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Relatively Prime Graph RP_n

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Abstract: *In this paper we introduce Relatively prime graph of order n , Further we investigate basic nature of this graph and we give an estimation of connectivity number of this graph.*

Keywords: *Relatively prime graph (RP_n - graph), coprime graph, prime graph and prime sub graph.*

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1. Introduction

Recently, investigation of various graphs on the integers has received significant attention. The most popular graphs being the prime (co prime) graphs. The definition of the prime (coprime) graphs is as follows: A graph of order n is coprime if one can objectively label its nodes with integers 1,2,…,n so that any two adjacent nodes get coprime labels[1]. SN Rao[2] made a review on Prime graphs. In this article, we have defined relatively prime graph of n (RP_n). The definition is as follows:

Let $n > 2$ be an integer, $\Omega_n = \{x \in Z : (x,n) = 1,\ 1 < x < n\}$. The Relatively prime graph of n (RP_n - graph) is the graph $G = (V, E)$, where $V(G) = \Omega_n$ and $E(G) = \{(x, y) : (x, y) = 1,\ x, y = \Omega_n\}$. Obviously, relatively prime graph of n is a subgraph of Prime graphs on n vertices. In this paper, We have considered non - hereditary property only. Notice that $|\Omega_n|$ is odd. A prime element $r \in \Omega_n$ is said to be maximal prime element of n if $(x, r) = 1$ for all $x \in \Omega_n$. For a given n, there exists more than one maximal prime element of n (for example, the maximal prime elements of 13 are 11 and 7), the number of maximal prime elements of given n is denoted by $\mu(n)$. For a given n, the number of prime numbers in Ω_n is denoted by $\rho(n)$.

A subset U of the vertex set $V(G)$ of G is said to be an independent set of G if the induced subgraph of $G(U)$ is a trivial graph. An independent set of G with maximum number of vertices is called a maximum independent set of G. The number of vertices of a maximum independent set of G is called the independence number of G. A vertex of a graph G is said to cover the edges incident with it, and a vertex cover of a graph G is a set of vertices cover all the edges of G. The minimum cardinality of a vertex

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cover of a graph G is called the vertex covering number of G [3]. Through this article RP_p denotes relatively prime graph on a prime number p.

2. Properties of RP_n graph

Theorem 1: RP_n graph is connected for every n.

Proof : If Ω_n contains exactly one element, then the proof is over. Let $|\Omega_n| \geq 2$, By Bertrand's Postulate there exists a prime number in $\left(\frac{n}{2}, n\right)$. Clearly this prime number belongs to Ω_n and it will act as maximal prime element of Ω_n. Since every vertex is connected with maximal prime element of n, RP_n is connected.

Remark 1: Since Ω_n contains atleast a maximal prime element of n, the domination number $\gamma(RP_n) = 1, \forall\ n.$

Theorem 2: RP_n graph is complete if and only if all the members of Ω_n are prime.

Proof: The second part is trivial. Let RP_n be complete. If a element $x \in \Omega_n$ is composite, then all the prime factors of x are in Ω_n. Which gives us that there exist a vertex which is not adjacent to x. Which conflicts the fact that if RP_n is complete, then all the members of Ω_n are prime.

Corollary 1: RP_n graph is not regular graph if and only if Ω_n consists atleast one composite number.

Proof: The fact that if RP_n graph is regular and Ω_n contains a maximal prime element of n gives RP_n must be complete. By theorem 2, We have all the members of Ω_n are prime.

Theorem 3: RP_n graph has a cycle if and only if Ω_n contains atleast three prime numbers.

Proof: If Ω_n contains atleast three prime elements namely p_1, p_2, p_3, then there exists a cycle $p_1 p_2 p_3 p_1$.

Conversely if Ω_n has atmost two prime numbers.

Subcase 1: Ω_n contains only one prime element p (say).

If there exists a cycle $U_1 U_2 U_3 ... U_n U_1$, then p divides every U_i, which is not possible.

Subcase 2: Assume Ω_n has only two prime elements p_1, p_2 (say).

Since Ω_n contains a maximal prime element, either p_1 or p_2 is a maximal prime element of n say p_2. Let there exists a cycle $U_1 U_2 U_3 ... U_n U_1$.

If p_1 divides all U_i, then there is no edge between any two pair of U_i and U_j, Which is not possible.

Now assume that p_2 divides U_i for some i. Then $U_i = p_2$. In this case also we get a contradiction. Hence the proof.

Corollary 2: If Ω_n contains atleast three prime elements, then RP_n is not bipartite.

Proof: If Ω_n contains atleast three prime elements, then we can form a cycle $p_1 p_2 p_3 p_1$. Therefore there exists an odd cycle.

Theorem 4: For RP_n, $\kappa(n) < |\Omega_n| - \rho(n)$, where $\kappa(n)$ is the connectivity number of RP_n.

Proof: Since for a given n, the degree of prime vertex of Ω_n is more than the degree of composite vertex of Ω_n, it is worthful that elimination of prime vertex only while calculating the connectivity number of RP_n. In this proof G^* denotes the resulting graph after elimination of some prime vertex from RP_n. In particular, if $\kappa(n) = |\Omega_n| - \rho(n)$, assume that G^* is connected. Then there exists a $u-v$ path if $u, v \in V(G^*)$. Now consider the graph G with vertex set $V(G) = V(G^*) \cup \{p\} (a \text{ max}. \textit{ prime element of } n)$ .Clearly there exists a cycle $pu...vp$. But $V(G)$ Contains only one prime element. Which is a contradiction to Ω_n contains atleast three prime numbers. Therefore G^* is disconnected. This completes our proof.

In RP_p graph, where p is a prime, $2 \in V(G)$ and this vertex adjacent to all odd members of vertex set. If $p+1$ is not divisible by 4, then degree of 2 is odd. It follows that:

Theorem 5: If p is a prime and $p \geq 3$, then RP_p is not Eulerian.

Remark 2: Since there exist a path $2, 3, 4, \ldots, p-2, p-1$ in RP_p, RP_p has Hamiltonian path.

Theorem 6: For RP_p graph , where p is a prime

$$d(x)=(p-2)-\left\{\left[\frac{p-1}{p_1}\right]+\left[\frac{p-1}{p_2}\right]+\ldots+\left[\frac{p-1}{p_r}\right]\right\}+\left[\frac{p-1}{p_1p_2}\right]+\left[\frac{p-1}{p_1p_3}\right]+\ldots+\left[\frac{p-1}{p_1p_r}\right]$$
$$+\left[\frac{p-1}{p_1p_2p_3}\right]+\left[\frac{p-1}{p_1p_2p_4}\right]+\ldots+\left[\frac{p-1}{p_1p_2p_r}\right]+\ldots+\left[\frac{p-1}{p_1p_2\cdots p_r}\right]$$

where $x\in\Omega_n$ and $x=p_1^{\alpha_1}p_2^{\alpha_2}\ldots p_r^{\alpha_r}$, $[x]$ is the greatest integer less than or equal to x.

Proof: For every $x\in\Omega_n$, there exist y such that $(x,y)=1$. Since $|\Omega_n|=p-2$, $\deg(x)<p-2$. Suppose $x=p_1^{\alpha_1}p_2^{\alpha_2}\ldots p_r^{\alpha_r}$. Then y cannot be a multiple of p_1 , $p_2,\ldots,p_r$. Enumerate the integers which are relatively prime to x we have

$$d(x)=(p-2)-\left\{\left[\frac{p-1}{p_1}\right]+\left[\frac{p-1}{p_2}\right]+\ldots+\left[\frac{p-1}{p_r}\right]\right\}+\left[\frac{p-1}{p_1p_2}\right]+\left[\frac{p-1}{p_1p_3}\right]+\ldots+\left[\frac{p-1}{p_1p_r}\right]$$
$$+\left[\frac{p-1}{p_1p_2p_3}\right]+\left[\frac{p-1}{p_1p_2p_4}\right]+\ldots+\left[\frac{p-1}{p_1p_2p_r}\right]+\ldots+\left[\frac{p-1}{p_1p_2\cdots p_r}\right]$$

Theorem 7: The independence number of RP_p graph is $\frac{p-1}{2}$.

Proof: Since every even vertex is adjacent to only odd vertices , the set of even vertices is the maximum independent set and its cardinality is $\frac{p-1}{2}$.

Theorem 8: The covering number of RP_p graph is $\frac{p-3}{2}$.

Proof: It follows from sum of independence number with covering number is equal to the cardinality of the vertex set [4].

3. RP_n^* graph

Let $n\geq 2$ be an integer, $\Omega_n=\{x\in Z:(x,n)=1,\ 1\leq x<n\}$. The RP_n^* graph of n is the graph $G=(V,E)$ where $V(G)=\Omega_n$ and $E(G)=\{(x,y):(x,y)=1\ and\ x+y=1,\ x,y\in\Omega_n\}$. Observe that since $1\in\Omega_n$ and 1 is connected with all the vertices of RP_n^*, RP_n^* is connected for every n.

Theorem 9: RP_n^* is not regular if and only if it contains more than two vertex.

Proof: The proof is trivial.

Theorem 10: RP_n^* is not complete if and only if and only if $|\Omega_n|$ has atleast three members.

Proof: It follows from the fact that $n-1\in\Omega_n$ and degree of this vertex is one.

Theorem 11: RP_n^* is not bipartite if and only if contains more than two vertex.

Proof: Let us assume the vertices set contains more than two elements. If $k\in\Omega_n$, then $n-k\in\Omega_n$. Now we have an odd cycle $1,k,n-k,1$. This completes the proof.

Theorem 12: For RP_p^* graph, where p is a prime, $d(1)=p-2$ and

$$d(x)=(p-x)-\left\{\left[\left[\frac{p-x}{p_1}\right]+\left[\frac{p-x}{p_2}\right]+\ldots+\left[\frac{p-x}{p_r}\right]\right]\right\}+\left[\frac{p-x}{p_1p_2}\right]+\left[\frac{p-x}{p_1p_3}\right]+\ldots+\left[\frac{p-x}{p_1p_r}\right]$$
$$+\left[\frac{p-x}{p_1p_2p_3}\right]+\left[\frac{p-x}{p_1p_2p_4}\right]+\ldots+\left[\frac{p-x}{p_1p_2p_r}\right]+\ldots+\left[\frac{p-x}{p_1p_2\cdots p_r}\right]$$

where $x\in\Omega_p$ and $x=p_1^{\alpha_1}p_2^{\alpha_2}\ldots p_r^{\alpha_r}$, $[x]$ is the greatest integer less than or equal to x and $x\neq 1$.

Proof: Since $(x,1)=1$, where $x\in\Omega_p-\{1\}$ and $1+x\le p$, $d(1)=p-2$. For every $x(\neq 1)\in\Omega_p$, there exist y such that $(x,y)=1$ and $x+y\le p$. Let $x=p_1^{\alpha_1}p_2^{\alpha_2}\ldots p_r^{\alpha_r}$. Then y cannot be a multiple of $p_1, p_2,\ldots, p_r$. Enumerate the integers which are relatively prime to x we have

$$d(x)=(p-x)-\left\{\left[\left[\frac{p-x}{p_1}\right]+\left[\frac{p-x}{p_2}\right]+\ldots+\left[\frac{p-x}{p_r}\right]\right]\right\}+\left[\frac{p-x}{p_1p_2}\right]+\left[\frac{p-x}{p_1p_3}\right]+\ldots+\left[\frac{p-x}{p_1p_r}\right]$$
$$+\left[\frac{p-x}{p_1p_2p_3}\right]+\left[\frac{p-x}{p_1p_2p_4}\right]+\ldots+\left[\frac{p-x}{p_1p_2p_r}\right]+\ldots+\left[\frac{p-x}{p_1p_2\cdots p_r}\right]$$

4. Conclusion

Further, it is worth to investigating the relation between RP_n and RP_n^*, the number of edges in RP_n and RP_n^*, the connectivity number and cardinality of cur set of RP_n.

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