n$  may denote the  $n$ -vertex graph with an  $(n-1)$ -cycle on its rim. For a wheel graph  $W_n, n \geq 5$ , it has  $|V(W_n)| = n$  and  $|E(W_n)| = 2(n-1)$ .

Also the degree of end vertices of  $(n-1)$  edges of a wheel graph  $W_n$  is given by  $[3, (n-1)]$  and the end vertices of the remaining  $(n-1)$  edges has the degree as  $[3, 3]$ . Then

$$\begin{aligned} (i) X\pi(W_n) &= \prod_{uv \in E(W_n)} \frac{1}{\sqrt{d_u + d_v}} \\ &= \left[ \frac{1}{\sqrt{3+(n-1)}} \right]^{(n-1)} \times \left[ \frac{1}{\sqrt{3+3}} \right]^{(n-1)} = \left[ \frac{1}{\sqrt{6(n+2)}} \right]^{(n-1)} \end{aligned}$$

$$\begin{aligned} (ii) R\pi(W_n) &= \prod_{uv \in E(W_n)} \frac{1}{\sqrt{d_u d_v}} \\ &= \left[ \frac{1}{\sqrt{3 \times (n-1)}} \right]^{(n-1)} \times \left[ \frac{1}{\sqrt{3 \times 3}} \right]^{(n-1)} = \left[ \frac{1}{3\sqrt{3(n-1)}} \right]^{(n-1)} \end{aligned}$$

$$\begin{aligned} (iii) H\pi(W_n) &= \prod_{uv \in E(W_n)} \frac{2}{d_u + d_v} \\ &= \left[ \frac{2}{3+(n-1)} \right]^{(n-1)} \times \left[ \frac{2}{3+3} \right]^{(n-1)} = \\ &= \left[ \frac{2}{n+2} \times \frac{1}{3} \right]^{(n-1)} = \left[ \frac{2}{3(n+2)} \right]^{(n-1)} \end{aligned}$$

**Lemma 3.1.5:**

If  $F_n, n \geq 2$  is the fan graph on  $n$  vertices, then

(i) The multiplicative Sum connectivity Index is

$$X\pi(F_n) = \frac{1}{5} \left( \frac{1}{2+n} \right) \left[ \frac{1}{\sqrt{3+n}} \right]^{n-2} \left[ \frac{1}{\sqrt{3+3}} \right]^{n-3} \quad (19)$$

(ii) The multiplicative Randić Index

$$R\pi(F_n) = \frac{1}{6} \left( \frac{1}{2n} \right) \left[ \frac{1}{\sqrt{3n}} \right]^{n-2} \left[ \frac{1}{3} \right]^{n-3} \quad (20)$$

(iii) The multiplicative Harmonic Index of

$$H\pi(F_n) = \frac{4}{25} \left( \frac{2}{(2+n)^2} \right) \left[ \frac{2}{3+n} \right]^{n-2} \times \left[ \frac{1}{3} \right]^{n-3} \quad (21)$$

**Proof:** For a fan graph  $F_n, n \geq 2$ ,  $|V(F_n)| = n + 1$  and  $|E(F_n)| = 2n - 1$ . Also in  $F_n$  the degrees of end vertices of  $n - 2$  edges are  $[3, n]$ , 2 edges have the degrees of the end vertices as  $[3, 2]$ , 2 edges have the degrees of the end vertices as  $[n, 2]$  and the end vertices of the remaining  $n - 3$  edges has the degree  $[3, 3]$ . Then

$$(i) X\pi(F_n) = \prod_{uv \in E(F_n)} \frac{1}{\sqrt{d_u + d_v}}$$

$$= \left[ \frac{1}{\sqrt{2+3}} \right]^2 \times \left[ \frac{1}{\sqrt{3+n}} \right]^{n-2} \times \left[ \frac{1}{\sqrt{2+n}} \right]^2 \times \left[ \frac{1}{\sqrt{3+3}} \right]^{n-3}$$

$$= \frac{1}{5} \left( \frac{1}{2+n} \right) \left[ \frac{1}{\sqrt{3+n}} \right]^{n-2} \left[ \frac{1}{\sqrt{3+3}} \right]^{n-3}$$

$$(ii) R\pi(F_n) = \prod_{uv \in E(F_n)} \frac{1}{\sqrt{d_u d_v}}$$

$$= \left[ \frac{1}{\sqrt{2 \times 3}} \right]^2 \times \left[ \frac{1}{\sqrt{3 \times n}} \right]^{n-2} \times \left[ \frac{1}{\sqrt{2 \times n}} \right]^2 \times \left[ \frac{1}{\sqrt{3 \times 3}} \right]^{n-3}$$

$$= \frac{1}{6} \left( \frac{1}{2n} \right) \left[ \frac{1}{\sqrt{3n}} \right]^{n-2} \left[ \frac{1}{3} \right]^{n-3}$$

$$(iii) H\pi(F_n) = \prod_{uv \in E(F_n)} \frac{2}{d_u + d_v}$$

$$= \left[ \frac{2}{2+3} \right]^2 \times \left[ \frac{2}{3+n} \right]^{n-2} \times \left[ \frac{2}{2+n} \right]^2 \times \left[ \frac{2}{3+3} \right]^{n-3}$$

$$= \frac{4}{25} \left( \frac{2}{(2+n)^2} \right) \left[ \frac{2}{3+n} \right]^{n-2} \times \left[ \frac{1}{3} \right]^{n-3}$$

**Lemma 3.1.6:**

If  $K_{m,n}, n, m \geq 2$  is the Complete bipartite graph, then

(i) The multiplicative Sum connectivity Index is

$$X\pi(K_{m,n}) = \left[ \frac{1}{\sqrt{n+m}} \right]^{mn} \quad (22)$$

(ii) The multiplicative Randić Index

$$R\pi(K_{m,n}) = \left[ \frac{1}{\sqrt{nm}} \right]^{mn} \quad (23)$$

(iii) The multiplicative Harmonic Index of

$$H\pi(K_{m,n}) = \left[ \frac{2}{n+m} \right]^{mn} \quad (24)$$

**Proof:** For a Complete bipartite graph  $K_{m,n}$ ,  $n, m \geq 2$ , it has  $|V(K_{m,n})| = m + n$  and  $|E(K_{m,n})| = mn$ . Also the degrees of end vertices of all the edges of a wheel graph  $K_{m,n}$  is given by  $[n, m]$ . Then

$$(i) X\pi(K_{m,n}) = \prod_{uv \in E(K_{m,n})} \frac{1}{\sqrt{d_u + d_v}} \\ = \left[ \frac{1}{\sqrt{n+m}} \right]^{mn}$$

$$(ii) R\pi(K_{m,n}) = \prod_{uv \in E(K_{m,n})} \frac{1}{\sqrt{d_u d_v}} \\ = \left[ \frac{1}{\sqrt{nm}} \right]^{mn}$$

$$(iii) H\pi(K_{m,n}) = \prod_{uv \in E(K_{m,n})} \frac{2}{d_u + d_v} \\ = \left[ \frac{2}{n+m} \right]^{mn}$$

**Lemma 3.1.7:**

If  $P_n, n \geq 2$  is the Path on  $n$  vertices, then

The multiplicative Sum connectivity Index is

$$X\pi(P_n) = \frac{1}{3} \left[ \frac{1}{2} \right]^{n-3} \quad (25)$$

(i) The multiplicative Randić Index

$$R\pi(P_n) = \left[ \frac{1}{2} \right]^{n-2} \quad (26)$$

(ii) The multiplicative Harmonic Index of

$$H\pi(P_n) = \frac{4}{9} \left[ \frac{1}{2} \right]^{n-3} \quad (27)$$

**Proof:** A path graph  $P_n$  has vertices  $v_1, v_2, v_3, \dots, v_n$  and edges  $e_1, e_2, e_3, \dots, e_{n-1}$ , such that edge  $e_k$  joins vertices  $v_k$  and  $v_{k+1}$ . For a Path graph  $P_n, n \geq 2$ , it has  $|V(P_n)| = n$  and  $|E(P_n)| = n - 1$ .

Also in  $P_n$  the degrees of end vertices of 2 edges are  $[2,1]$  and the end vertices of the remaining  $n - 3$  edges has the degree  $[2,2]$ . Then

$$(i) X\pi(P_n) = \prod_{uv \in E(P_n)} \frac{1}{\sqrt{d_u + d_v}} \\ = \left[ \frac{1}{\sqrt{2+1}} \right]^2 \left[ \frac{1}{\sqrt{2+2}} \right]^{n-3} = \frac{1}{3} \left[ \frac{1}{2} \right]^{n-3}$$

$$(ii) R\pi(P_n) = \prod_{uv \in E(P_n)} \frac{1}{\sqrt{d_u d_v}} \\ = \left[ \frac{1}{\sqrt{2 \cdot 1}} \right]^2 \left[ \frac{1}{\sqrt{2 \cdot 2}} \right]^{n-3} = \left[ \frac{1}{2} \right]^{n-2}$$

$$(iii) H\pi(P_n) = \prod_{uv \in E(P_n)} \frac{2}{d_u + d_v} \\ = \left[ \frac{2}{2+1} \right]^2 \left[ \frac{2}{2+2} \right]^{n-3} = \frac{4}{9} \left[ \frac{1}{2} \right]^{n-3}$$



**Lemma 3.1.8:**

If  $G$  is a regular graph of degree  $r > 0$ , then

- (i) The multiplicative Sum connectivity Index is

$$X\pi(G) = \left[ \frac{1}{\sqrt{2r}} \right]^{\frac{nr}{2}} \quad (25)$$

- (ii) The multiplicative Randić Index

$$R\pi(G) = \left[ \frac{1}{r} \right]^{\frac{nr}{2}} \quad (26)$$

- (iii) The multiplicative Harmonic Index of

$$H\pi(G) = \left[ \frac{1}{r} \right]^{\frac{nr}{2}} \quad (27)$$

**Proof.** A regular graph  $G$  on  $n$  vertices, having degree  $r$ , possesses  $\frac{nr}{2}$  edges. Thus

$$\begin{aligned} (i) X\pi(G) &= \prod_{uv \in E(G)} \frac{1}{\sqrt{d_u + d_v}} \\ &= \left[ \frac{1}{\sqrt{r+r}} \right]^{\frac{nr}{2}} = \left[ \frac{1}{\sqrt{2r}} \right]^{\frac{nr}{2}}. \end{aligned}$$

$$\begin{aligned} (ii) R\pi(G) &= \prod_{uv \in E(G)} \frac{1}{\sqrt{d_u d_v}} \\ &= \left[ \frac{1}{\sqrt{r \cdot r}} \right]^{\frac{nr}{2}} = \left[ \frac{1}{r} \right]^{\frac{nr}{2}} \end{aligned}$$

$$\begin{aligned} (iii) H\pi(G) &= \prod_{uv \in E(G)} \frac{2}{d_u + d_v} \\ &= \left[ \frac{2}{r+r} \right]^{\frac{nr}{2}} = \left[ \frac{1}{r} \right]^{\frac{nr}{2}} \end{aligned}$$

#### 4. On $X\pi$ , $R\pi$ , and $H\pi$ of Nanostar Dendrimers

Consider a graph  $G$  on  $n$  vertices, where  $n \geq 2$ . The maximum possible vertex degree in such a graph is  $n - 1$ . Suppose  $d_{ij}$  denotes the number of edges of  $G$  connecting vertices of degrees  $i$  and  $j$ . Clearly,  $d_{ij} = d_{ji}$ . We now consider two infinite classes  $NS_1[n]$  and  $NS_2[n]$  of Nano star dendrimers as in Figure.1 and Figure.2. In this paper our aim is to compute the Multiplicative Sum connectivity  $X\pi(G)$  index, Multiplicative Randić ( $R\pi(G)$ ) index and multiplicative Harmonic index ( $H\pi(G)$ ) of for two classes of these Nanostar dendrimers Figure.3.

We consider the molecular graph of  $NS_1[n]$  with four similar branches and three extra edges, where  $n$  denotes the steps of growth as shown in the Figure.1.

Define  $d_{23}$  to be the number of edges connecting a vertex of degree 2 with a vertex of degree 3,  $d_{13}$  to be the number of edges connecting a vertex of degree 1 with a vertex of degree 3,  $d_{22}$  to be the number of edges connecting two vertices of degree 2 and  $d_{12}$  to be the number of edges connecting a vertex of degree 1 with a vertex of degree 2. Also

$d'_{ij}$  denotes the number of edges connecting vertices of degrees  $i$  and  $j$  in each branch ( $i, j \leq 4$ ). It is obvious that  $d_{12} = 4d'_{12}, d_{22} = 4d'_{22} + 1, d_{13} = 4d'_{13}, d_{23} = 4d'_{23} + 2$ . On the other hand a simple calculation shows that  $d'_{12} = 2^{n-1}$ . Therefore  $d_{12} = 4d'_{12} = 2^n$ . Using a similar argument, one can see that  $d'_{22} = 3n - 3$  then  $d_{22} = 12 \times 2^{n-1} - 11, d'_{13} = 2^n - 1$ , then  $d_{13} = 4d'_{13} = 4 \times 2^n - 4$ , and finally  $d'_{23} = 3(2^n - 1) + (2^{n-1} - 1) = 3(2n-1) + (2n-1-1)$ , then  $d_{23} = 4d'_{23} + 2 = 14 \times 2^n - 14$ .

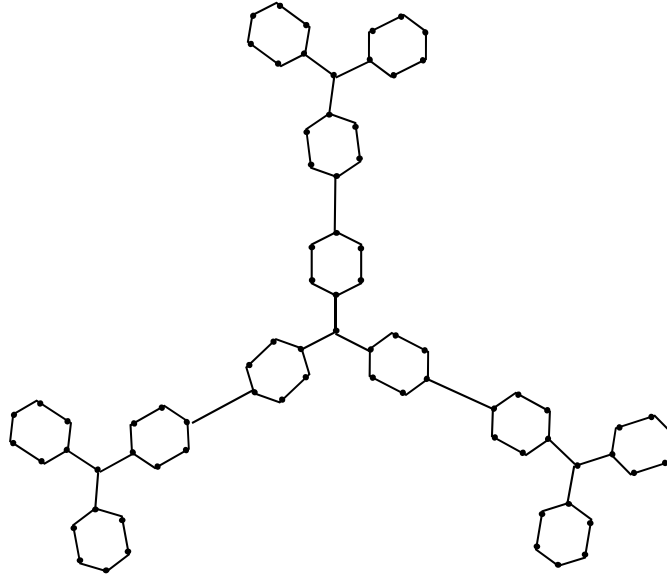


Figure.4 The nanostar dendrimer  $D_n$  for  $n = 2$ .

We compute the Multiplicative Sum connectivity ( $X\pi(G)$ ) index, Multiplicative Randić ( $R\pi(G)$ ) index and Multiplicative Harmonic index ( $H\pi(G)$ ) for  $NS_1[n]$  in the following theorem.

**Theorem 4.1**

Let  $NS_1[n]$  be a nanostar dendrimer. Then

$$(i) X\pi(NS_1[n]) = \left[\frac{1}{\sqrt{3}}\right]^{2^{n+1}} \times \left[\frac{1}{2}\right]^{4(2^n-1)} \times \left[\frac{1}{2}\right]^{12 \cdot 2^n - 11} \times \left[\frac{1}{\sqrt{5}}\right]^{14 \cdot 2^n - 14} \quad (28)$$

$$(ii) R\pi(NS_1[n]) = \left[\frac{1}{\sqrt{2}}\right]^{2^{n+1}} \times \left[\frac{1}{\sqrt{3}}\right]^{4(2^n-1)} \times \left[\frac{1}{2}\right]^{12 \cdot 2^n - 11} \times \left[\frac{1}{\sqrt{6}}\right]^{14 \cdot 2^n - 14} \quad (29)$$

$$(iii) H\pi(NS_1[n]) = \left[\frac{2}{\sqrt{3}}\right]^{2^{n+1}} \times \left[\frac{1}{2}\right]^{4(2^n-1)} \times \left[\frac{1}{2}\right]^{12 \cdot 2^n - 11} \times \left[\frac{2}{\sqrt{5}}\right]^{14 \cdot 2^n - 14} \quad (30)$$

**Proof:**

Let  $G$  be a  $NS_1[n]$  nanostar dendrimer. The edge set  $E(NS_1[n])$  divided into four edge partitions based on degrees of end vertices. The first edge partition  $E_1(NS_1[n])$  contains  $2^{n+1}$  edges  $uv$ , where  $deg(u) = 1, deg(v) = 2$ . The second edge partition  $E_2(NS_1[n])$  contains  $4(2^n - 1)$  edges  $uv$ , where  $deg(u) = 1, deg(v) = 3$ . The third edge partition  $E_3(NS_1[n])$  contains  $12 \cdot 2^n - 11$  edges  $uv$ , where  $deg(u) = deg(v) = 2$ . The fourth edge partition  $E_4(NS_1[n])$  contains  $14 \cdot 2^n - 14$  edges  $uv$ , where  $deg(u) = 2, deg(v) = 3$ . It is easy to see that  $|E_1(NS_1[n])| = d_{12}, |E_2(NS_1[n])| = d_{13}, |E_3(NS_1[n])| = d_{22}, |E_4(NS_1[n])| = d_{23}$ . Now using Eqs.(4)–(6), we have

$$\begin{aligned} (i) X\pi(NS_1[n]) &= \prod_{uv \in E(NS_1[n])} \frac{1}{\sqrt{d_u+d_v}} \\ &= \prod_{uv \in E_1(NS_1[n])} \frac{1}{\sqrt{d_u+d_v}} \times \prod_{uv \in E_2(NS_1[n])} \frac{1}{\sqrt{d_u+d_v}} \times \\ &\quad \prod_{uv \in E_3(NS_1[n])} \frac{1}{\sqrt{d_u+d_v}} \times \prod_{uv \in E_4(NS_1[n])} \frac{1}{\sqrt{d_u+d_v}} \\ &= |E_1(NS_1[n])|^{2^{n+1}} \times |E_2(NS_1[n])|^{4(2^n-1)} \times \\ &\quad |E_3(NS_1[n])|^{12 \cdot 2^n - 11} \times |E_4(NS_1[n])|^{14 \cdot 2^n - 14} \\ &\quad \times \left[\frac{1}{\sqrt{1+3}}\right]^{4(2^n-1)} \times \left[\frac{1}{\sqrt{2+2}}\right]^{12 \cdot 2^n - 11} \times \left[\frac{1}{\sqrt{3+2}}\right]^{14 \cdot 2^n - 14} \\ &= \left[\frac{1}{\sqrt{3}}\right]^{2^{n+1}} \times \left[\frac{1}{2}\right]^{4(2^n-1)} \times \left[\frac{1}{2}\right]^{12 \cdot 2^n - 11} \times \\ &\quad \left[\frac{1}{\sqrt{5}}\right]^{14 \cdot 2^n - 14} \end{aligned}$$

$$\begin{aligned} (ii) R\pi(NS_1[n]) &= \prod_{uv \in E(NS_1[n])} \frac{1}{d_u d_v} \\ &= \prod_{uv \in E_1(NS_1[n])} \frac{1}{d_u d_v} \times \prod_{uv \in E_2(NS_1[n])} \frac{1}{d_u d_v} \times \\ &\quad \prod_{uv \in E_3(NS_1[n])} \frac{1}{d_u d_v} \times \prod_{uv \in E_4(NS_1[n])} \frac{1}{d_u d_v} \\ &= |E_1(NS_1[n])|^{2^{n+1}} \times |E_2(NS_1[n])|^{4(2^n-1)} \times \\ &\quad |E_3(NS_1[n])|^{12 \cdot 2^n - 11} \times |E_4(NS_1[n])|^{14 \cdot 2^n - 14} \\ &= \left[\frac{1}{\sqrt{1.2}}\right]^{2^{n+1}} \times \left[\frac{1}{\sqrt{1.3}}\right]^{4(2^n-1)} \times \left[\frac{1}{\sqrt{2.2}}\right]^{12 \cdot 2^n - 11} \times \\ &\quad \left[\frac{1}{\sqrt{3.2}}\right]^{14 \cdot 2^n - 14} \\ &= \left[\frac{1}{\sqrt{2}}\right]^{2^{n+1}} \times \left[\frac{1}{\sqrt{3}}\right]^{4(2^n-1)} \times \left[\frac{1}{2}\right]^{12 \cdot 2^n - 11} \times \\ &\quad \left[\frac{1}{\sqrt{6}}\right]^{14 \cdot 2^n - 14} \end{aligned}$$

$$(iii) H\pi(NS_1[n]) = \prod_{uv \in E(NS_1[n])} \frac{2}{d_u+d_v}$$

$$\begin{aligned}
&= \prod_{uv \in E_1(NS_1[n])} \frac{2}{d_u+d_v} \times \prod_{uv \in E_2(NS_1[n])} \frac{2}{d_u+d_v} \times \\
&\quad \prod_{uv \in E_3(NS_1[n])} \frac{2}{d_u+d_v} \times \prod_{uv \in E_4(NS_1[n])} \frac{2}{d_u+d_v} \\
&= |E_1(NS_1[n])|^{2^{n+1}} \times |E_2(NS_1[n])|^{4(2^n-1)} \times \\
&\quad |E_3(NS_1[n])|^{12 \cdot 2^n - 11} \times |E_4(NS_1[n])|^{14 \cdot 2^n - 14} \\
&= \left[ \frac{2}{\sqrt{1+2}} \right]^{2^{n+1}} \times \left[ \frac{2}{\sqrt{1+3}} \right]^{4(2^n-1)} \times \\
&\quad \left[ \frac{2}{\sqrt{2+2}} \right]^{12 \cdot 2^n - 11} \times \left[ \frac{2}{\sqrt{3+2}} \right]^{14 \cdot 2^n - 14} \\
&= \left[ \frac{2}{\sqrt{3}} \right]^{2^{n+1}} \times \left[ \frac{1}{2} \right]^{4(2^n-1)} \times \left[ \frac{1}{2} \right]^{12 \cdot 2^n - 11} \times \\
&\quad \left[ \frac{2}{\sqrt{5}} \right]^{14 \cdot 2^n - 14}
\end{aligned}$$

We consider the second class of Nanostar dendrimers  $NS_2[n]$ , where  $n$  is steps of growth. Since the molecular graph of  $G$  has four similar branches and five extra edges (see Figure.2), in which we have that  $d_{12} = 4d'_{12}$ ,  $d_{22} = 4d'_{22} + 3$ , and  $d_{23} = 4d'_{23} + 2$ . By a routine calculation we have  $d'_{12} = 2^{n-1}$ ,  $d'_{22} = 2(2^n - 1)$ ,  $d'_{23} = 3 \cdot 2^{n-1} - 2$ . It is easy to compute  $d_{12} = 2^{n+1}$ ,  $d_{22} = 8 \cdot 2^n - 5$  and  $d_{23} = 6 \cdot 2^n - 6$ . Now we compute the multiplicative Sum connectivity ( $X\pi(G)$ ) index, Multiplicative Randić ( $R\pi(G)$ ) index and Multiplicative Harmonic index ( $H\pi(G)$ ) for  $NS_2[n]$  in the following theorem.

**Theorem 4.2**

Let  $NS_2[n]$  be a nanostar dendrimer. Then

$$(i) X\pi(NS_2[n]) = \left[ \frac{1}{\sqrt{3}} \right]^{2^{n+1}} \times \left[ \frac{1}{2} \right]^{8 \cdot 2^n - 5} \times \left[ \frac{1}{\sqrt{5}} \right]^{6 \cdot 2^n - 6} \quad (31)$$

$$(ii) R\pi(NS_2[n]) = \left[ \frac{1}{\sqrt{2}} \right]^{2^{n+1}} \times \left[ \frac{1}{2} \right]^{8 \cdot 2^n - 5} \times \left[ \frac{1}{\sqrt{6}} \right]^{6 \cdot 2^n - 6} \quad (32)$$

$$(iii) H\pi(NS_2[n]) = \left[ \frac{2}{3} \right]^{2^{n+1}} \times \left[ \frac{1}{2} \right]^{8 \cdot 2^n - 5} \times \left[ \frac{2}{5} \right]^{6 \cdot 2^n - 6} \quad (33)$$

**Proof:**

Let  $G$  be a  $NS_2[n]$  Nanostar dendrimer. The edge set  $E(NS_2[n])$  divided into three edge partitions based on degrees of end vertices. The first edge partition  $E_1(NS_2[n])$  contains  $2^{n+1}$  edges  $uv$ , where  $deg(u) = 1, deg(v) = 2$ . The second edge partition  $E_2(NS_2[n])$  contains  $8 \cdot 2^n - 5$  edges  $uv$ , where  $deg(u) = deg(v) = 2$ . The third edge partition  $E_3(NS_2[n])$  contains  $6 \cdot 2^n - 6$  edges  $uv$ , where  $deg(u) = 2, deg(v) = 3$ . It is easy to see that  $|E_1(NS_2[n])| = d_{12}$ ,  $|E_2(NS_2[n])| = d_{22}$ ,  $|E_3(NS_2[n])| = d_{23}$ . Now using Eqs.(4)-(6), we have

$$\begin{aligned}
 (i) X\pi(NS_2[n]) &= \prod_{uv \in E(NS_2[n])} \frac{1}{\sqrt{d_u+d_v}} \\
 &= \prod_{uv \in E_1(NS_2[n])} \frac{1}{\sqrt{d_u+d_v}} \times \prod_{uv \in E_2(NS_2[n])} \frac{1}{\sqrt{d_u+d_v}} \times \\
 &\quad \prod_{uv \in E_3(NS_2[n])} \frac{1}{\sqrt{d_u+d_v}} \\
 &= |E_1(NS_2[n])|^{2^{n+1}} \times |E_2(NS_2[n])|^{8 \cdot 2^{n-5}} \times \\
 &\quad |E_3(NS_2[n])|^{6 \cdot 2^{n-6}} \\
 &= \left[\frac{1}{\sqrt{1+2}}\right]^{2^{n+1}} \times \left[\frac{1}{\sqrt{2+2}}\right]^{8 \cdot 2^{n-5}} \times \left[\frac{1}{\sqrt{2+3}}\right]^{6 \cdot 2^{n-6}} \\
 &= \left[\frac{1}{\sqrt{3}}\right]^{2^{n+1}} \times \left[\frac{1}{2}\right]^{8 \cdot 2^{n-5}} \times \left[\frac{1}{\sqrt{5}}\right]^{6 \cdot 2^{n-6}}
 \end{aligned}$$

$$\begin{aligned}
 (ii) R\pi(NS_1[n]) &= \prod_{uv \in E(NS_2[n])} \frac{1}{\sqrt{d_u d_v}} \\
 &= \prod_{uv \in E_1(NS_2[n])} \frac{1}{\sqrt{d_u+d_v}} \times \prod_{uv \in E_2(NS_2[n])} \frac{1}{\sqrt{d_u+d_v}} \times \\
 &\quad \prod_{uv \in E_3(NS_2[n])} \frac{1}{\sqrt{d_u+d_v}} \\
 &= |E_1(NS_2[n])|^{2^{n+1}} \times |E_2(NS_2[n])|^{8 \cdot 2^{n-5}} \times \\
 &\quad |E_3(NS_2[n])|^{6 \cdot 2^{n-6}} \\
 &= \left[\frac{1}{\sqrt{1 \cdot 2}}\right]^{2^{n+1}} \times \left[\frac{1}{\sqrt{2 \cdot 2}}\right]^{8 \cdot 2^{n-5}} \times \left[\frac{1}{\sqrt{2 \cdot 3}}\right]^{6 \cdot 2^{n-6}} \\
 &= \left[\frac{1}{\sqrt{2}}\right]^{2^{n+1}} \times \left[\frac{1}{2}\right]^{8 \cdot 2^{n-5}} \times \left[\frac{1}{\sqrt{6}}\right]^{6 \cdot 2^{n-6}}
 \end{aligned}$$

$$\begin{aligned}
 (iii) H\pi(NS_2[n]) &= \prod_{uv \in E(NS_2[n])} \frac{2}{d_u+d_v} \\
 &= \prod_{uv \in E_1(NS_2[n])} \frac{2}{d_u+d_v} \times \prod_{uv \in E_2(NS_2[n])} \frac{2}{d_u+d_v} \times \\
 &\quad \prod_{uv \in E_3(NS_2[n])} \frac{2}{d_u+d_v} \\
 &= |E_1(NS_2[n])|^{2^{n+1}} \times |E_2(NS_2[n])|^{8 \cdot 2^{n-5}} \times \\
 &\quad |E_3(NS_2[n])|^{6 \cdot 2^{n-6}} \\
 &= \left[\frac{2}{1+2}\right]^{2^{n+1}} \times \left[\frac{2}{2+2}\right]^{8 \cdot 2^{n-5}} \times \left[\frac{2}{2+3}\right]^{6 \cdot 2^{n-6}} \\
 &= \left[\frac{2}{3}\right]^{2^{n+1}} \times \left[\frac{1}{2}\right]^{8 \cdot 2^{n-5}} \times \left[\frac{2}{5}\right]^{6 \cdot 2^{n-6}}
 \end{aligned}$$

Next we compute the multiplicative Sum connectivity ( $X\pi(G)$ ) index, multiplicative Randić ( $R\pi(G)$ ) index and multiplicative Harmonic index ( $H\pi(G)$ ) for nanostar dendrimer  $D_n$  in the following theorem.

**Theorem 4.3**

Let  $D_n$  be a nanostar dendrimer. Then

$$(i) X\pi(D_n) = \left[\frac{1}{2}\right]^{24 \times 2^{n-1}} \times \left[\frac{1}{\sqrt{5}}\right]^{30 \times 2^{n-1} - 24} \times \left[\frac{1}{\sqrt{6}}\right]^{12 \times 2^{n-1} - 9} \tag{34}$$

$$(ii) R\pi(D_n) = \left[\frac{1}{2}\right]^{24 \times 2^{n-1}} \times \left[\frac{1}{\sqrt{6}}\right]^{30 \times 2^{n-1} - 24} \times \left[\frac{1}{3}\right]^{12 \times 2^{n-1} - 9} \quad (35)$$

$$(iii) H\pi(D_n) = \left[\frac{2}{\sqrt{5}}\right]^{30 \times 2^{n-1} - 24} \times \left[\frac{2}{\sqrt{3}}\right]^{12 \times 2^{n-1} - 9} \quad (36)$$

**Proof.**

Let  $G$  be a  $D_n$  Nanostar dendrimer. One can see that the Nanostar dendrimer graph is a partial cube. It is easy to see that this graph  $G = D_n$  has  $57 \times 2^{n-1} - 38$  vertices and  $33 \times 2^n - 45$  edges. The edge set  $E(D_n)$  divides into three edge partitions based on degrees of end vertices. The first edge partition  $E_1(D_n)$  contains  $24 \times 2^{n-1} - 12$  edges  $uv$ , where  $\deg(u) = \deg(v) = 2$ . The second edge partition  $E_2(D_n)$  contains  $30 \times 2^{n-1} - 24$  edges  $uv$ , where  $\deg(u) = 2, \deg(v) = 3$ . The third edge partition  $E_3(D_n)$  contains  $12 \times 2^{n-1} - 9$  edges  $uv$ , where  $\deg(u) = \deg(v) = 3$ . Now using Eqs.(4)-(6), we have

$$\begin{aligned} (i) \chi\pi(D_n) &= \prod_{uv \in E(D_n)} \frac{1}{\sqrt{d_u + d_v}} \\ &= \prod_{uv \in E_1(D_n)} \frac{1}{\sqrt{d_u + d_v}} \times \prod_{uv \in E_2(D_n)} \frac{1}{\sqrt{d_u + d_v}} \times \\ &\quad \prod_{uv \in E_3(D_n)} \frac{1}{\sqrt{d_u + d_v}} \\ &= |E_1(D_n)|^{24 \times 2^{n-1} - 12} \times |E_2(D_n)|^{30 \times 2^{n-1} - 24} \times \\ &\quad |E_3(D_n)|^{12 \times 2^{n-1} - 9} \\ &= \left[\frac{1}{\sqrt{2+2}}\right]^{24 \times 2^{n-1}} \times \left[\frac{1}{\sqrt{2+3}}\right]^{30 \times 2^{n-1} - 24} \times \\ &\quad \left[\frac{1}{\sqrt{3+3}}\right]^{12 \times 2^{n-1} - 9} \\ &= \left[\frac{1}{2}\right]^{24 \times 2^{n-1}} \times \left[\frac{1}{\sqrt{5}}\right]^{30 \times 2^{n-1} - 24} \times \\ &\quad \left[\frac{1}{\sqrt{6}}\right]^{12 \times 2^{n-1} - 9} \\ (ii) R\pi(D_n) &= \prod_{uv \in E(D_n)} \frac{1}{\sqrt{d_u d_v}} \\ &= \prod_{uv \in E_1(D_n)} \frac{1}{\sqrt{d_u d_v}} \times \prod_{uv \in E_2(D_n)} \frac{1}{\sqrt{d_u d_v}} \times \\ &\quad \prod_{uv \in E_3(D_n)} \frac{1}{\sqrt{d_u d_v}} \\ &= |E_1(D_n)|^{24 \times 2^{n-1} - 12} \times |E_2(D_n)|^{30 \times 2^{n-1} - 24} \times \\ &\quad |E_3(D_n)|^{12 \times 2^{n-1} - 9} \\ &= \left[\frac{1}{\sqrt{2.2}}\right]^{24 \times 2^{n-1}} \times \left[\frac{1}{\sqrt{2.3}}\right]^{30 \times 2^{n-1} - 24} \times \\ &\quad \left[\frac{1}{\sqrt{3.3}}\right]^{12 \times 2^{n-1} - 9} \\ &= \left[\frac{1}{2}\right]^{24 \times 2^{n-1}} \times \left[\frac{1}{\sqrt{6}}\right]^{30 \times 2^{n-1} - 24} \times \\ &\quad \left[\frac{1}{3}\right]^{12 \times 2^{n-1} - 9} \\ (iii) H\pi(D_n) &= \prod_{uv \in E(D_n)} \frac{2}{d_u + d_v} \end{aligned}$$

$$\begin{aligned}
&= \prod_{uv \in E_1(D_n)} \frac{2}{d_u + d_v} \times \prod_{uv \in E_2(D_n)} \frac{2}{d_u + d_v} \times \prod_{uv \in E_3(D_n)} \frac{2}{d_u + d_v} \\
&= |E_1(D_n)|^{24 \times 2^{n-1} - 12} \times |E_2(D_n)|^{30 \times 2^{n-1} - 24} \times |E_3(D_n)|^{12 \times 2^{n-1} - 9} \\
&= \left[ \frac{2}{\sqrt{2+2}} \right]^{24 \times 2^{n-1}} \times \left[ \frac{2}{\sqrt{2+3}} \right]^{30 \times 2^{n-1} - 24} \times \left[ \frac{2}{\sqrt{3+3}} \right]^{12 \times 2^{n-1} - 9} \\
&= \left[ \frac{2}{\sqrt{5}} \right]^{30 \times 2^{n-1} - 24} \times \left[ \frac{2}{\sqrt{3}} \right]^{12 \times 2^{n-1} - 9}
\end{aligned}$$

## 5. Conclusion

In this paper we worked on a chemical structure Nanostar dendrimers and studied their Multiplicative topological indices. We determined the multiplicative Sum connectivity ( $X\pi(G)$ ) index, multiplicative Randić ( $R\pi(G)$ ) index and multiplicative Harmonic index ( $H\pi(G)$ ) for Nanostar dendrimers. In future, we are interested to study their higher orders of Multiplicative topological indices

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